Power Electronics (EIEN25)
Exercises with Solutions

1

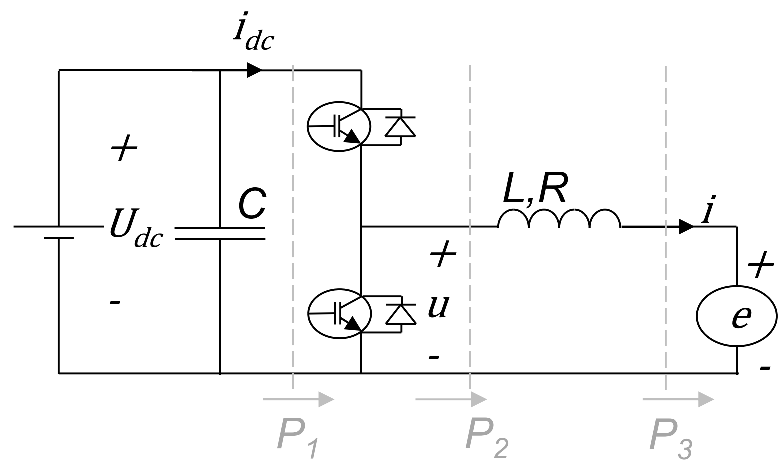
Exercises on Modulation

Exercise 1.1 2Q Boost / Buck converter no resistance

- **Data for the Boost / Buck converter**

U_{dc}	300 V
e	100 V
L	2 mH
R	0 ohm
f_{sw} (switch- freq)	3.33 kHz
i_{ave} (constant)	10 A

- **Determine**
  - *Load voltage (u) and load current (i) incl graphs*
  - *Dclink current (i_{dc}) incl graphs*
  - *The average powers at P_1, P_2, P_3*



Solution 1.1

- **Calculation steps**
  1. *Duty cycle*
  2. *Load current ripple, at positive or negative current slope. Max and min current*
  3. *Load current (i) graph, Load voltage (u) graph and dclink (i_{dc}) current graph*
  4. *Average current and average voltage at P_1, P_2, P_3 and P_4*
  5. *Average powers at P_1, P_2, P_3 and P_4*

1) $u_{avg} = e + R \cdot i_{avg} = 100 + 0 = 100V$

$$D = \frac{u_{avg}}{U_{dc}} = \frac{100}{300} = 0.33$$

Duty Cycle

2) Current ripple, max and min current

$$i_{ripple} = \frac{(U_{dc} - e)}{L} \cdot T_{per} \cdot D = \frac{(300 - 100)}{0.002} \cdot 0.0003 \cdot 0.33$$
$$= 10A$$

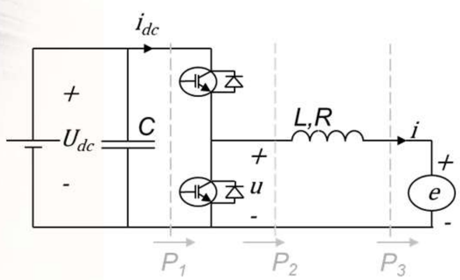
Period time
a.k.a T_{sw}

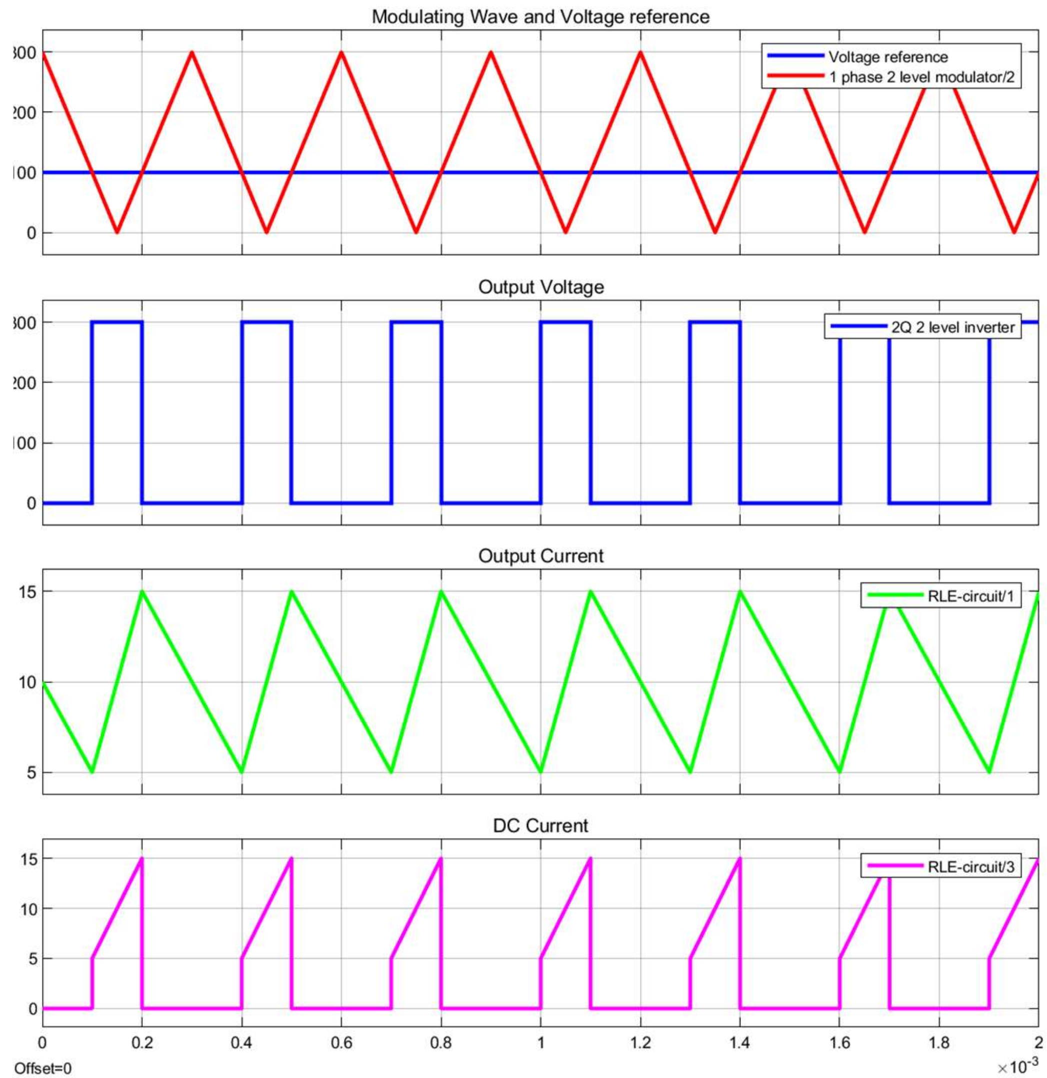
Solution 1.1

3. *The output voltage equals the DC link voltage ($u = U_{dc}$) during 33% of the period (called the pulse) time(the duty cycle D), and equals zero ($u = 0$) the rest of the time (called the pulse gap).*

   The output current (i) increases from 5 A to 15 A during the pulse, and returns from 15 A back to 5 A in the pulse gap.

   The dclink current = the output current ($i_{dc} = i$) during the pulse (= when the upper transistor is conducting) and is zero for the rest (when the lower diode is conducting).

