Torque Control of Induction Machines ...

... as compared to PMSM

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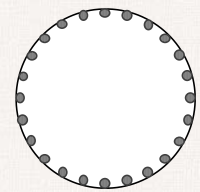
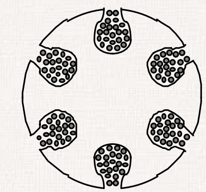
Lund University, Sweden

Mechanical design of IM

- **Stator same as PMSM**
- **Rotor:**

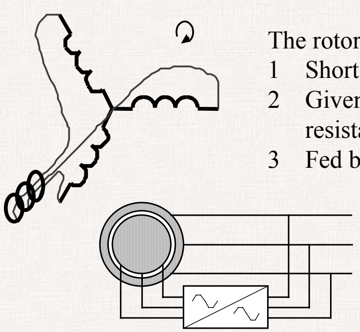


Cast aluminum



Wound copper /Slip ring

Slip ring rotor



The rotor can be:

1. Short circuited
2. Given an external rotor resistance
3. Fed by power electronics

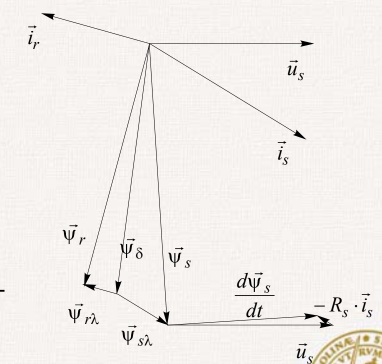
Mathematical model

- **Stator**

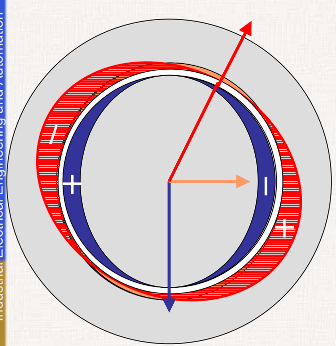
$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}$$

- **Rotor**

$$\vec{u}_r = R_s \cdot \vec{i}_r + \frac{d\vec{\psi}_r}{dt}$$



How does it work?



Imagine the following sequence :

1. A magnetizing stator current is stationary vs. the rotor.
2. The current is instantly moved and increased.
3. The rotor conserves the (rotor-)flux and thus the stator flux

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Torque

Rotor flux orientation

$$T = \vec{\psi}_s \times \vec{i}_s = \vec{\psi}_r \times \vec{i}_r = -\frac{L_m}{L_s} \cdot \vec{\psi}_s \times \vec{i}_r = \frac{L_m}{L_s} \cdot \vec{\psi}_r \times \vec{i}_s = \vec{\psi}_\delta \times \vec{i}_s$$

Stator flux orientation

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Vector control of Induction motors

- **Flux estimation (rotor- or stator- the most difficult part)**

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Rotor flux estimation 1

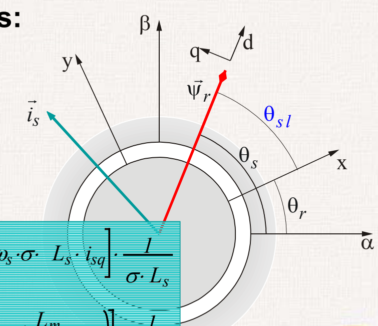
- **2 vector equations -> 4 equations.**
- **Rewriting a lot gives:**

$$\frac{d\psi_{rd}}{dt} = \frac{L_m}{\tau_r} \cdot i_{sd} - \frac{1}{\tau_r} \cdot \psi_{rd}$$

$$\frac{d\theta_{sl}}{dt} = \frac{L_m \cdot i_{sq}}{\tau_r \cdot \psi_{rd}}$$

$$\frac{di_{sd}}{dt} = \left[u_{sd} - R_s \cdot i_{sd} - \frac{L_m}{L_r} \cdot \frac{d\psi_{rd}}{dt} + \omega_s \cdot \sigma \cdot L_s \cdot i_{sq}\right] \cdot \frac{1}{\sigma \cdot L_s}$$

$$\frac{di_{sq}}{dt} = \left[u_{sq} - R_s \cdot i_{sq} - \omega_s \cdot \left(\sigma \cdot L_s \cdot i_{sd} + \frac{L_m}{L_r} \cdot \psi_{rd}\right)\right] \cdot \frac{1}{\sigma \cdot L_s}$$



