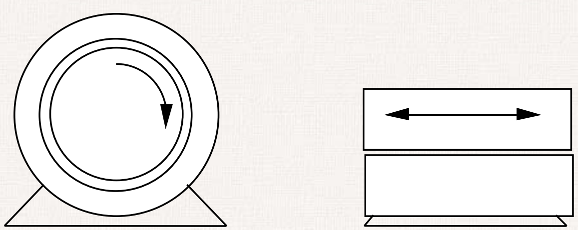
Torque generation with Electrical Machines

Industrial Electrical Engineering and Automation
Lund University, Sweden

Torque generating phenomena

1. Conductor in magnetic field
2. Iron shape in magnetic field
3. Electrostatic
4. Piezostriction
   Magnetostriction

Linear / rotating

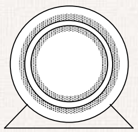


Energy density

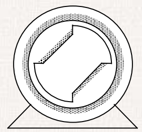
Magnetic energy density : $\frac{B^2}{2\mu_0} \; J/m^3$

Electric energy density : $\frac{E^2}{2\varepsilon_0} \; J/m^3$

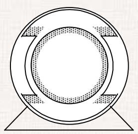
Geometry



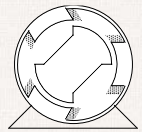
IM



SM



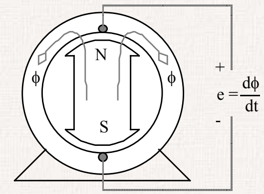
DCM

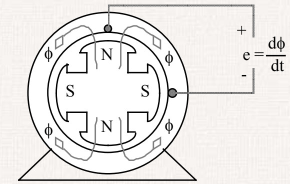


RM

Industrial Electrical Engineering and Automation

of poles





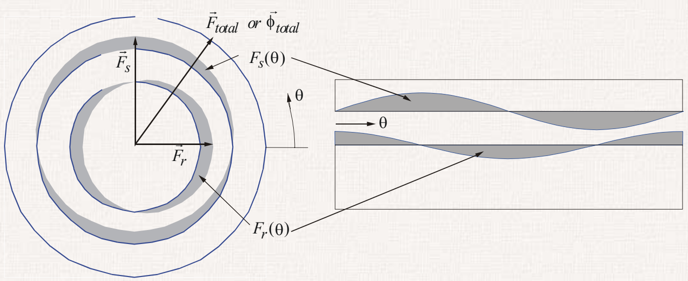
$$\omega_{el} = \frac{p}{2}\cdot \omega_{mec}$$

$$T_{mech} = \frac{p}{2}\cdot T_{el}$$

$$p(t) = \omega_{el}\cdot T_{el} = \frac{p}{2}\cdot \omega_{mec}\cdot \frac{2}{p}\cdot T_{mec} = \omega_{mec}\cdot T_{mec}$$

Industrial Electrical Engineering and Automation

Sinusoidal mmf & flux



1. Superposition
2. Vectors

Industrial Electrical Engineering and Automation

MMF, Flux & Reluctance

$$\phi_s = \int_0^{\pi} B_s(\theta)\cdot l\cdot r\cdot d\theta \quad \text{where} \quad \begin{array}{l} l = \text{stator length} \\ r = \text{stator radius} \end{array}$$

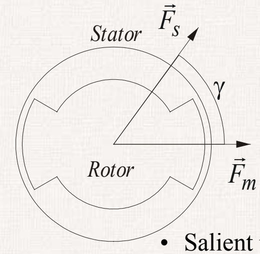
$$R = \frac{F_{total,average}}{\phi_{total}} = \frac{\frac{2}{\pi}\cdot \hat{F}_{total}}{\phi_{total}}$$

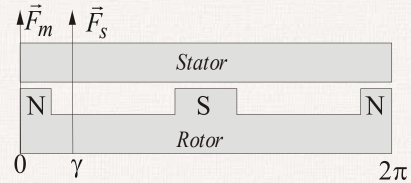
Summary so far ...