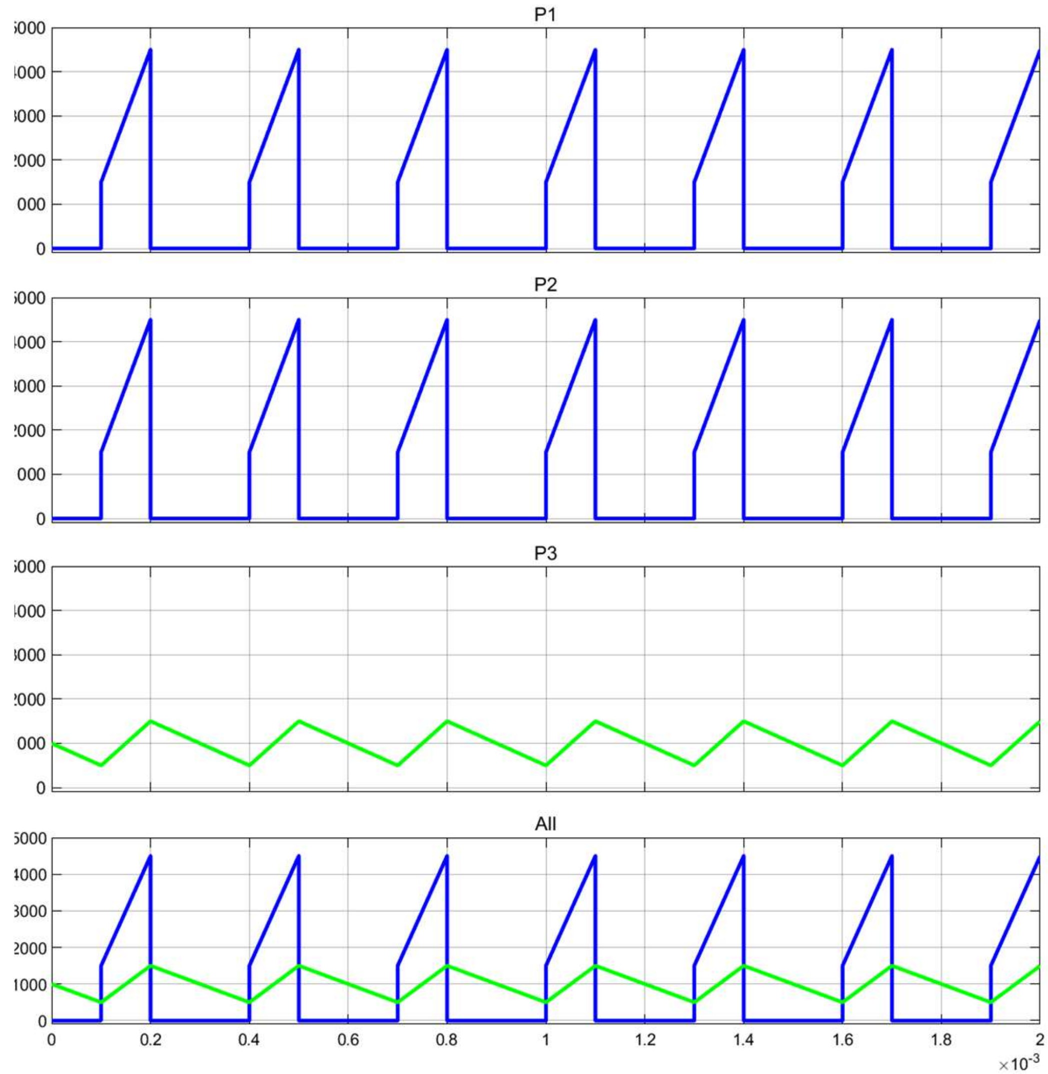




Solution 1.1

4. *Average current, average voltage and Power at P_1, P_2, P_3 and P_4*

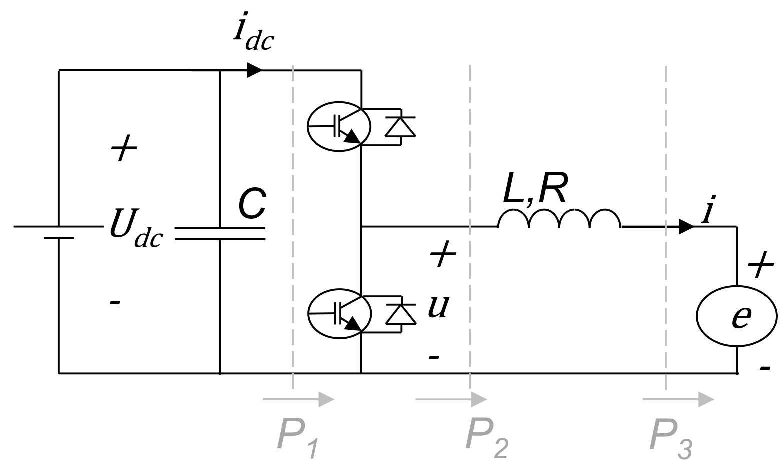
- *P1*
  - The voltage is the DC link voltage U_{dc}=300 V
  - The current equals the load current while the transistor is on and zero for the rest $\rightarrow$ $i_{dc,ave} = 3.33\ A$
  - The average power is $P_1 = U_{dc} \cdot i_{dc,ave} = 300 \cdot 3.33 = 1000\ W = 1\ kW$
- *P2*
  - The voltage is the is the average output voltage $u_{ave} =$ 100 V
  - The current is the average load current $i_{ave} =$ 10 A
  - The average power is $P_3 = U_{dc} \cdot i_{dc,ave} = 100 \cdot 10 = 1000\ W = 1\ kW$
- *P3*
  - The voltage is the is the load voltage $e = 100$ V
  - The current is the average load current $i_{ave} =$ 10 A
  - The average power is $P_4 = e \cdot i_{ave} = 100 \cdot 10 = 1000\ W = 1\ kW$



Exercise 1.2 2Q Boost/Buck converter with resistance

- **Data for the Boost / Buck converter**

U_{dc}	300 V
e	100 V
L	Very large
R	1 ohm
f_{sw} (switch- freq)	3.33 kHz
i_{avg} (constant)	10 A



- **Determine**
  - *Load voltage (u) incl graphs*
  - *DC-link current (i_{dc}) incl graphs*
  - *Power at P_1, P_2, and P_3*

Solution 1.2

Calculation steps

1. Avg phase voltage
2. Duty cycle
3. Load current. Ripple and min and max current
4. Load current graph, Load voltage graph and dclink current graph
5. Average current and average voltage at $P_{1,}$ P_2 *and* P_3
6. Power at $P_{1,}$ P_2 *and* P_3

1) Average load voltage

$$u_{ave} = e + R \cdot i_{av} \quad = 100 + 1 \cdot 10 = 110V$$

2) Duty cycle (D)

$$D = \frac{u_{avg}}{U_{dc}} = \frac{110}{300} = 0.37$$

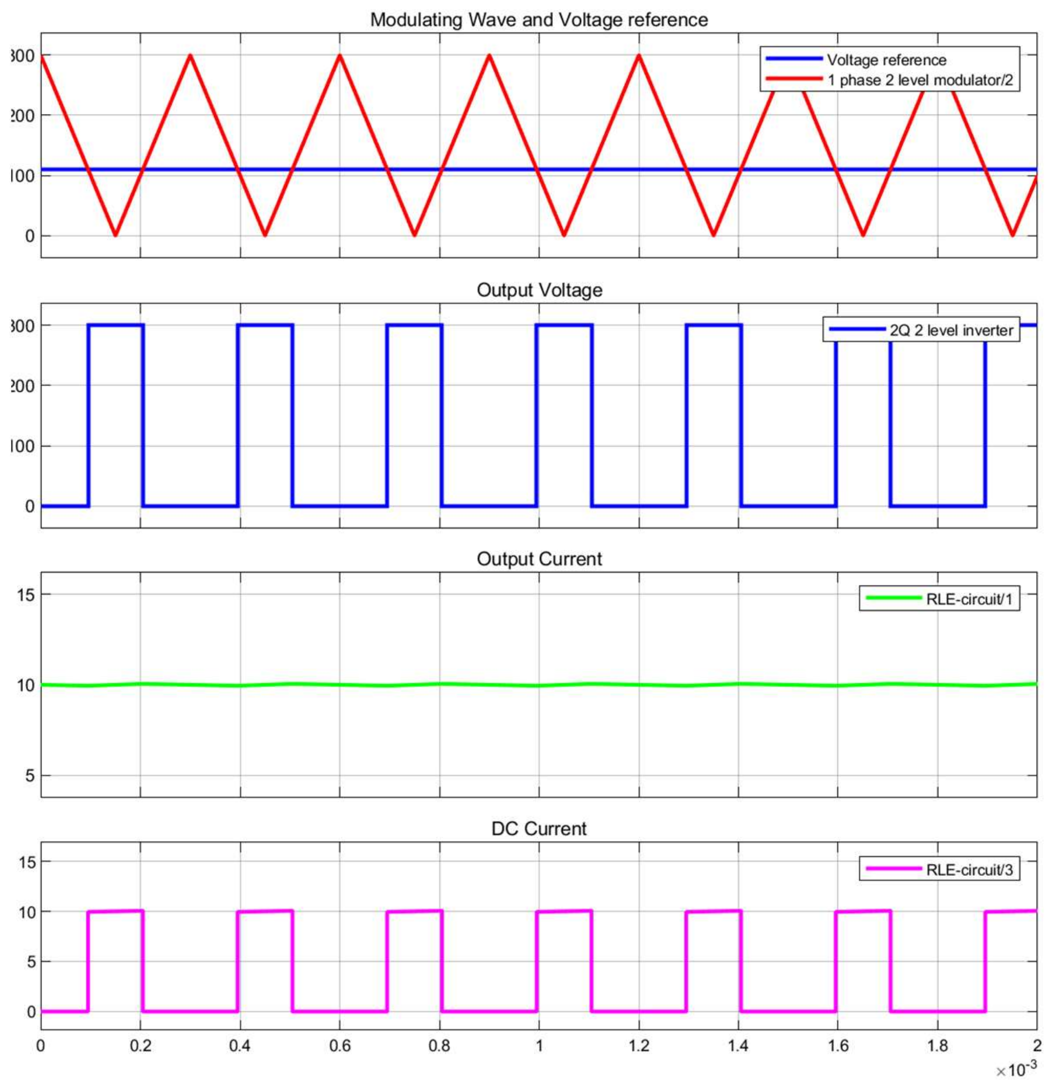
Solution 1.2

3) Load current. Ripple and min and max current

$$i_{ripple} = \frac{(U_{dc} - R \cdot i_{av} \ - e)}{\infty} \cdot \frac{1}{f_{sw}} \cdot D = 0.0A$$

4) Phase current graph, phase voltage graph and dclink current graph

$$i(t) = \{L\ is\ very\ high, i.e.\ no\ ripple\} = 10A$$



Solution 1.2

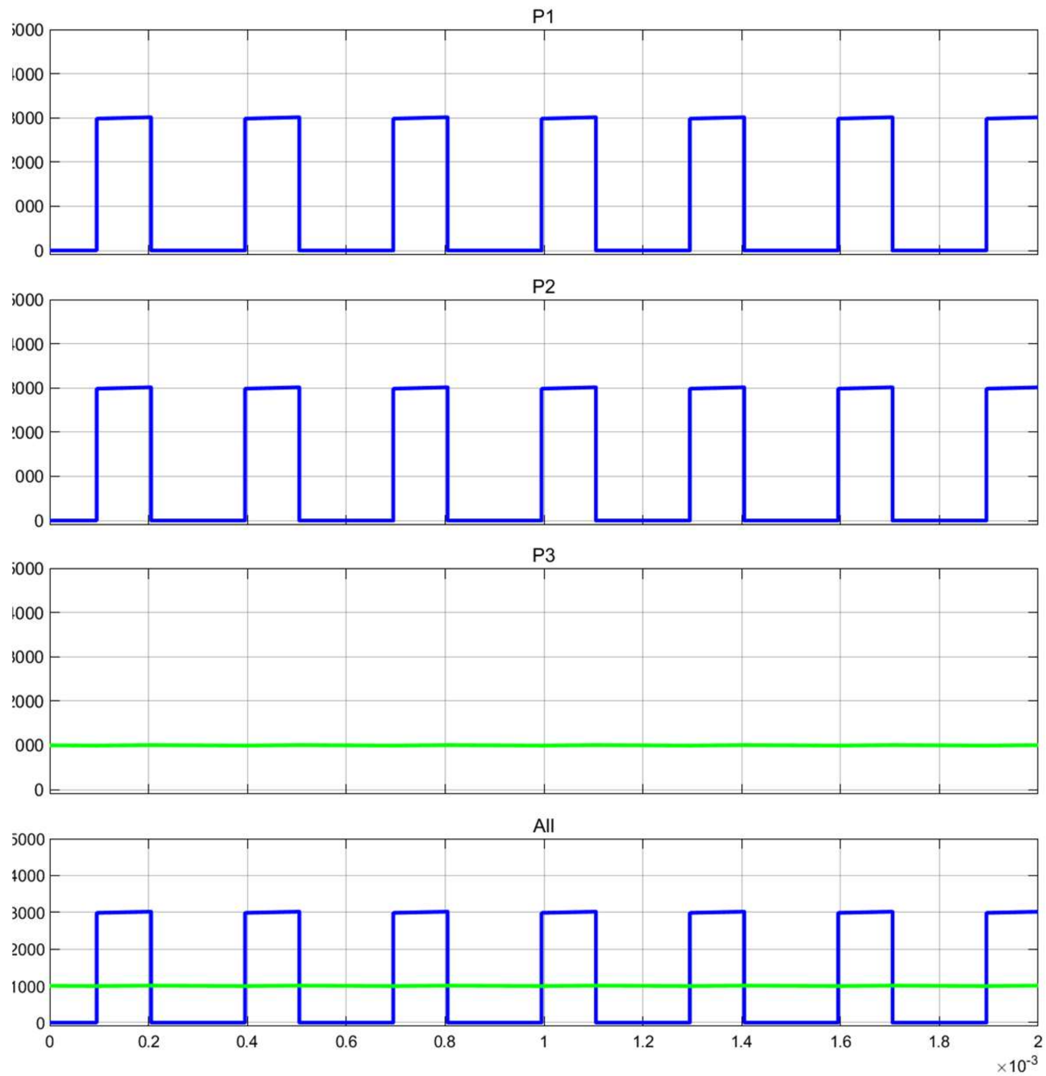
5) Average current and average voltage at P_1, P_2 and P_3

$$\begin{cases} i_{avg\_P_1} = D \cdot 10A = 3.7A \\ i_{avg\_P_2} = 10A \\ i_{avg\_P_3} = 10A \end{cases}$$

$$\begin{cases} u_{ave\_P_1} = 300V \\ u_{ave\_P_2} = 110V \\ u_{ave\_P_3} = 100V \end{cases}$$