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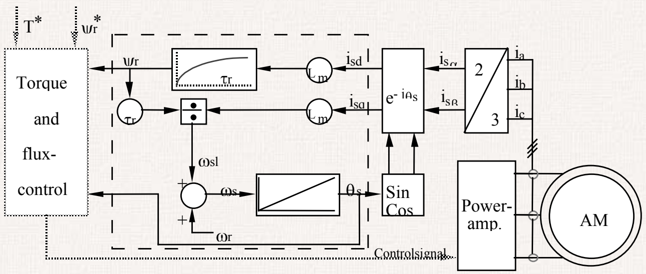
Rotor flux estimation 2



Full Flux Observer 1

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}$$

$$0 = R_r \cdot \vec{i}_r + \frac{d\vec{\psi}_r}{dt} - j \cdot \omega_r \cdot \vec{\psi}_r$$

$$\Downarrow$$

$$\underbrace{\begin{bmatrix} \vec{u}_s \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_s \\ \vec{i}_r \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -j \cdot \omega_r \end{bmatrix} \cdot \begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix}}_{u = R \cdot i + \frac{d\psi}{dt} + \Omega \cdot \psi}$$

Full Flux Observer 2

$$\frac{d\psi}{dt} = u - R \cdot i - \Omega \cdot \psi = \begin{Bmatrix} \psi = L \cdot i \\ i = L^{-1} \cdot \psi \end{Bmatrix} = -\left(R \cdot L^{-1} + \Omega\right) \cdot \psi + u$$

$$\frac{d\hat{\psi}}{dt} = -\left(R \cdot L^{-1} + \Omega\right) \cdot \hat{\psi} + u + R \cdot k \cdot \left(i_s - C \cdot \hat{\psi}\right) =$$

$$= -\left(R \cdot L^{-1} + \Omega - R \cdot k \cdot C\right) \cdot \hat{\psi} + u + R \cdot k \cdot i_s =$$

$$= A_{obs} \cdot \hat{\psi} + B \cdot u + R \cdot k \cdot i_s$$

where

$$R = \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix}; L = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix}; \psi = \begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$u = \begin{bmatrix} \vec{u}_s \\ 0 \end{bmatrix}; C = L^{-1}(1,:); \Omega = \begin{bmatrix} 0 & 0 \\ 0 & -j \cdot \omega_r \end{bmatrix}$$

Voltage measurement

- **Estimation by software**
  - **Error prone at low speeds**
- **Analogue filtering and sampling**
  - **slow**
- **Direct integration**
  - **Needs galvanic isolation**

Split equation solving

$$\begin{bmatrix} \frac{d\vec{\psi}_s}{dt} \\ \frac{d\vec{\psi}_r}{dt} \end{bmatrix} = \begin{bmatrix} A_{obs,11} & A_{obs,12} \\ A_{obs,21} & A_{obs,22} \end{bmatrix} \cdot \begin{bmatrix} \hat{\vec{\psi}}_s \\ \hat{\vec{\psi}}_r \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \vec{u}_s + \begin{bmatrix} R_s \cdot k_1 \cdot \vec{i}_s \\ R_r \cdot k_2 \cdot \vec{i}_s \end{bmatrix}$$

$$\frac{d\vec{\psi}_s}{dt} = A_{obs,11} \cdot \hat{\vec{\psi}}_s + A_{obs,12} \cdot \hat{\vec{\psi}}_r + \vec{u}_s + R_s \cdot k_1 \cdot \vec{i}_s$$

$$= A_{obs,11} \cdot \hat{\vec{\psi}}_s + A_{obs,12} \cdot \hat{\vec{\psi}}_r + R_s \cdot k_1 \cdot \vec{i}_s \quad + \quad \vec{u}_s$$

