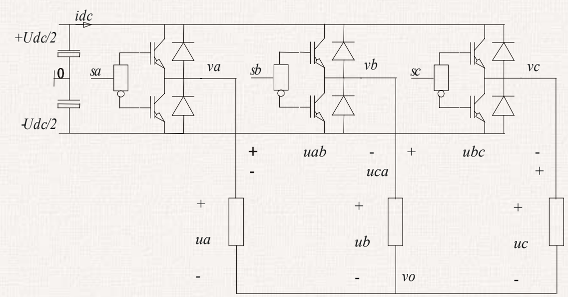
Modulation of 3-phase converters

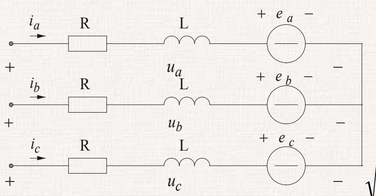
Industrial Electrical Engineering and Automation
Lund University, Sweden

3 – phase converters



- In AC motor drives
- In grid connected converters

The generic 3-phase load



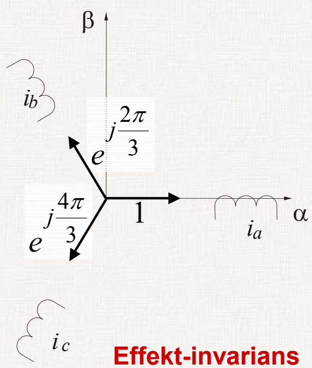
$$\sqrt{\frac{2}{3}}\left(u_a = R\cdot i_a + L\cdot\frac{di_a}{dt} + e_a\right)$$

$$\sqrt{\frac{2}{3}}\cdot e^{j\frac{2\pi}{3}}\cdot\left(u_b = R\cdot i_b + L\cdot\frac{di_a}{dt} + e_b\right)$$

$$+\sqrt{\frac{2}{3}}\cdot e^{j\frac{4\pi}{3}}\cdot\left(u_c = R\cdot i_c + L\cdot\frac{di_a}{dt} + e_c\right)$$

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{di}{dt} + \vec{e}$$

Vectors in 3-phase systems



$$\sqrt{\frac{2}{3}}\left(u_a = R\cdot i_a + L\cdot\frac{di_a}{dt} + e_a\right)$$

$$\sqrt{\frac{2}{3}}\cdot e^{j\frac{2\pi}{3}}\cdot\left(u_b = R\cdot i_b + L\cdot\frac{di_a}{dt} + e_b\right)$$

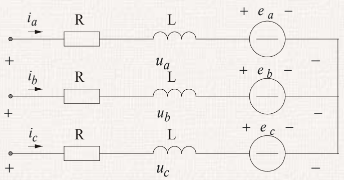
$$+\sqrt{\frac{2}{3}}\cdot e^{j\frac{4\pi}{3}}\cdot\left(u_c = R\cdot i_c + L\cdot\frac{di_a}{dt} + e_c\right)$$

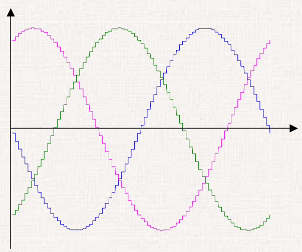
$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{di}{dt} + \vec{e}$$

Effekt-invarians

$$p(t) = u_a\cdot i_a + u_b\cdot i_b + u_c\cdot i_c = u_\alpha\cdot i_\alpha + u_\beta\cdot i_\beta$$

Symmetric emf





$$\begin{cases} e_a = \hat{e}\cdot\cos(\omega\cdot t) \\ e_b = \hat{e}\cdot\cos\left(\omega\cdot t-\frac{2\pi}{3}\right) \\ e_c = \hat{e}\cdot\cos\left(\omega\cdot t-\frac{4\pi}{3}\right) \end{cases}$$

Example, grid voltage vector

$$\vec{e} = \sqrt{\frac{2}{3}}\cdot\left(e_a + e_b\cdot e^{j\frac{2\pi}{3}} + e_c\cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\hat{e}\cdot\left(\cos(\omega\cdot t)+\cos\left(\omega\cdot t-\frac{2\pi}{3}\right)\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)+\cos\left(\omega\cdot t-\frac{4\pi}{3}\right)\cdot\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\hat{e}\cdot\left(\begin{array}{l}\cos(\omega\cdot t)+\left(\cos(\omega\cdot t)\cdot\cos\left(\frac{2\pi}{3}\right)+\sin(\omega\cdot t)\cdot\sin\left(\frac{2\pi}{3}\right)\right)\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right) \\ +\left(\cos(\omega\cdot t)\cdot\cos\left(\frac{4\pi}{3}\right)+\sin(\omega\cdot t)\cdot\sin\left(\frac{4\pi}{3}\right)\right)\cdot\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\end{array}\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\hat{e}\cdot\left(\cos(\omega\cdot t)\cdot\left(1+\frac{1}{4}+\frac{1}{4}\right)+j\cdot\sin(\omega t\cdot t)\cdot\left(\frac{3}{4}+\frac{3}{4}\right)\right) =$$