6) Power at p_1, p_2 and p_3

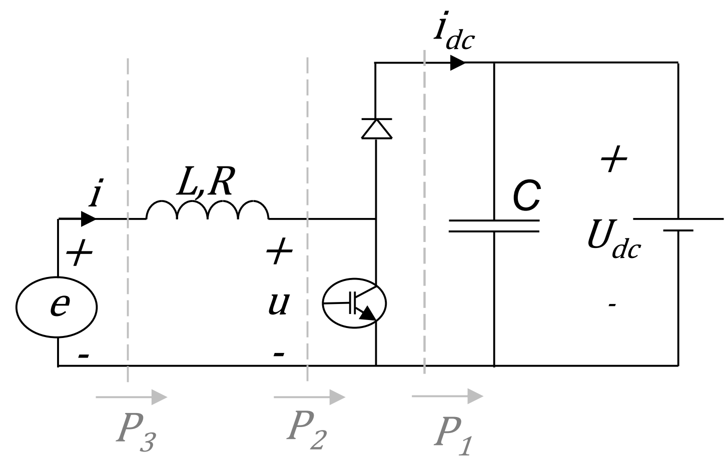
$$\begin{cases} P_{p1} = U_{dc} \cdot i_{avg} = 300 \cdot 3.7 = 1.1kW \\ P_{p2} = u_{ave} \cdot i_{avg} = 110 \cdot 10 = 1.1kW \\ P_{p3} = e \cdot i_{avg} = 100 \cdot 10 = 1\ kW \end{cases}$$



Exercise 1.3 1Q Boost converter with resistance

- **Data for the Boost converter**

U_{dc}	300 V
e	100 V
L	Very large
R	1 ohm
f_{sw} (switch- freq)	3.33 kHz
i_{ave} (constant)	10 A



- **Determine**
  - *Input voltage (u) incl graphs*
  - *DC-link current (i_{dc}) incl graphs*
  - *Power at P_1, P_2, and P_3*

Solution 1.3

Calculation Steps

1. Avg phase voltage
2. Duty cycle
3. Phase current. Ripple and min and max current
4. Phase current graph, phase voltage graph and dclink current graph
5. Average current and average voltage at $P_{1,} P_2$ *and* P_3
6. Power at $P_{1,} P_2$ *and* P_3

1) $u_{ave} = e - R \cdot i_{ave} = 100 - 1 \cdot 10 = 90V$

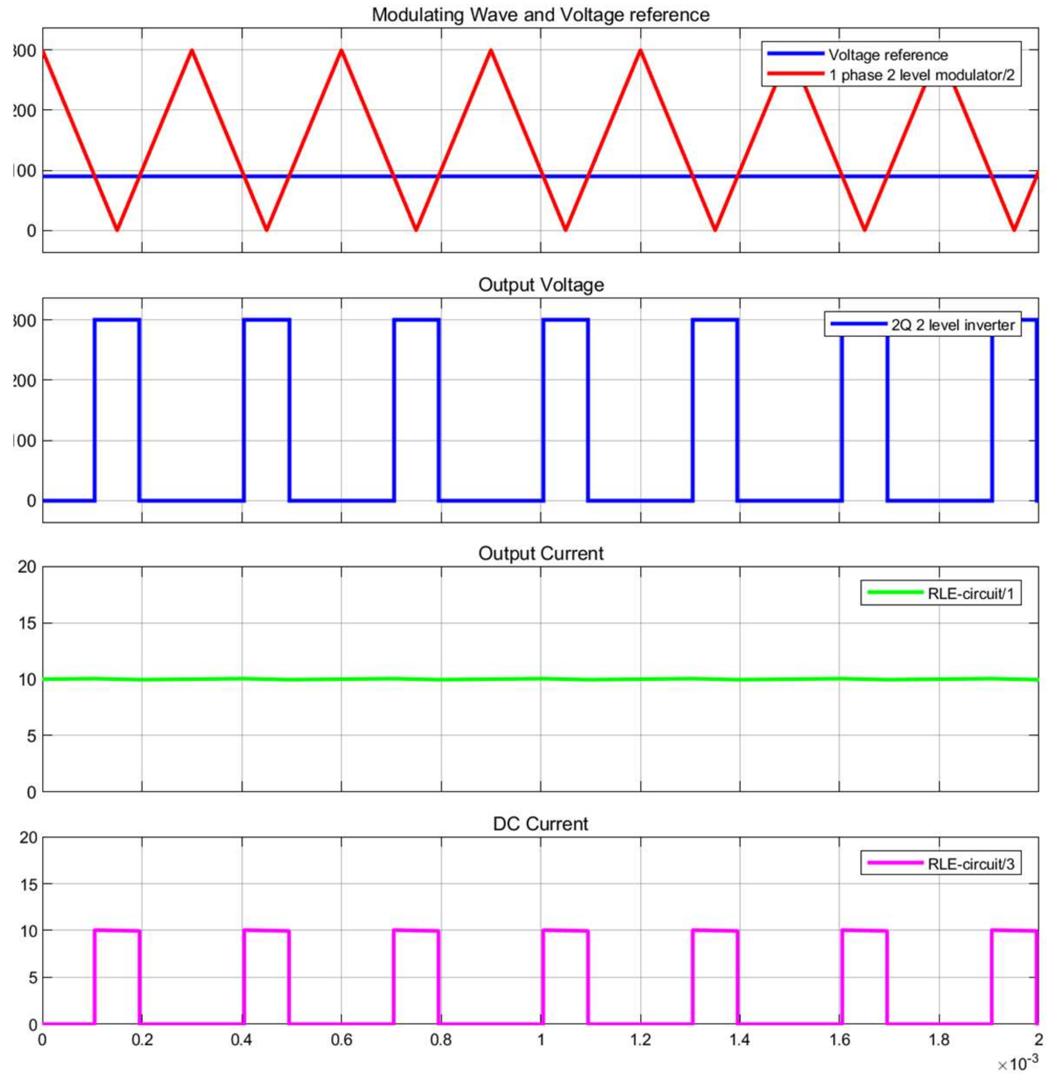
2) $D = \dfrac{u_{av}}{U_{dc}} = \dfrac{90}{300} = 0.30$

Solution 1.3

3) Phase current. Ripple and min and max current

$$i_{ripple} = \frac{(U_{dc} + R \cdot i_{ave} - e)}{\infty} \cdot \frac{1}{f_{sw}} \cdot D = 0.0A$$

4) Load current graph, load voltage graph and DC-link current graph



Solution 1.3

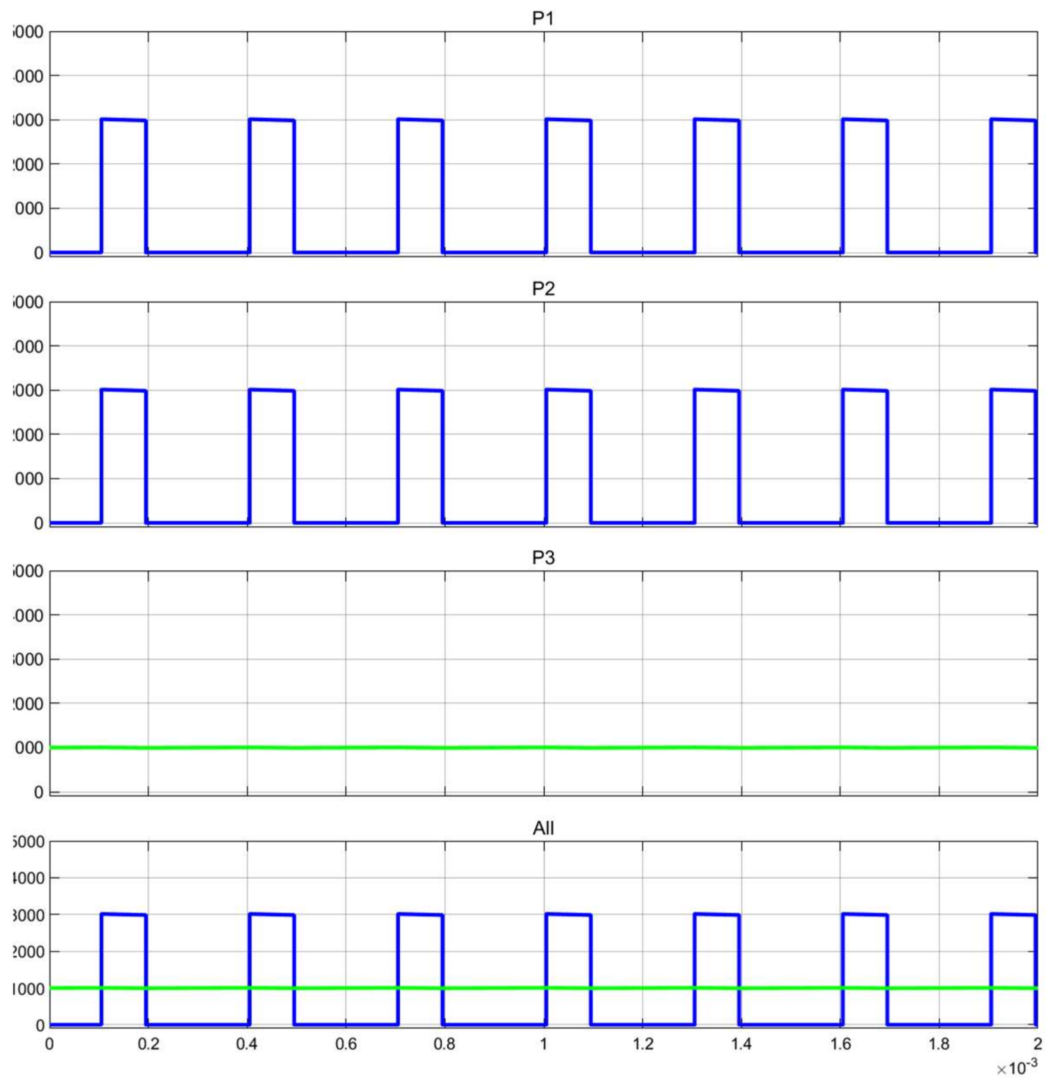
5) Average current and average voltage at P_1, P_2 and P_3

$$\begin{cases} i_{av\ \_P_1} = dutycycle \cdot 10A = 3.0A \\ i_{ave\_P_2} = 10A \\ i_{ave\_P_3} = 10A \end{cases}$$

$$\begin{cases} u_{ave\_P_1} = 300V \\ u_{ave\_P_2} = 90V \\ u_{ave\_P_3} = 100V \end{cases}$$