Solved partly outside the microprocessor

$$\frac{d\vec{\psi}_r}{dt} = A_{obs,21} \cdot \hat{\vec{\psi}}_s + A_{obs,22} \cdot \hat{\vec{\psi}}_r + 0 + R_r \cdot k_2 \cdot \vec{i}_s$$

Solved entirely in the microprocessor

Direct integration implemented

Stator Flux Oriented Vector Control of Torque and Flux

- **3 advantages:**
  - **In a synchronously rotating reference frame, there are no stationary errors since all reference values are constant in stationarity**
  - **The stator flux vector can in most cases be observed with high accuracy and low parameter sensitivity**
  - **Direct flux control can be applied**

Equations, again ...

- **2 vector equation, i.e. 4 scalar equations, rewritten gives :**

$$\frac{d\psi_{sd}}{dt} = \frac{L_s}{\tau_r} \cdot i_{sd} - \frac{1}{\tau_r} \cdot \psi_{sd} + \left[\sigma \cdot L_s \cdot \frac{di_{sd}}{dt} - \omega_{sl} \cdot \sigma \cdot L_s \cdot i_{sq} \right] = u_{sd} - R_s \cdot i_{sd}$$

$$\frac{d\theta_{sl}}{dt} = \frac{L_s \cdot i_{sq} + \sigma \cdot L_s \cdot \tau_r \cdot \frac{di_{sq}}{dt}}{\tau_r \cdot \psi_{sd} - \sigma \cdot L_s \cdot \tau_r \cdot i_{sd}} = \frac{u_{sq} - R_s \cdot i_{sq}}{\psi_{sd}} - \omega_r$$

Direct Flux Vector Control

$$u_{sd}(t) = R_s \cdot i_{sd}(t) + \frac{d\psi_{sd}(t)}{dt}$$

$$u_{sd}^*(k) = R_s \cdot \bar{i}_{sd}(k,k+1) + \frac{\psi_{sd}^*(k) - \psi_{sd}(k)}{T_s} =$$

$$= \{\psi_{sd} = L_s \cdot i_{sd} \ \ in\ stationarity\} =$$

$$\approx R_s \cdot \frac{\psi_{sd}(k)}{L_s} + \frac{\psi_{sd}^*(k) - \psi_{sd}(k)}{T_s}$$

Control of the Torque producing current component : 1

- **The remaining two rewritten scalar equations:**

$$u_{sq} = R_s \cdot i_{sq} + \omega_r \cdot \psi_{sd} + \left[\frac{L_s \cdot i_{sq} + \sigma \cdot L_s \cdot \tau_r \cdot \frac{di_{sq}}{dt}}{\tau_r \cdot \psi_{sd} - \sigma \cdot L_s \cdot \tau_r \cdot i_{sd}}\right] \cdot \psi_{sd}$$

$$u_{sq} = (R_s + R_r) \cdot i_{sq} + \sigma \cdot L_s \cdot \frac{di_{sq}}{dt} + \omega_r \cdot \psi_{sd} \approx$$

$$\approx (R_s + R_r) \cdot i_{sq} + (L_{s\lambda} + L_{r\lambda}) \cdot \frac{di_{sq}}{dt} + \omega_r \cdot \psi_{sd}$$

Control of the Torque producing current component : 2

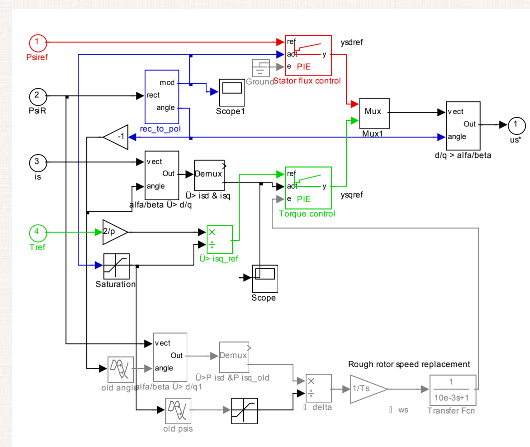
Use generic 1-phase load ...