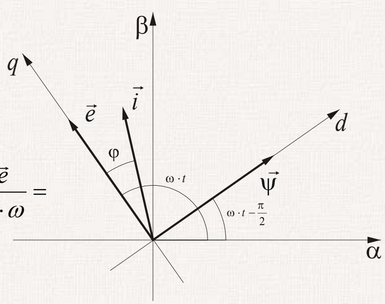
$$= \sqrt{\frac{3}{2}}\cdot\hat{e}\cdot\left(\cos(\omega\cdot t)+j\cdot\sin(\omega t\cdot t)\right) = E\cdot e^{j\omega t}$$

Rotating reference frame

Use the integral of The grid back emf vector:

$$\vec{\psi} = \int_0^t \vec{e}\cdot dt = \int_0^t E\cdot e^{j\omega\cdot t}dt = \frac{\vec{e}}{j\cdot\omega} =$$

$$= \frac{E}{\omega}e^{j\left(\omega\cdot t-\frac{\pi}{2}\right)}$$



Voltage equation in the (d,q)-frame

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{d\vec{i}}{dt} + j\cdot\omega\cdot L\cdot\vec{i} + \vec{e}$$

Active power ...

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{d\vec{i}}{dt} + j\cdot\omega\cdot L\cdot\vec{i} + \vec{e}$$

$$p(t) = \mathrm{Re}\left\{\vec{u}\cdot\vec{i}^{\,*}\right\} = \mathrm{Re}\left\{R\cdot\vec{i}\cdot\vec{i}^{\,*} + L\cdot\frac{d\vec{i}}{dt}\cdot\vec{i}^{\,*} + j\cdot\omega\cdot L\cdot\vec{i}\cdot\vec{i}^{\,*} + \vec{e}\cdot\vec{i}^{\,*}\right\} =$$

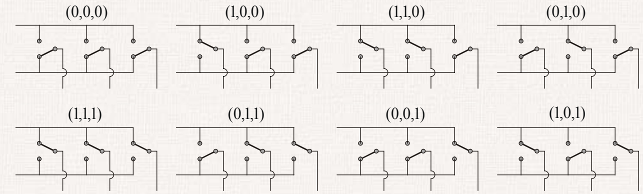
$$= \underbrace{Ri_d^2 + Ri_q^2}_{1} + \underbrace{L\frac{di_d}{dt}i_d + L\frac{di_q}{dt}i_q}_{2} + \underbrace{e_q i_q}_{3}$$

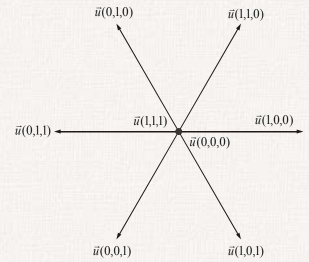
1. Resistive losses
2. Energizing inductances
3. Power absorbed by the grid back emf

Stationarity:

$$p(t) = E\cdot\sqrt{\frac{3}{2}}\cdot\left|\vec{i}\right|\cdot\cos(\varphi) = \sqrt{3}\cdot E\cdot I_{rms,\,phase}\cdot\cos(\varphi)$$

3-phase converters – 8 switch states





$$\vec{u}(1,0,0) = \sqrt{\frac{2}{3}}U_{dc} = -\vec{u}(0,1,1)$$

$$\vec{u}(0,1,0) = \sqrt{\frac{2}{3}}U_{dc}\cdot e^{j\frac{2\pi}{3}} = -\vec{u}(1,0,1)$$

$$\vec{u}(0,0,1) = \sqrt{\frac{2}{3}}U_{dc}\cdot e^{j\frac{4\pi}{3}} = -\vec{u}(1,1,0)$$

$$\vec{u}(0,0,0) = 0 = \vec{u}(1,1,1)$$

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3-phase converters - sinusoidal references

$$\vec{u}^{\,*} = u^*\cdot e^{j\omega t} = u^*\cdot(\cos(\omega t) + j\cdot\sin(\omega t)) = u_\alpha^* + j\cdot u_\beta^*$$