6) Power at P_1, P_2 and P_3

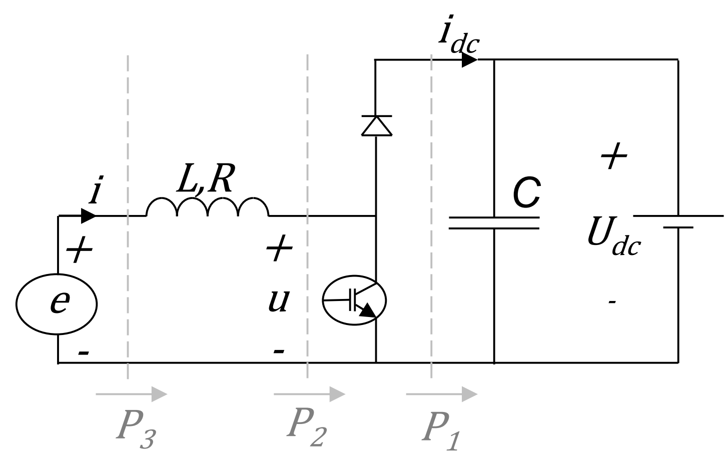
$$\begin{cases} P_1 = U_{dc} \cdot i_{ave} \cdot D = 300 \cdot 10 \cdot 0.3 = 0.9kW \\ P_2 = u_{avg} \cdot i_{ave} = 90 \cdot 10 = 0.9kW \\ P_3 = e \cdot i_{av} \ \ = 100 \cdot 10 = 1kW \end{cases}$$



Exercise 1.4 1Q Boost converter no resistance

- **Data for the Boost converter**

U_{dc}	300 V
e	100 V
L	2 mH
R	0 ohm
f_{sw} (switch- freq)	3.33 kHz
i_{ave} (constant)	5 A



- **Determine**
  - *Input voltage (u) incl graphs*
  - *DC-link current (i_{dc}) incl graphs*
  - *Power at P_1, P_2, and P_3*

Solution 1.4

Calculation steps

1. Duty cycle

2. Source current ripple, at positive or negative current slope.

3. Medium, max and min current

4. Source current graph, phase voltage graph and dclink current graph

5. Average source current voltage and power at $P_{1,}$ $P_{2,}$ P_3

1) Duty cycle

$$u_{ave} = e - R \cdot i_{av} \quad = 100 + 0 = 100V$$

$$D = \frac{u_{ave}}{U_d} = \frac{100}{300} = 0.33$$

2) Source current ripple, at positive or negative current slope

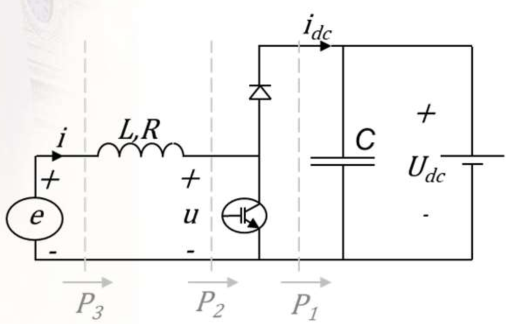
$$\frac{di}{dt} = \begin{cases} \frac{e - U_{dc}}{L} \text{ if Transistor OFF (Negative slope!)} \\ \frac{e}{L} \text{ if Transistor ON(Positive slope!)} \end{cases}$$

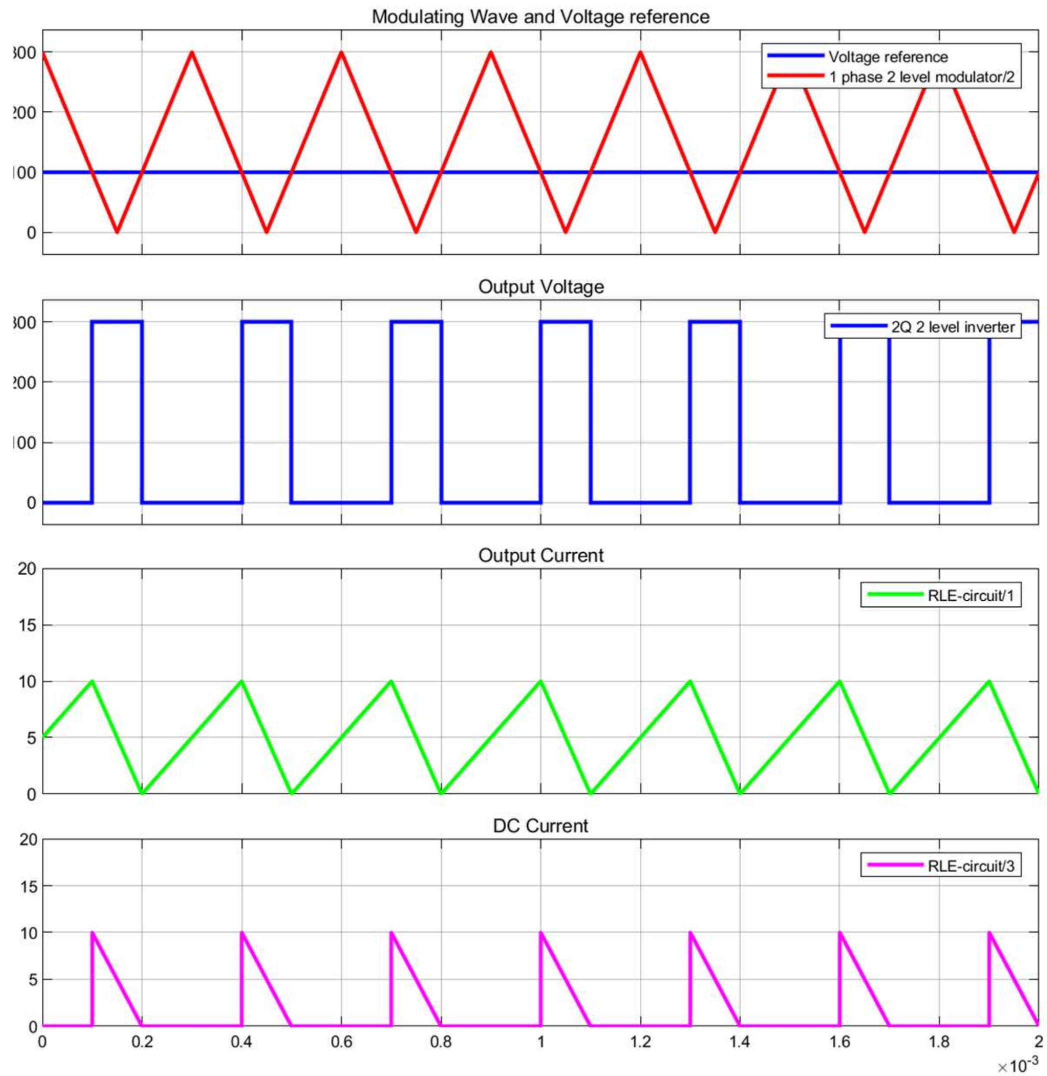
$$\Delta i = \begin{cases} \frac{e - U_d}{L} \cdot \Delta t = \frac{e - U_{dc}}{L} \cdot D \cdot T_{sw} = \frac{100 - 300}{0.002} \cdot \frac{1}{3} \cdot 300 \cdot 10^{-6} = -10 \rightarrow \frac{di}{dt} = \frac{-10}{100e^{-6}} = -100\ kA/s \\ \frac{e}{L} \cdot \Delta t = \frac{e}{L} \cdot (1 - D) \cdot T_{sw} = \frac{100}{0.002} \cdot \frac{2}{3} \cdot 300 \cdot 10^{-6} = 10 \rightarrow \frac{di}{dt} = \frac{10}{200e^{-6}} = 50\ kA/s \end{cases}$$

Solution 1.4

3) The current ripples between 0 and 10 A, with an average value of 5 A.

4) Phase current, Phase voltage and dclink current graph

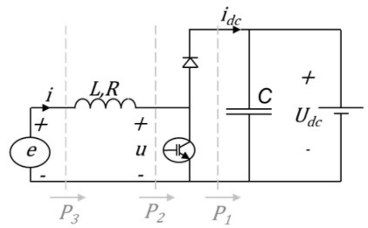




Solution 1.4

5) Average current and average voltage at P_1, P_2 and P_3

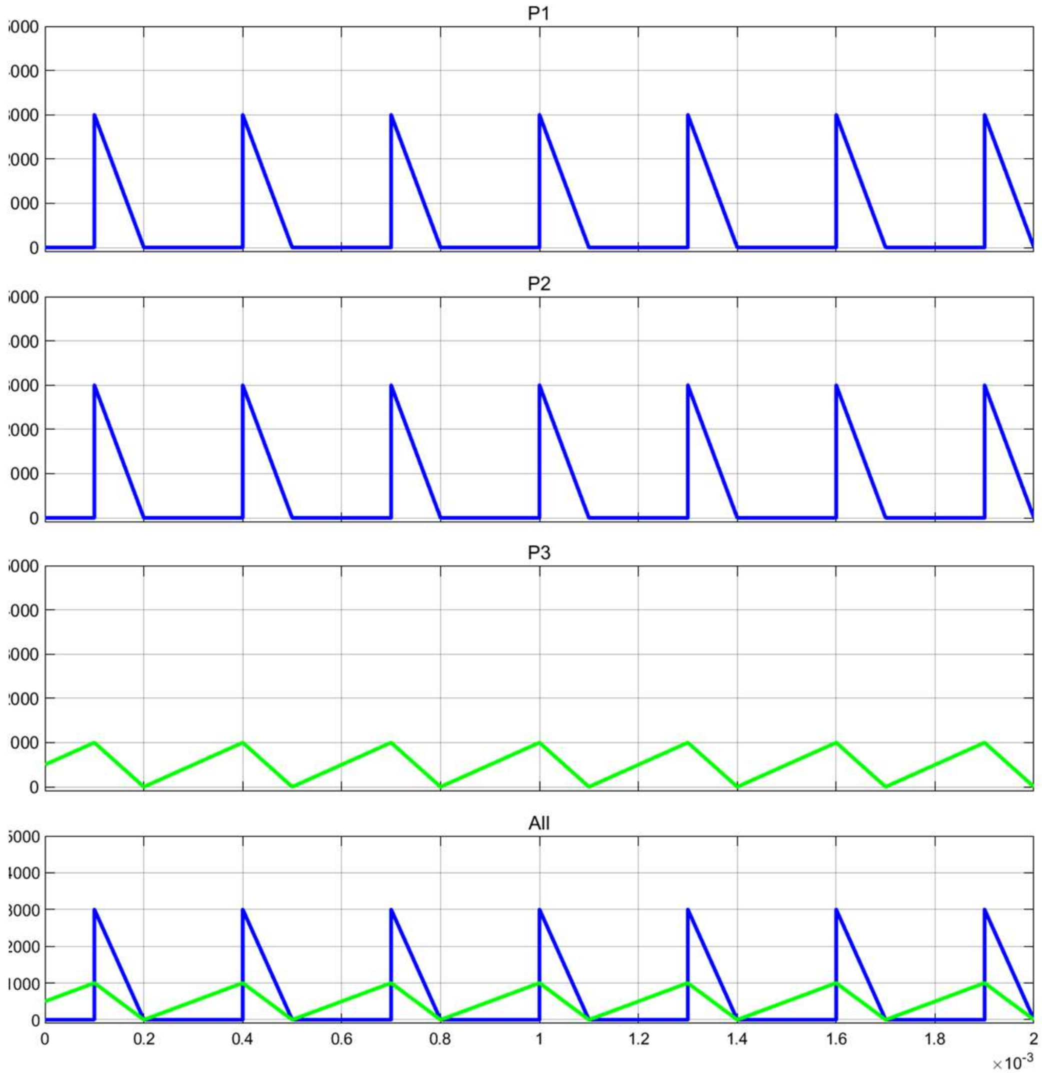
$$\begin{cases} i_{ave_{P_1}} = dutycycle \cdot 5A = 1.67A \\ i_{ave\_P_2} = 5A \\ i_{ave\_P_3} = 5A \end{cases}$$