- Magnetic fields for energy transfer
- Salient poles on one side (stator or rotor)
- 2 poles, but can be scaled later
- Sinusoidal mmf-distribution
- Flux=integral of flux density
- Reluctance = MMF/Flux

- Now, we create a machine with these properties ...

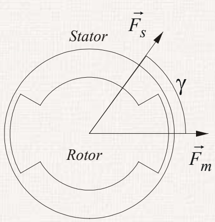
The PM / PM –machine





- Salient poles in the rotor
- Rotor reference frame (*x/y*)
- Rotor mmf $\vec{F}_m$ and stator mmf $\vec{F}_s$ (sinusiodal)
- Reluctances R_x and R_y
- Ideal iron (no magnetic saturation effects)

Air gap magnetic energy



$$\vec{F}_\delta = \vec{F}_m + \vec{F}_s = F_{\delta x} + F_{\delta y}$$

$$F_{\delta x} = \hat{F}_s \cdot \cos(\gamma) + \hat{F}_m = \hat{F}_{sx} + \hat{F}_m$$

$$F_{\delta y} = \hat{F}_s \cdot \sin(\gamma) = \hat{F}_{sy}$$

$$W_{magn} = \frac{1}{2}\frac{\hat{F}_x^2}{R_x} + \frac{1}{2}\frac{\hat{F}_y^2}{R_y} = \frac{1}{2}\cdot\left(\frac{\hat{F}_s^2 \cdot \cos^2\gamma + 2\cdot\hat{F}_s\cdot\hat{F}_m\cdot\cos\gamma + \hat{F}_m^2}{R_x}\right) + \frac{1}{2}\cdot\left(\frac{\hat{F}_s^2\cdot\sin^2\gamma}{R_y}\right)$$

Torque : Derivative of magnetic energy

$$\frac{dW_{mec}}{d\gamma} = T$$

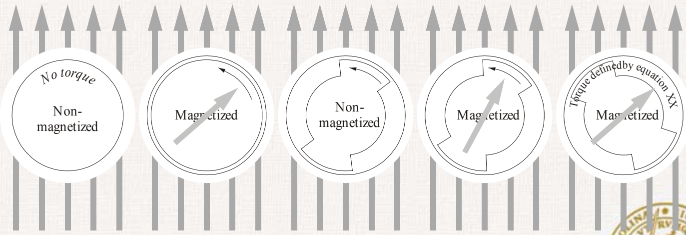
Compare to linear movement
(*F=dW/dx* or *W=F*x*)

$W_{magn} + W_{mec}$ = Constant, i.e. no energy supplied

$$T = -\frac{dW_{magn}}{d\gamma} = \ldots = \frac{\hat{F}_{sy}\cdot\hat{F}_m}{R_x} + \hat{F}_{sx}\cdot\hat{F}_{sy}\cdot\left(\frac{1}{R_x} - \frac{1}{R_y}\right)$$

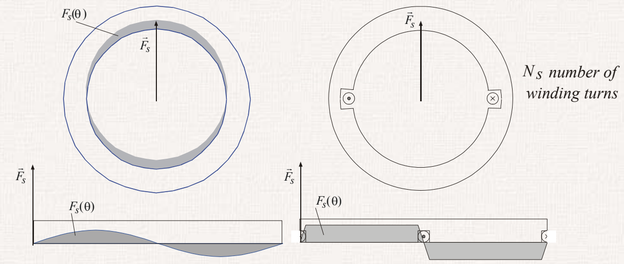
Torque : Components

$$T = -\frac{dW_{magn}}{d\gamma} = \ldots = \frac{\hat{F}_{sy} \cdot \hat{F}_m}{R_x} + \hat{F}_{sx} \cdot \hat{F}_{sy} \cdot \left(\frac{1}{R_x} - \frac{1}{R_y}\right)$$