$$u_{sq}^*(k) =$$

$$= \left(\frac{L_{s\lambda} + L_{r\lambda}}{T_s} + \frac{R_s + R_r}{2}\right) \cdot \left(\left(i_{sq}^*(k) - \hat{i}_{sq}(k)\right) + \frac{T_s}{\left(\frac{L_{s\lambda} + L_{r\lambda}}{R_s + R_r} + \frac{T_s}{2}\right)} \cdot \sum_{n=0}^{n=k-1} \left(i_{sq}^*(n) - \hat{i}_{sq}(n)\right) \right) +$$

$$+ \omega_r \cdot \psi_{sd}(k)$$

Vector Control System



DTC

- Compare to DCC.
- Replace i_{sd} with ψ_{sd}
- Replace i_{sq} with T

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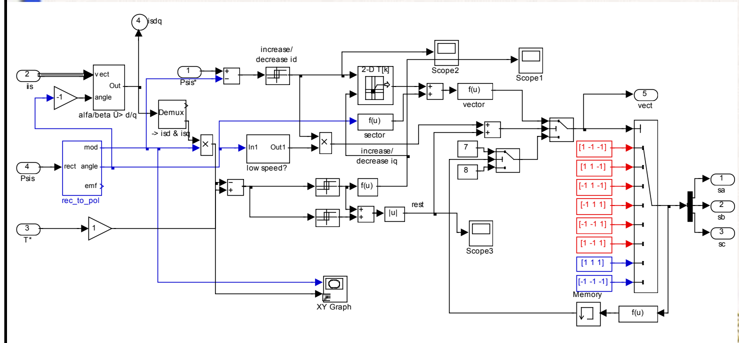
DTC system



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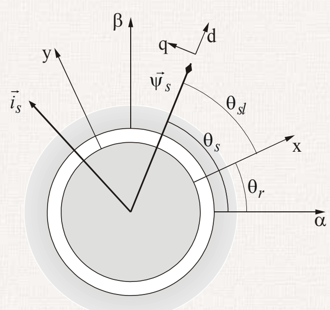
Simplified Stator Flux Model 1



$$\vec{u}_s = R\cdot\vec{i}_s + \frac{d\vec{\psi}_s}{dt} = R\cdot\left(\vec{i}_s-\vec{i}_{s0}\right) + R\cdot\vec{i}_{s0} + \frac{d\vec{\psi}_s}{dt}$$

$$\frac{d\vec{\psi}_s}{dt} = -R\cdot\vec{i}_{s0} + \left[\vec{u}_s - R\cdot\left(\vec{i}_s-\vec{i}_{s0}\right)\right] = \left\{\begin{array}{c} \text{no load running} \\ \vec{i}_s = \vec{i}_{s0} = \dfrac{\vec{\psi}_{s0}}{L_s}\end{array}\right\} = -R\cdot\frac{\vec{\psi}_{s0}}{L_s} + \vec{u}_s = -\frac{\vec{\psi}_{s0}}{\tau_s} + \vec{u}_s$$

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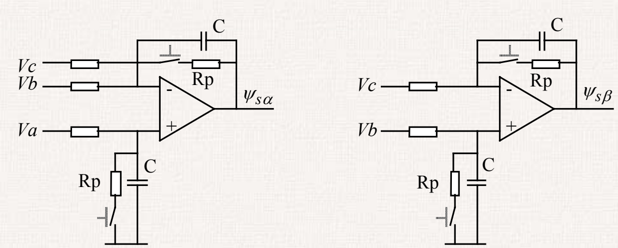
Simplified Stator flux model 2