$$u_a^* = \sqrt{\frac{2}{3}}u_\alpha^*$$

$$u_b^* = \frac{1}{\sqrt{2}}u_\beta^* - \frac{1}{\sqrt{6}}u_\alpha^*$$

$$u_c^* = -\frac{1}{\sqrt{2}}u_\beta^* - \frac{1}{\sqrt{6}}u_\alpha^*$$

⇒

$$u_a^* = \sqrt{\frac{2}{3}}\cdot u^*\cos(\omega t)$$

$$u_b^* = \sqrt{\frac{2}{3}}\cdot u^*\cos(\omega t - \frac{2\pi}{3})$$

$$u_c^* = \sqrt{\frac{2}{3}}\cdot u^*\cos(\omega t - \frac{4\pi}{3})$$

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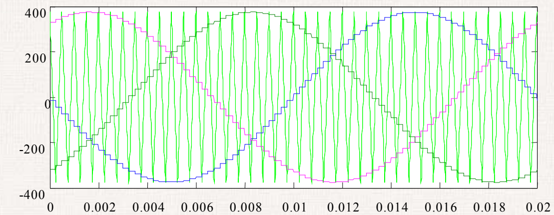
3-phase converters modulation

Simplest with sinusoidal references...



... but the DC link voltage is badly utilized.

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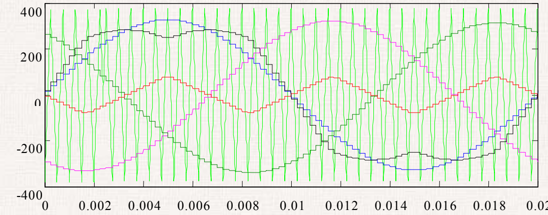
3-phase converters – symmetrization

3 phase potentials, only 2 vector components. One degree of freedom to be used for other purposes.

$$v_{az}^* = u_a^* - v_z^*$$
$$v_{bz}^* = u_b^* - v_z^*$$
$$v_{cz}^* = u_c^* - v_z^*$$

3-phase symmetrized modulation

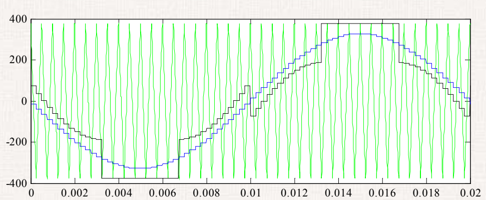
$$v_z^* = \frac{\max(u_a^*, u_b^*, u_c^*) + \min(u_a^*, u_b^*, u_c^*)}{2}$$



Maximum phase voltage with sinusoidal modulation : Udc/2
Maximum phase-to phase voltage with symmetrized modulation : Udc -> Phase voltage Udc/sqrt(3), i.e. 2/sqrt(3)=1.15 times larger than with sinusoidal modulation.

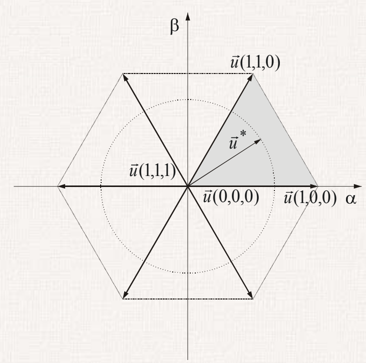
3-phase minimum switching modulation

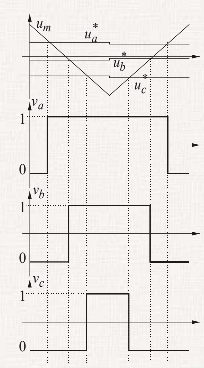
$$v_z{}^* = -\min\left(\frac{U_{dc}}{2} - \max\left(u_a^*, u_b^*, u_c^*\right), -\frac{U_{dc}}{2} - \min\left(u_a^*, u_b^*, u_c^*\right)\right)$$



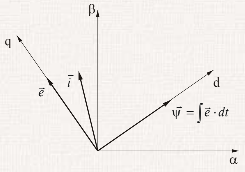
One phase is not switching for 2 60 degree intervals ...

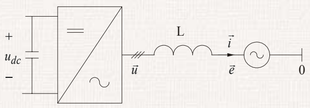
Modulation sequence vs. ripple





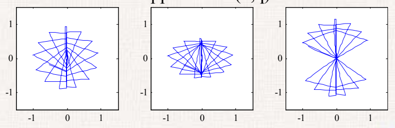
Modulation sequence vs. ripple





$$\frac{d\vec{i}}{dt} = \frac{\vec{u} - \vec{e}}{L}$$

Current ripple in the (d,q)-frame



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