External magnetic field generated by Fs

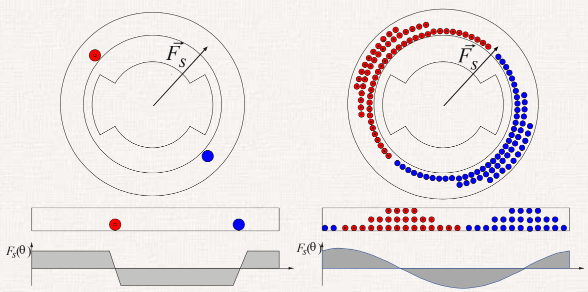
Electrically magnetized stator



$$\hat{F}_{s1} = \frac{4}{\pi} \cdot \frac{N_s \cdot i_s}{2} = \frac{2}{\pi} \cdot N_s \cdot i_s \qquad \text{fundamental}$$

$$\hat{F}_{sn} = \frac{4}{\pi} \cdot \frac{N_s \cdot i_s}{2 \cdot n} = \frac{2}{\pi} \cdot \frac{N_s \cdot i_s}{n} \qquad n = 3,5,7,\ldots \qquad \text{harmonics}$$

Sinusoidally distributed winding



$$\hat{F}_{s1} = \frac{2}{\pi} \cdot N_s \cdot i_s \cdot k_{r1} = \left(\frac{2}{\pi} \cdot N_s \cdot k_{r1}\right) \cdot i_s$$

$$\hat{F}_{sn} = \frac{2}{\pi} \cdot \frac{N_s \cdot i_s}{n} \cdot k_{r1} = \left(\frac{2}{\pi} \cdot N_s \cdot k_{rn}\right) \cdot \frac{1}{n} \cdot i_s \qquad n = 3,5,7,\ldots$$

Torque : expressed in flux and mmf

$$T = \frac{1}{R_x} \cdot \left(\hat{F}_m + \hat{F}_{sx}\right) \cdot \hat{F}_{sy} - \frac{1}{R_y} \cdot \hat{F}_{sy} \cdot \hat{F}_{sx} =$$

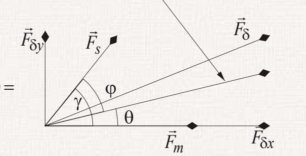
$$= \frac{\pi}{2} \cdot \left(\phi_{\delta x} \cdot \hat{F}_{sy} - \phi_{\delta y} \cdot \hat{F}_{sx}\right) =$$

$$= \frac{\pi}{2} \cdot \phi_\delta \cdot \hat{F}_s \cdot (\cos\theta \sin\gamma - \sin\theta \cos\gamma) =$$

$$= \frac{\pi}{2} \cdot \phi_\delta \cdot \hat{F}_s \cdot \sin(\gamma - \theta) = \frac{\pi}{2} \cdot \phi_\delta \cdot \hat{F}_s \cdot \sin(\varphi) =$$

$$= \frac{\pi}{2} \cdot \vec{\phi}_\delta \times \vec{F}_s$$

$$\vec{\phi}_\delta = \frac{\frac{2}{\pi} \cdot \hat{F}_{\delta x}}{R_x} + j \cdot \frac{\frac{2}{\pi} \cdot \hat{F}_{\delta y}}{R_y}$$



Remember: Flux=MMF/Reluctance

Important conclusion

Torque : expressed in flux linkage and mmf

$N_{s,eff} = N_s \cdot k_{r1}$ Effective number of turns, (to create the fundamental mmf wave)

$$\vec{F}_s = \hat{F}_{sx} + j\hat{F}_{sy} = \frac{2}{\pi} \cdot N_{s,eff} \cdot \vec{i}_s = \frac{2}{\pi} \cdot N_{s,eff} \cdot (i_{sx} + ji_{sy})$$