$$\begin{cases} u_{av\ \_P_1} = 300V \\ u_{av\ \ P_2} = 100V \\ u_{av\ \_P_3} = 100V \end{cases}$$

6) Power at P_1, P_2 and P_3

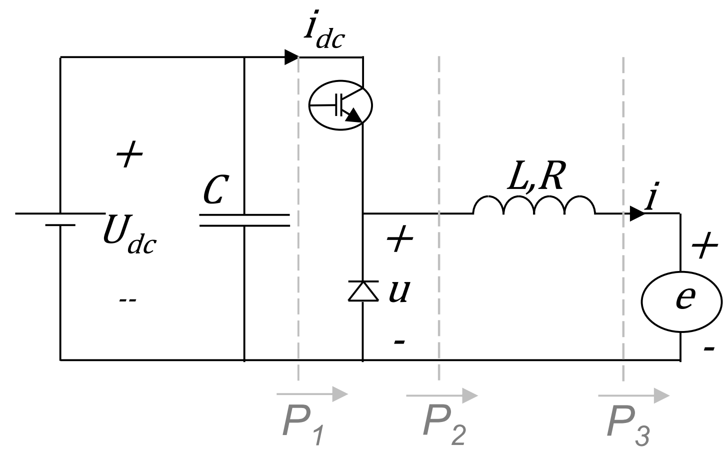
$$\begin{cases} P_1 = U_{dc} \cdot i_{ave} \cdot D = 300 \cdot 1.67 = 500\ W \\ P_2 = u_{avg} \cdot i_{ave} = 100 \cdot 5 = 500\ W \\ P_3 = e \cdot i_{av} \ \ = 100 \cdot 5 = 500\ W \end{cases}$$



Exercise 1.5 1Q Buck converter no resistance

- **Data for the Buck converter**

U_{dc}	300 V
e	100 V
L	2 mH
R	0 ohm
f_{sw}(switch- freq)	3.33 kHz
i_{start}	0 A, i.e. a switching period starts with zero load current
t_{on}	50 ms, the time the transistor is activated



- **Determine**
  - *Output voltage (u) incl graphs*
  - *DC-clink current (i_{dc}) incl graphs*
  - *Power at $P_1, P_2,$ and P_3*

Calculation steps

1. *How high do the load current rise during the time the transistor is on?*
2. *How long time does it take for the load current to fall back to zero?*
3. *The load voltage (u) when the transistor is off and current =0*
4. *The load current, load voltage and dclink current (i_{dc}) graph*
5. *The average current and average voltage at P_1, P_2 and P_3*
6. *The powers at P_1, P_2 and P_3*

Solution 1.5

1) *How high do the load current rise during the time the transistor is on?*

$$\frac{di}{dt} = \frac{u-e}{L}$$

$$\Delta i = \frac{u-e}{L} \cdot t_{on} = \frac{300-100}{0.002} \cdot 50e^{-6} = 5A$$

2) *Time for load current (i) current to fall back to zero.*

$$\frac{di}{dt} = \frac{u-e}{L} = \frac{-e}{L}$$

$$\Delta t = -\Delta i \cdot \frac{L}{e} = -(-5) \cdot \frac{0.002}{100} = 100\mu s$$

3) *Phase voltage when load current is zero (i=0)*

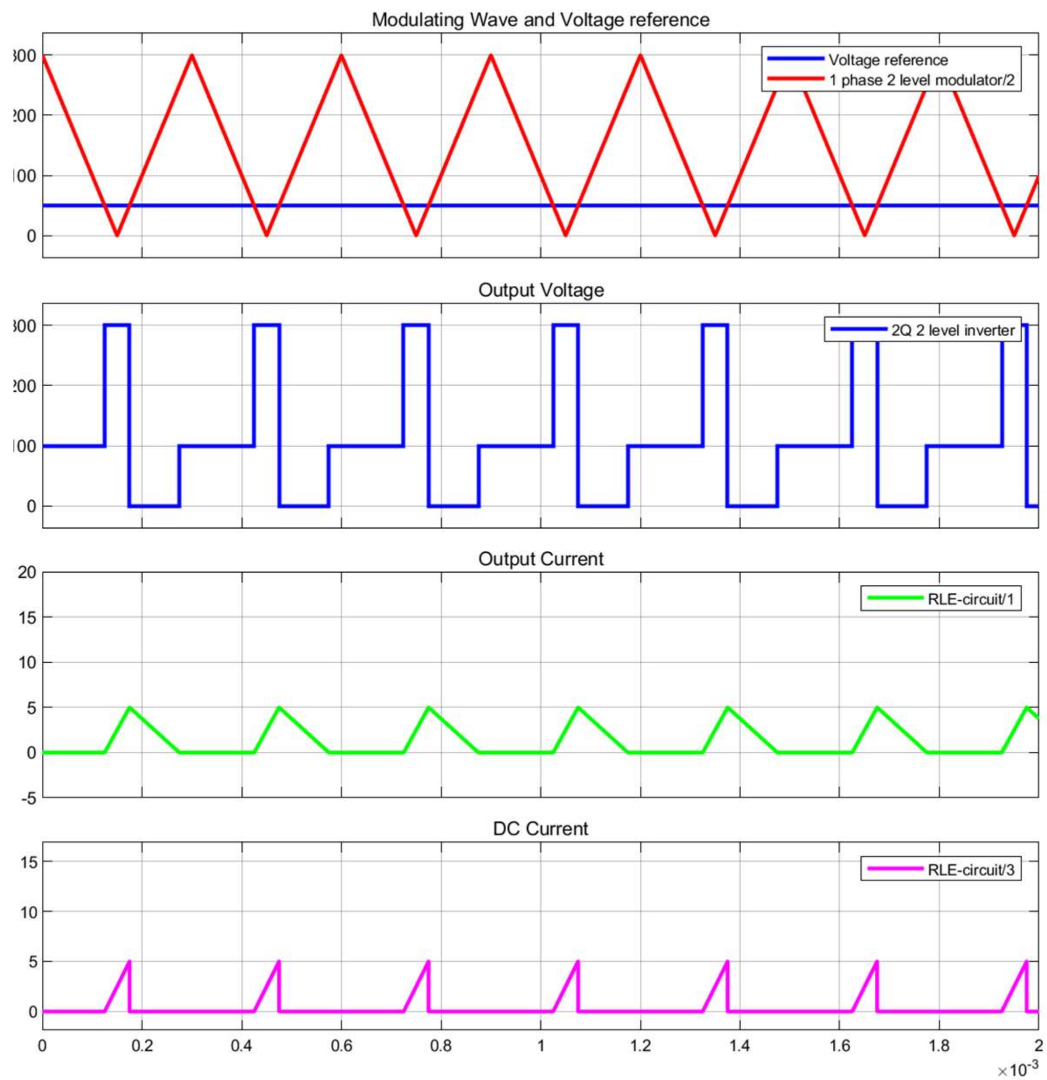
When the load current (i) is zero, neither of the transistor or diode are conducting. This means that neither of them ties the bridge output potential to the positive or negative side of the DC link. Since there is no voltage drop over the resistor either, the load voltage is "floating" and equal to the back-emf,

$$u = e = 100\ V$$

Solution 1.5

4) *The load current, load voltage and dclink current (i_{dc}) graph*

5) *Notice that with this 1Q (1 Quadrant) Buck cponverter the current cannot be negative and thus "stops" on its way down when it reaches zero. Then the load becomes "floating" and the output voltage equal to the load back-emf.*