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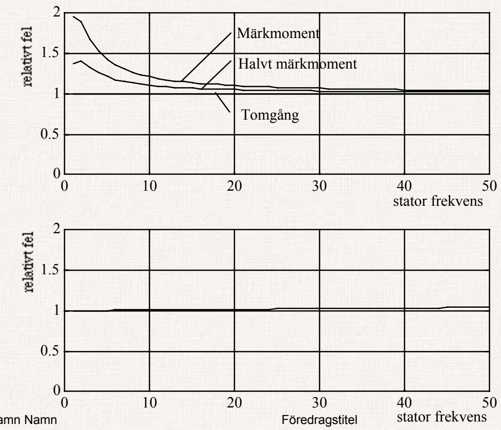
Simplified Stator Flux Model 3

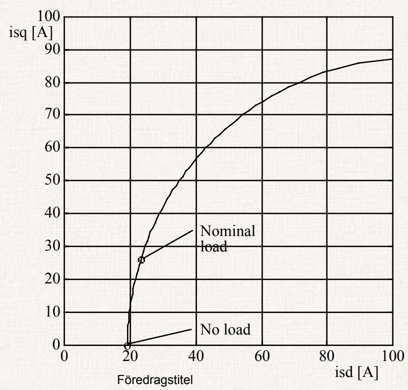
$$\frac{d\psi_{sd}}{dt} = \frac{L_s}{\tau_r}\cdot i_{sd} - \frac{1}{\tau_r}\cdot \psi_{sd} + \left[\sigma\cdot L_s\cdot\frac{di_{sd}}{dt} - \omega_{sl}\cdot\sigma\cdot L_s\cdot i_{sq}\right] = u_{sd} - R_s\cdot i_{sd}$$

$$\frac{d\theta_{sl}}{dt} = \frac{L_s\cdot i_{sq} + \sigma\cdot L_s\cdot\tau_r\cdot\frac{di_{sq}}{dt}}{\tau_r\cdot\psi_{sd} - \sigma\cdot L_s\cdot\tau_r\cdot i_{sd}} = \frac{u_{sq} - R_s\cdot i_{sq}}{\psi_{sd}} - \omega_r$$

Simplified estimation error



Relation iq/id in stator flux frame



Full Flux Observer 3

$$\hat{\psi}(k+1) = \boldsymbol{F}_{obs}\cdot\hat{\psi}(k) + \boldsymbol{G}_{obs}\cdot\bar{\boldsymbol{u}}(k,k+1) + \boldsymbol{K}_{obs}\cdot\bar{i}_s(k,k+1)$$

$$\boldsymbol{F}_{obs} = e^{\boldsymbol{A}_{obs}\cdot T_s} = e^{-(\boldsymbol{R}\cdot\boldsymbol{L}^{-1}+\boldsymbol{\Omega}+\boldsymbol{R}\cdot\boldsymbol{k}\cdot\boldsymbol{C})\cdot T_s} \approx$$

$$\approx e^{-\boldsymbol{\Omega}\cdot T_s/2}\cdot e^{-(\boldsymbol{R}\cdot\boldsymbol{L}^{-1}+\boldsymbol{R}\cdot\boldsymbol{k}\cdot\boldsymbol{C})\cdot T_s}\cdot e^{-\boldsymbol{\Omega}\cdot T_s/2} =$$

$$= \begin{bmatrix}0 & 0\\ 0 & e^{j\cdot\omega_r\cdot T_s/2}\end{bmatrix}\cdot\boldsymbol{F}_{obs,\omega_r=0}\cdot\begin{bmatrix}0 & 0\\ 0 & e^{j\cdot\omega_r\cdot T_s/2}\end{bmatrix}$$

$$\boldsymbol{F}_{obs} = \left[I - \boldsymbol{A}_{obs}\cdot\frac{T_s}{2}\right]^{-1}\cdot\left[I + \boldsymbol{A}_{obs}\cdot\frac{T_s}{2}\right] \approx \left[I + \boldsymbol{A}_{obs}\cdot\frac{T_s}{2}\right]^2$$