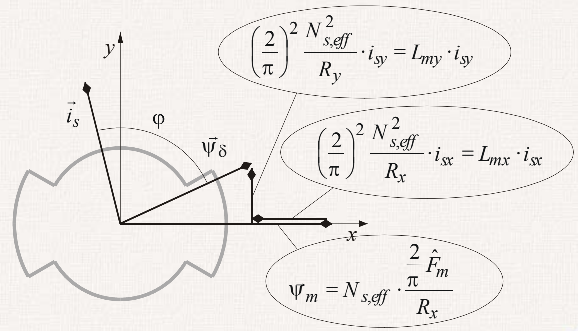
$$T = \frac{\pi}{2} \cdot \left(\phi_{\delta x} \cdot \hat{F}_{sy} - \phi_{\delta y} \cdot \hat{F}_{sx}\right) = \frac{\pi}{2} \cdot \left(\phi_{\delta x} \cdot \frac{2}{\pi} \cdot N_{s,eff} \cdot i_{sy} - \phi_{\delta y} \cdot \frac{2}{\pi} \cdot N_{s,eff} \cdot i_{sx}\right) =$$

$$= \phi_{\delta x} \cdot N_{s,eff} \cdot i_{sy} - \phi_{\delta y} \cdot N_{s,eff} \cdot i_{sx} = \psi_{\delta x} \cdot i_{sy} - \psi_{\delta y} \cdot i_{sx}$$

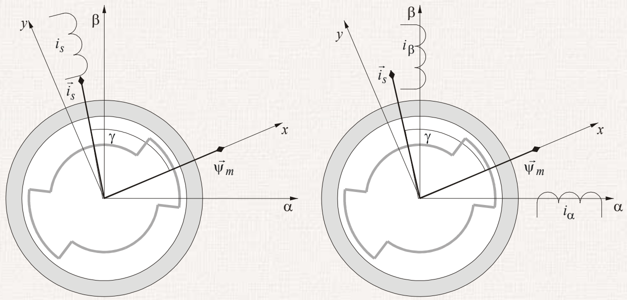
$$= \vec{\psi}_\delta \times \vec{i}_s$$

Important conclusion

Flux vectors, and inductances ...



2 Phases

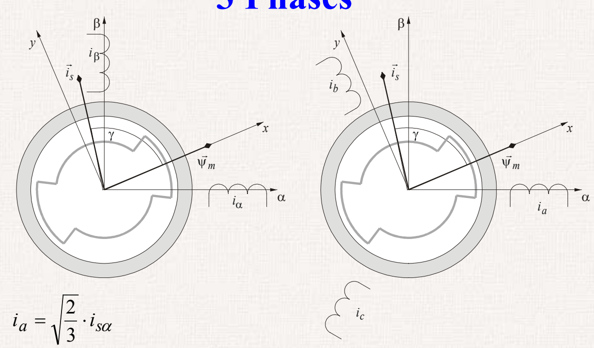


$$\vec{i}_s^{\,xy} = i_{sx} + ji_{sy} = i_s \cdot e^{j\gamma} \qquad \vec{i}_s^{\,\alpha\beta} = \vec{i}_s^{\,xy} \cdot e^{j\theta_r} = i_s \cdot e^{j(\theta_r + \gamma)} = i_{s\alpha} + ji_{s\beta}$$

Stationary operation:

$$i_{s\alpha} + ji_{s\beta} = i_s \cdot \cos(\omega_r t + \gamma) + ji_s \cdot \sin(\omega_r t + \gamma)$$

3 Phases

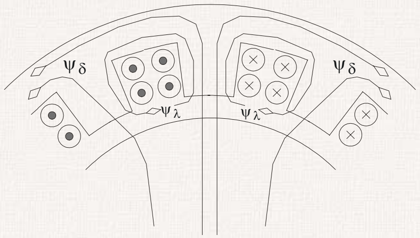


$$i_a = \sqrt{\frac{2}{3}} \cdot i_{s\alpha}$$

$$i_b = \sqrt{\frac{2}{3}} \cdot \left(-\frac{1}{2} i_{s\alpha} + \frac{\sqrt{3}}{2} i_{s\beta}\right)$$

$$i_c = \sqrt{\frac{2}{3}} \cdot \left(-\frac{1}{2} i_{s\alpha} - \frac{\sqrt{3}}{2} i_{s\beta}\right)$$