Solution 1.5

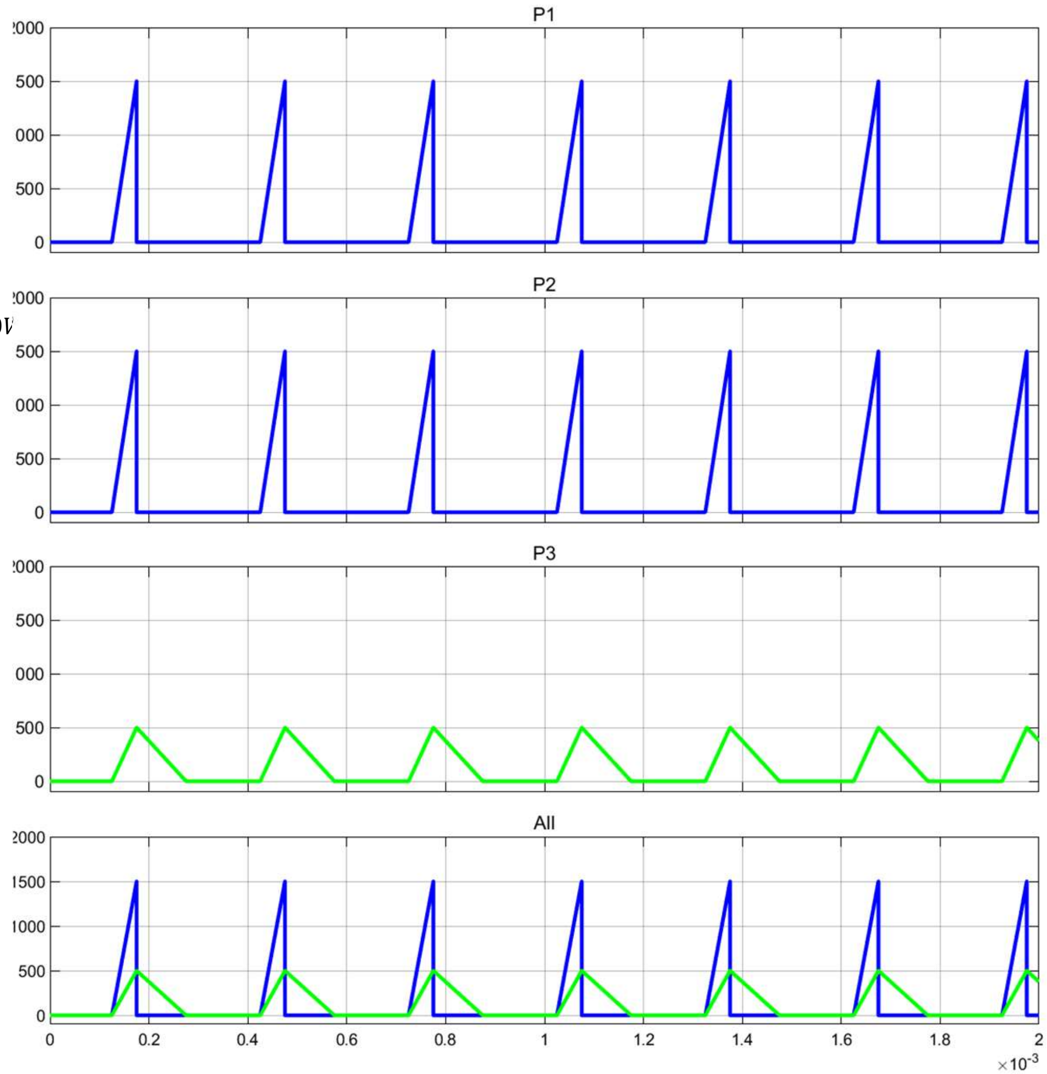
5) Average current and average voltage at $P_{1,}$ P_2 and P_3

$$\begin{cases} u_{ave\_P_1} = U_{dc} = 300V \\ u_{ave\_P_2} = \dfrac{(300 \cdot 50\mu s + 0 \cdot 100\mu s + 100 \cdot (300 - 50 - 100)\mu s)}{300\mu s} = 100V \\ u_{ave\_P_3} = e = 100V \end{cases}$$

$$\begin{cases} i_{ave\_P_1} = \dfrac{5 \cdot 50\mu s}{2} \cdot \dfrac{1}{300\mu s} = 0.4167A \\ i_{av\ \_P_2} = \dfrac{5 \cdot 150\mu s}{2} \cdot \dfrac{1}{300\mu s} = 1.25A \\ i_{av\ \_P_3} = i_{P_2} \end{cases}$$

6) Power at $P_{1,}$ P_2 and P_3

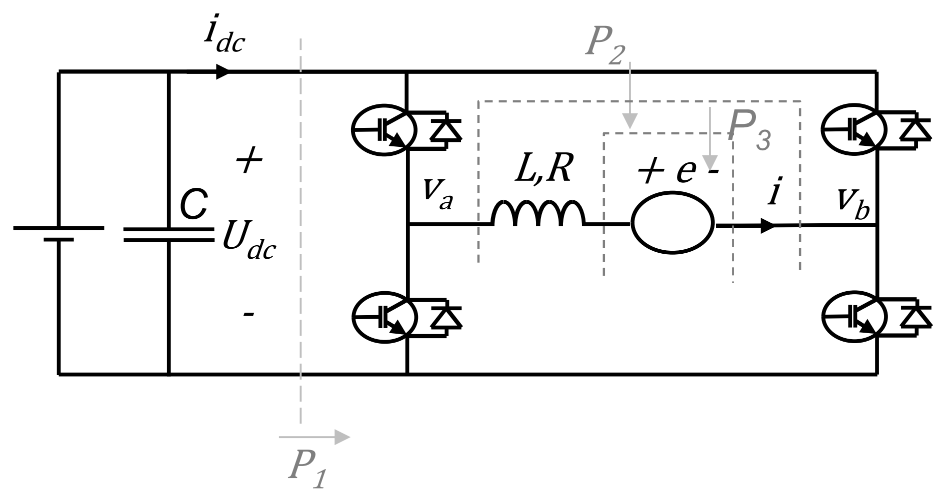
$$\begin{cases} P_{P_1} = U_{dc} \cdot i_{ave\_P_1} = 300 \cdot 0.4167 = 125W \\ P_{P_2} = u_{ave\_P_2} \cdot i_{ave\_P_2} = 100 \cdot 1.25 = 125W \\ P_{P_3} = e \cdot i_{av\ \_P_3} = 100 \cdot 1.25 = 125W \end{cases}$$



Exercise 1.6 4QC Bridge converter

Data for the Bridge converter

U_{dc}	300 V
e	100 V
L	2 mH
R	0 ohm
f_{sw}(switch- freq)	3.33 kHz
i_{ave} (constant)	10 A



Determine

- *Phase potentials (v_a & v_b) incl graphs*
- *DC-link current incl graphs*
- *Power at P_1, P_2 and P_3*

Solution 1.6

Calculation steps

1. Phase potential references

2. Phase potentials and Output voltage, pulse (t_p) width

3. Phase current. Ripple and min and max current

4. DC link current graph

5. Average current and average voltage at P_1, P_2 and P_3

6. Power at P_1, P_2 and P_3

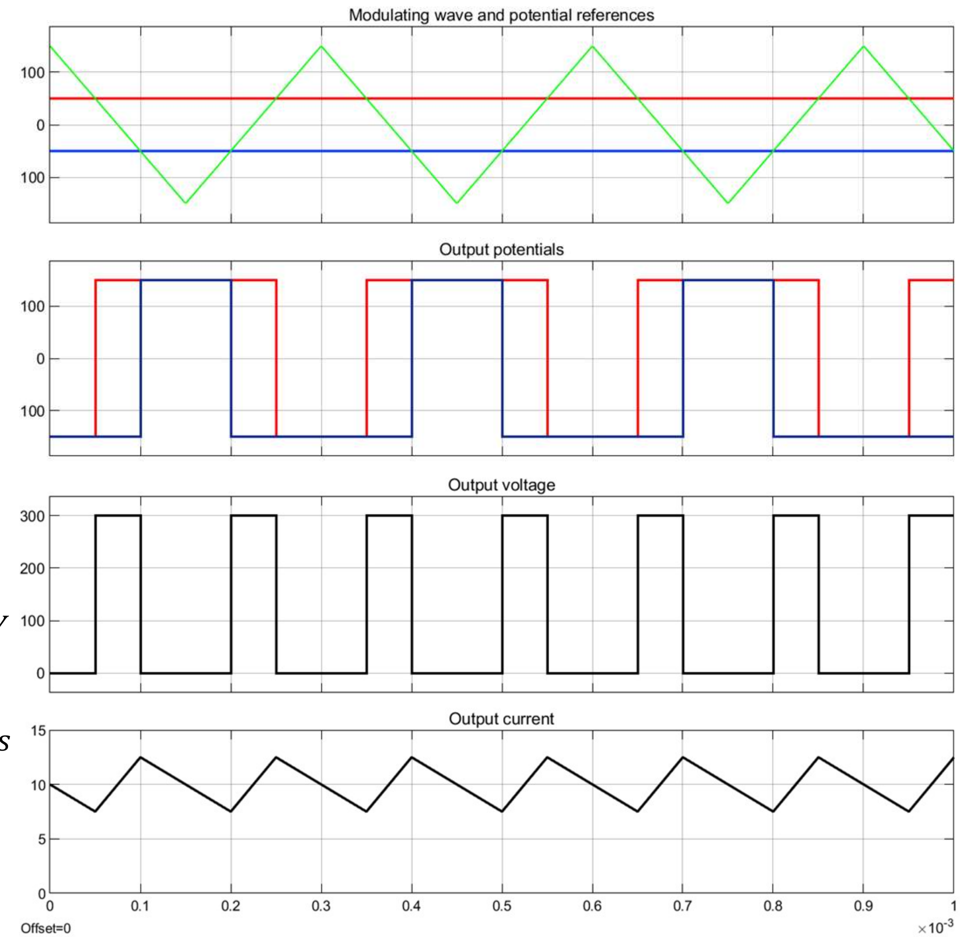
1) Assume symmetric modulation:

$\rightarrow v_a^* = \frac{u^*}{2} = \{\text{Stationary operation}\} = \frac{e+R\cdot i}{2} = \frac{100}{2} = 50V$

$\rightarrow v_b^* = -\frac{u^*}{2} = \{\text{Stationary operation}\} = -\frac{e+R\cdot i}{2} = -\frac{100}{2} = -50V$

Modulating Wave: $\pm\frac{U_{dc}}{2}$ @ $3.33\ kHz$

Pulse width of output voltage $t_p = \frac{u^*}{U_{dc}} \cdot \frac{1}{2\cdot f_{sw}} = \frac{100}{300\cdot 2\cdot 333} = 50\mu s$



Solution 1.6

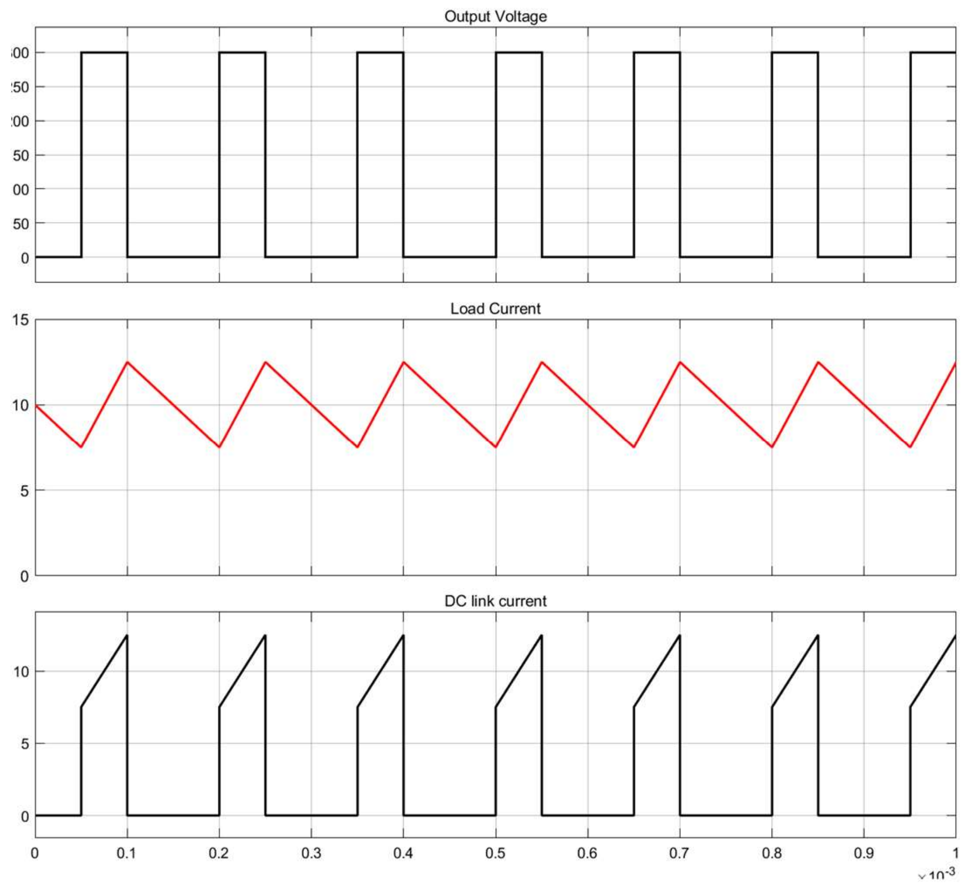
3) *Phase current*
   Average load current is zero (i_{ave}= 0), i.e., starting curre t is zero

   $$\frac{di}{dt} = \frac{u-e}{L} \rightarrow \Delta i = \frac{u-e}{L} \cdot t_p = \frac{300-100}{0.002} \cdot 50e^{-6} = 5A$$
   i.e., a current rippling between $10 \pm 2.5\ A$

4) DC current graph?
   The DC current, i_{dc}, equals the load current when u=U_{dc} and is zero when u=*0*.

5) Average Output voltage, $u_{ave} = 100\ V$
   Average Output current, $i_{ave} = 10\ A$
   Average Input voltage, $u_{dc} = 300\ V$
   Average Input current, $i_{dc,ave} = 10 \cdot \frac{t_p}{T} = 3.33\ A$

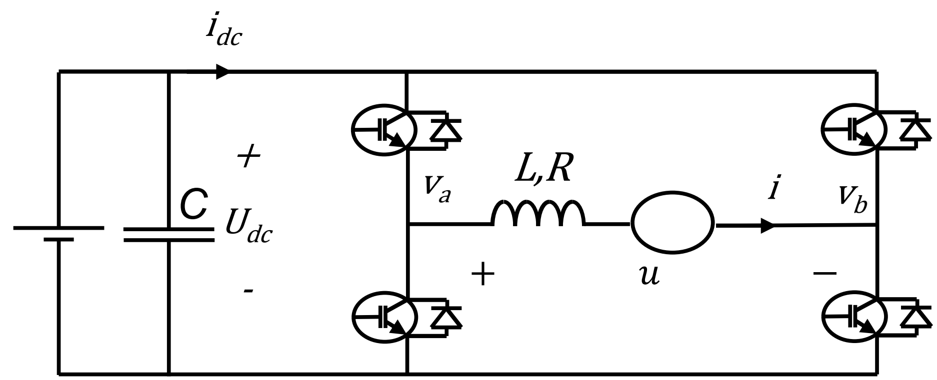
6) Average powers:
   $P_1 = u_{dc} \cdot i_{dc,ave}$= 300· 3.33 = 1000 W
   $P_2 = u_{ave} \cdot i_{ave}$= 100 · 10= 1000 W
   $P_3 = e \cdot i_{ave}$= 100 · 10 = 1000 W



Exercise 1.7 4QC Bridge converter

Data for the Bridge converter

U_{dc}	300 V
f_{sw}(switch- freq)	3.33 kHz



Determine

Draw the output potentials $(v_a \;\&\; v_b)$ *and Output voltage* (u) *with symmetric modulation and*

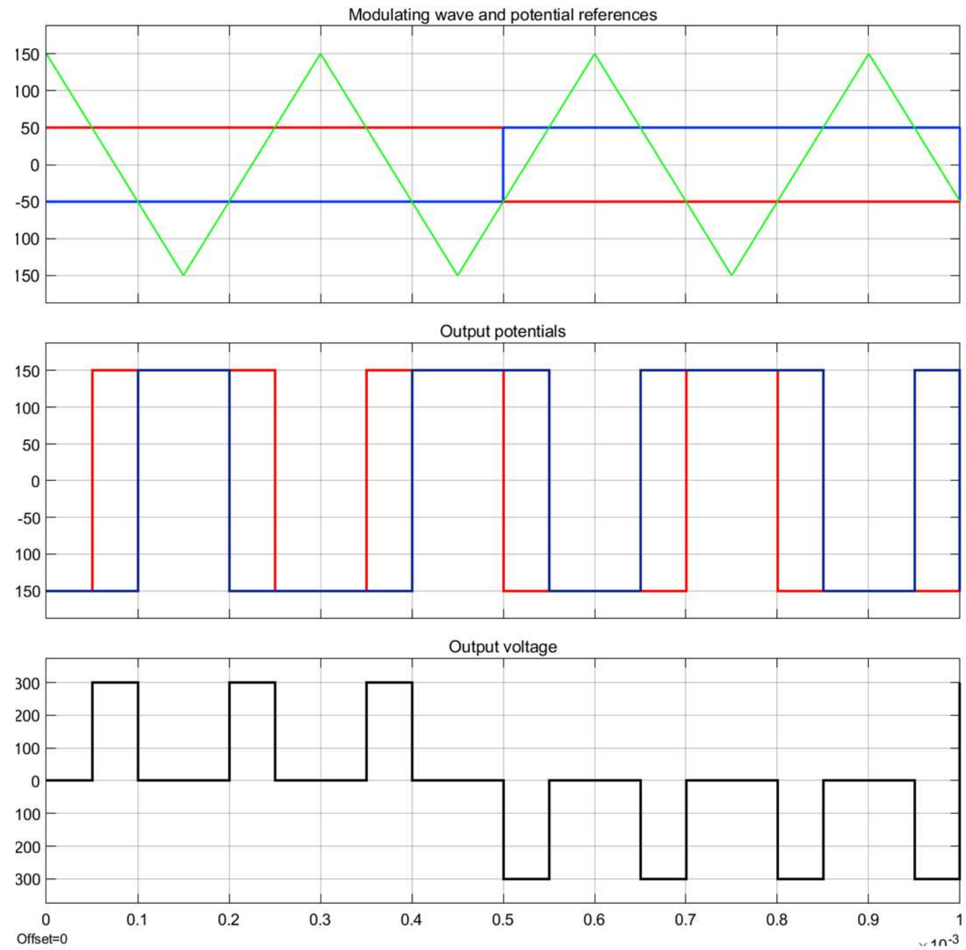
a) u^* *= 100 V,*

b) u^* *= -100 V*

Solution 1.7

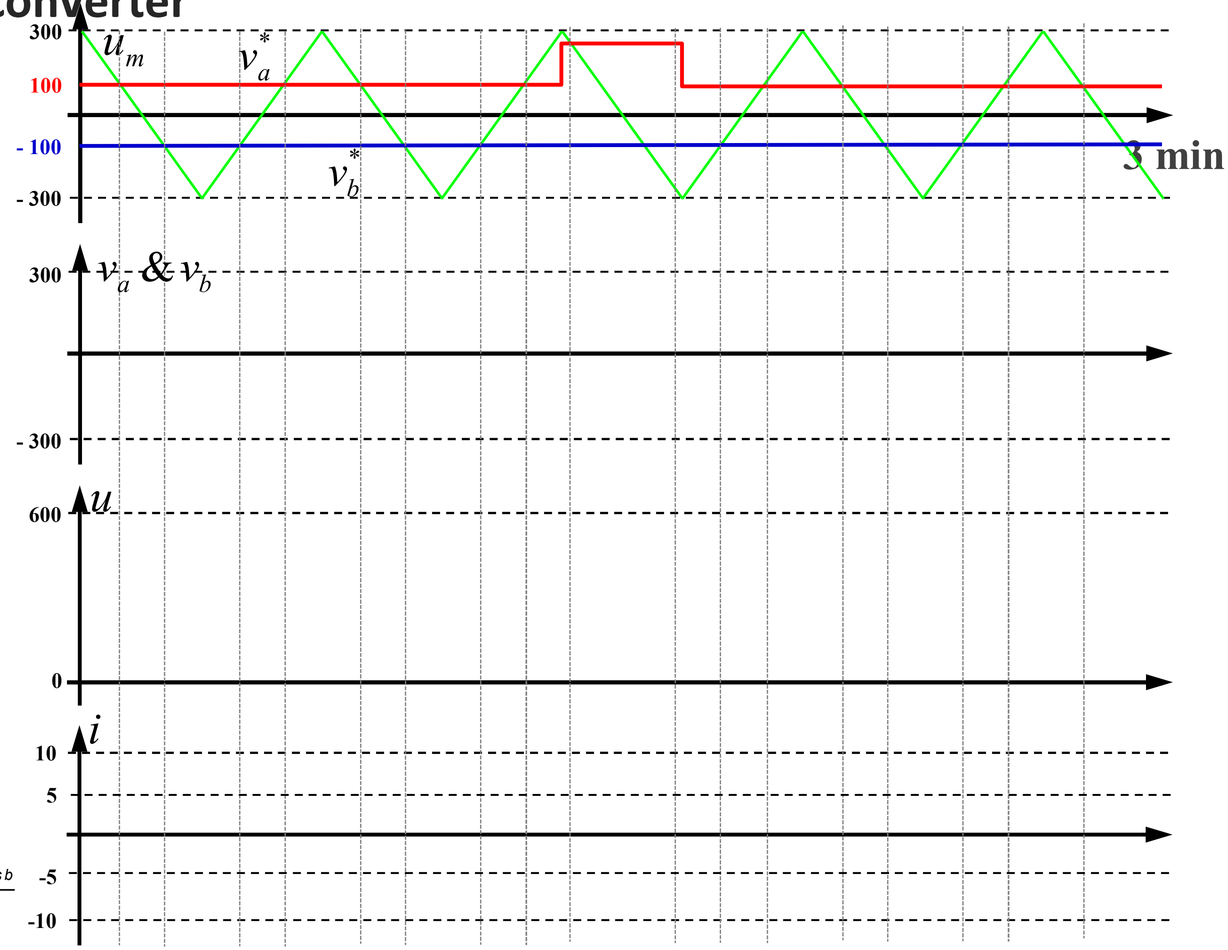
a) $u^* = 100\ V \rightarrow\ v_a^* = 50V\ \&\ v_b^* = -50V,$