Leakage inductances



$$\psi_{a\lambda} = N_{s,eff} \cdot \phi_{a\lambda} = L_{s\lambda} \cdot i_a$$
$$\psi_{b\lambda} = N_{s,eff} \cdot \phi_{b\lambda} = L_{s\lambda} \cdot i_b$$
$$\psi_{c\lambda} = N_{s,eff} \cdot \phi_{c\lambda} = L_{s\lambda} \cdot i_c$$

Stator voltage equations

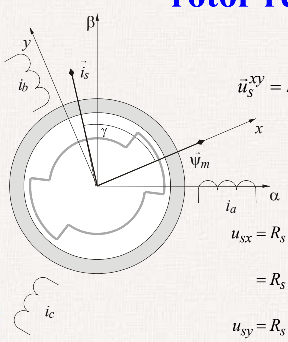
$$u_a = R_s \cdot i_a + \frac{d\psi_a}{dt} = R_s \cdot i_a + \frac{d}{dt}\left(\psi_{\delta a} + L_{s\lambda} \cdot i_a\right)$$
$$u_b = R_s \cdot i_b + \frac{d\psi_b}{dt} = R_s \cdot i_b + \frac{d}{dt}\left(\psi_{\delta b} + L_{s\lambda} \cdot i_b\right)$$

”+”

$$u_c = R_s \cdot i_c + \frac{d\psi_c}{dt} = R_s \cdot i_c + \frac{d}{dt}\left(\psi_{\delta c} + L_{s\lambda} \cdot i_c\right)$$

$$\vec{u}_s^{\alpha\beta} = R_s \cdot \vec{i}_s^{\alpha\beta} + \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = R_s \cdot \vec{i}_s^{\alpha\beta} + \frac{d}{dt}\left(\vec{\psi}_\delta^{\alpha\beta} + L_{s\lambda} \cdot \vec{i}_s^{\alpha\beta}\right)$$

Stator voltage in the rotor reference frame



$$\vec{u}_s^{xy} = R_s \cdot \vec{i}_s^{xy} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right) + j\omega_r \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)$$

$$u_{sx} = R_s \cdot i_{sx} + \frac{d}{dt}\left(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}\right) - \omega_r \cdot \left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) =$$
$$= R_s \cdot i_{sx} + \frac{d}{dt}\left(\psi_m + L_{sx} \cdot i_{sx}\right) - \omega_r \cdot L_{sy} \cdot i_{sy}$$
$$u_{sy} = R_s \cdot i_{sy} + \frac{d}{dt}\left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) + \omega_r \cdot \left(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}\right) =$$
$$= R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot \left(\psi_m + L_{sx} \cdot i_{sx}\right)$$

Control challenges

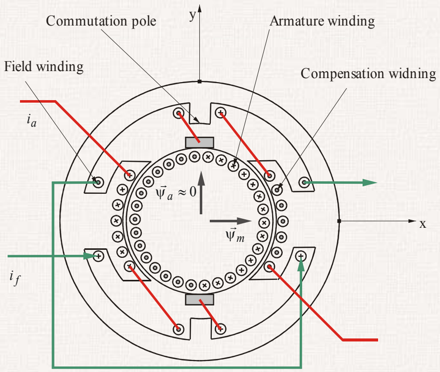
- **Priority:**
  - **Torque**
  - **Stator flux**
  - **Power factor**
  - **Field weakening**

DC Machine control

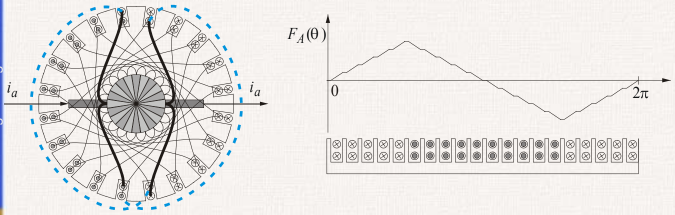
Why DC?

- Simple to control
- Cheap to produce – but only due to effective production and large series.

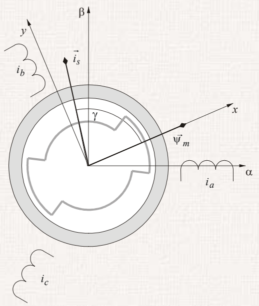
Mechanical design:I

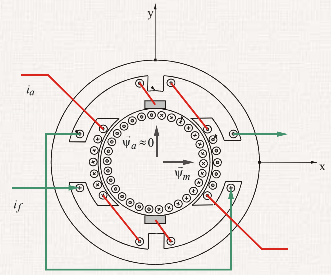


Mechanical design:II



Mathematical model:I



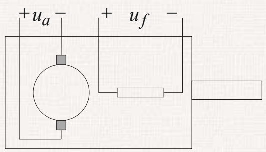


$$u_{sx} = R_s \cdot i_{sx} + \frac{d}{dt}(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}) - \omega_r \cdot (L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}) =$$
$$= R_s \cdot i_{sx} + \frac{d}{dt}(\psi_m + L_{sx} \cdot i_{sx}) - \omega_r \cdot L_{sy} \cdot i_{sy}$$
$$u_{sy} = R_s \cdot i_{sy} + \frac{d}{dt}(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}) + \omega_r \cdot (\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}) =$$
$$= R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot (\psi_m + L_{sx} \cdot i_{sx})$$

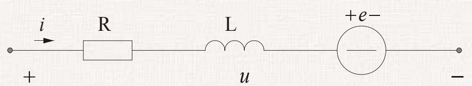
$$u_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + \omega_r \cdot \psi_m$$
$$T = \psi_m \cdot i_a$$

Mathematical model:II

$$u_f = R_f \cdot i_f + L_f \cdot \frac{di_f}{dt}$$
$$\psi_m = L_m \cdot i_f$$
$$L_f = L_m + L_{f\lambda}$$



Torque Control



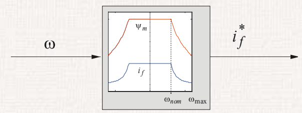
$$u^*(k) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot \left((i^*(k) - i(k)) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2}\right)} \cdot \sum_{n=0}^{n=k-1} (i^*(n) - i(n)) \right) + e(k)$$

$$u_a^*(k) = \left(\frac{L_a}{T_s} + \frac{R_a}{2}\right) \cdot \left((i_a^*(k) - i_a(k)) + \frac{T_s}{\left(\frac{L_a}{R_a} + \frac{T_s}{2}\right)} \cdot \sum_{n=0}^{n=k-1} (i_a^*(n) - i_a(n)) \right) + e_a(k)$$

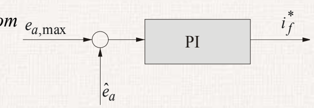
$$e_a(k) = \omega_r(k) \cdot \psi_m$$

Field weakening

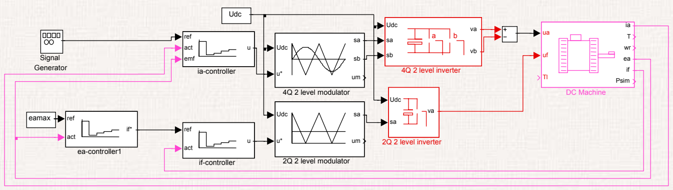
$$e_a = \omega_r \cdot \psi_m$$



$$\psi_m^* = \begin{cases} \psi_{m,nom} \text{ if } \omega_r < \omega_{r,nom} \\ \psi_{m,nom} \cdot \dfrac{\omega_{r,nom}}{\omega_r} \text{ if } \omega_r > \omega_{r,nom} \end{cases}$$



Example


Udc
Signal Generator
ref
act
emf
u
ia-controller
Udc
u*
sa
sb
um
4Q 2 level modulator
Udc
sa
sb
a
b
va
vb
4Q 2 level inverter
+
−
ua
uf
Tl
ia
T
wr
ea
if
Psim
DC Machine
eamax
ref
act
if*
ea-controller1
ref
act
u
if-controller
Udc
u*
sa
um
2Q 2 level modulator
Udc
sa
va
2Q 2 level inverter