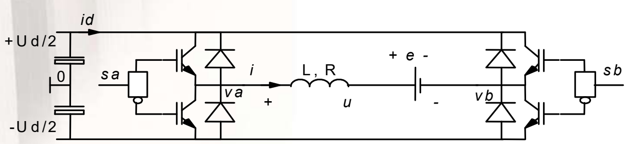
b) $u^* = -100\ V \rightarrow\ v_a^* = -50V\ \&\ v_b^* = 50V,$



1.8 : Modulation of a 4Q converter

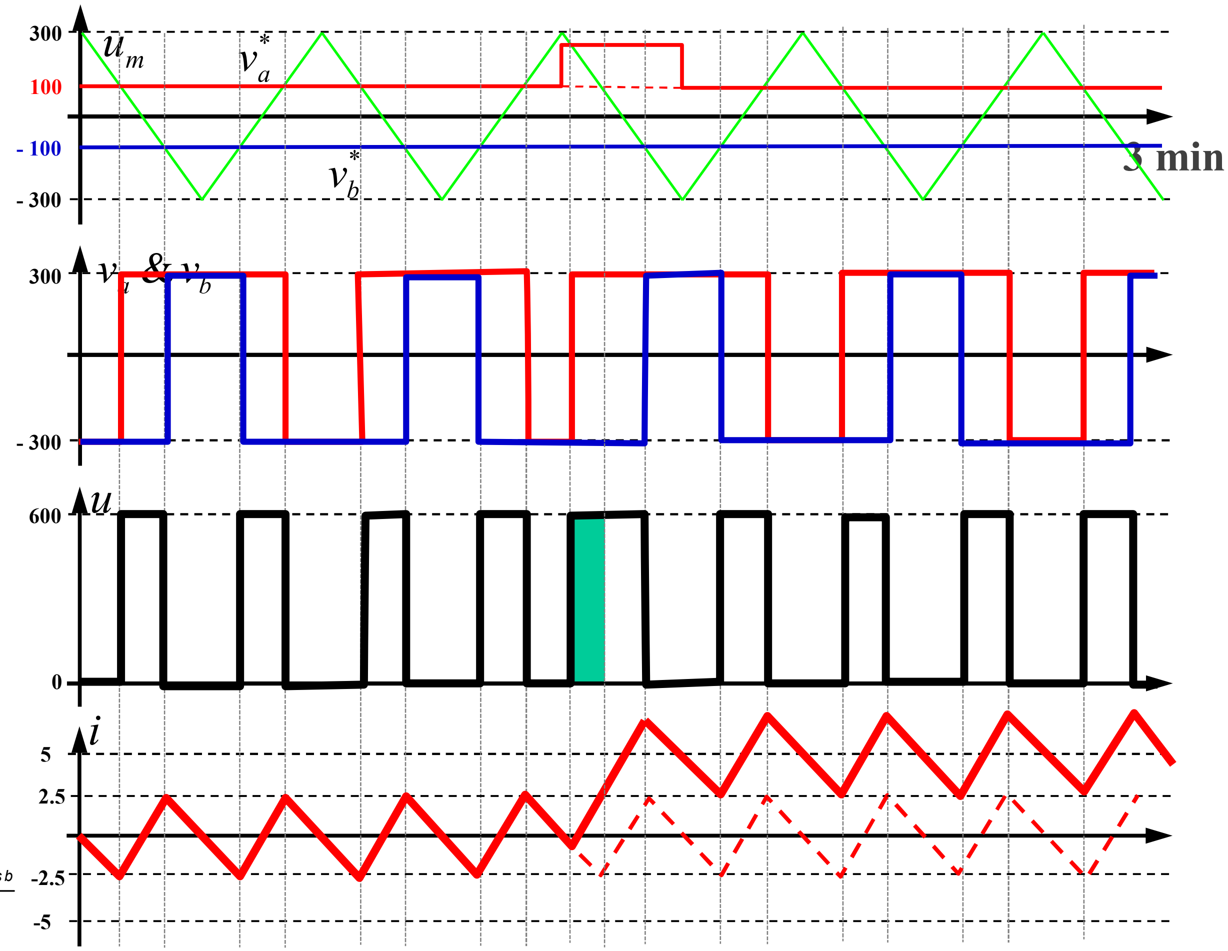
- **Given:**
  - *Udc = 600 V*
  - *e=200 V*
  - *i(t=0)=0*
  - *Voltage reference given*
- **Parameters:**
  - *L= 2 [mH]*
  - *Switchfrekvens: 6.67 [kHz]*
- **Draw:**
  - *Potentials v_a and v_b*
  - *Load voltage u*
- **Calculate**
  - *Positive current derivative*
  - *Negative current derivative*
- **Draw**
  - *Load current i*

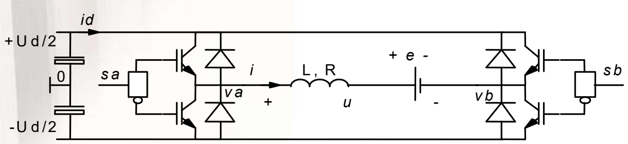




1.8 Solution

- Given:
  - *Udc = 600 V*
  - *e=200 V*
  - *i(t=0)=0*
  - *Voltage reference given*
- Parameters:
  - *L= 2 [mH]*
  - *Switchfrekvens: 6.67 [kHz]*
- Draw:
  - *Potentials v_a and v_b*
  - *Load voltage u*
- Calculate
  - *Positive current derivative*
  - *Negative current derivative*
- Draw
  - *Load current i*

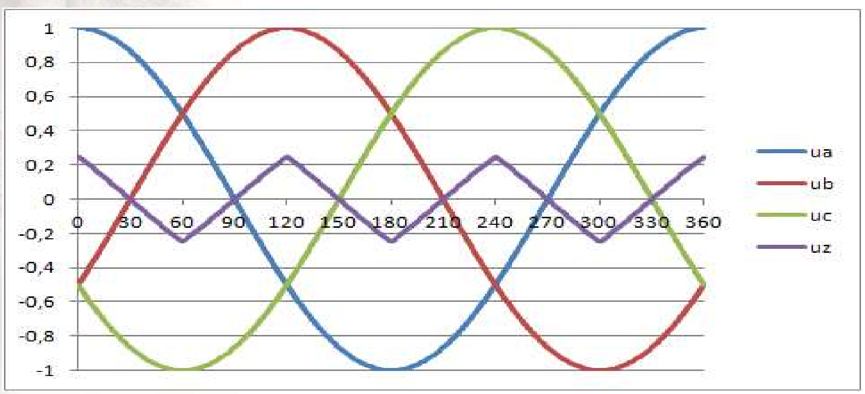


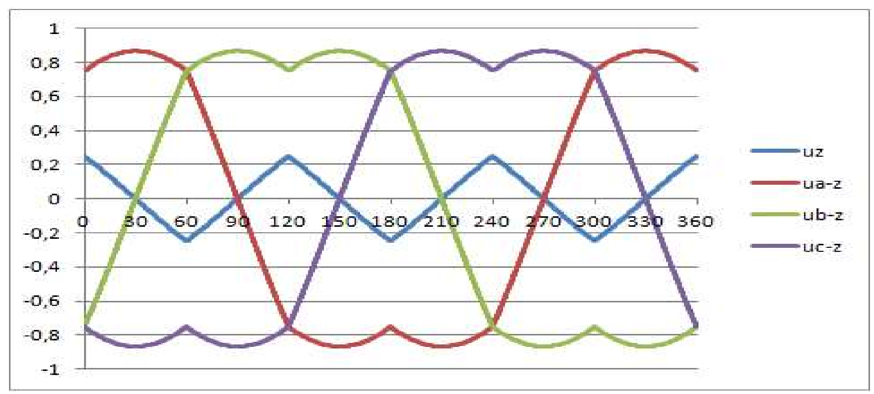


Exercise 1.9 Symmetrized 3phase voltage

- The sinusoidal reference curves (v_a*, v_b*, v_c*) for a three phase constant voltage converter can be modified with a zero-sequence signal:
  - *v_z* = [max(a, b, c)+min(a, b, c)]/2*

  according to the figure below.
- Determine the analytical expression for e.g. a-z in one of the 60˚ intervals!
- Determine the ratio between the maxima of the input (e.g. va*) and output signals (e.g. vaz*) !





Solution 1.9

The interval 0-60 deg is used (any such 60 degree can be used)
Find the maximum in this interval.

$$u_{az,0-60} = u_a - u_z = u_a - \left(\frac{u_a + u_c}{2}\right) = \frac{u_a}{2} - \frac{u_c}{2} = \frac{\cos(x)}{2} - \frac{\cos\left(x - \frac{4\pi}{3}\right)}{2}$$

$$\frac{du_{az,0-60}}{dx} = -\sin(x) + \sin\left(x - \frac{4\pi}{3}\right) = \sin(x) \cdot \cos\left(\frac{4\pi}{3}\right) - \cos(x) \cdot \sin\left(\frac{4\pi}{3}\right) - \sin(x) = -\frac{3}{2} \cdot \sin(x) + \frac{\sqrt{3}}{2} \cdot \cos(x)$$

$$\frac{du_{az,0-60}}{dx} = 0 \Rightarrow \frac{3}{2} \cdot \sin(x) = \frac{\sqrt{3}}{2} \cdot \cos(x) \Rightarrow \tan(x) = \frac{2 \cdot \sqrt{3}}{2 \cdot 3} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6} = 30^o$$

$$Check\ if\ \max \frac{d^2 u_{az,0-60}}{dx^2} = -\frac{3}{2} \cdot \cos(x) - \frac{\sqrt{3}}{2} \cdot \sin(x) = \left\{x = \frac{\pi}{6}\right\} = -\frac{3\sqrt{3}}{4} - \frac{\sqrt{3}}{4} < 0 \Rightarrow \max$$

$$u_{az,0-60}\left(\frac{\pi}{6}\right) = \frac{\cos\left(\frac{\pi}{6}\right)}{2} - \frac{\cos\left(\frac{\pi}{6} - \frac{4\pi}{3}\right)}{2} = \frac{\cos\left(\frac{\pi}{6}\right) - \cos\left(-\frac{7\pi}{6}\right)}{2} = \frac{\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{\sqrt{3}}{2} \approx 0.866$$

$$The\ ratio\ between\ the \rightleftarrows input\ and\ the\ output\ signals = \frac{1}{0.866} = 1.155$$

Exercise 1.10 Sinusoidal 3phase voltage

The following data is given for a 3-phase carrier wave modulated 2-level converter:

- Phase voltage reference amplitude: 350 V
- Phase voltage reference frequency: 50 Hz
- DC link voltage 700 VDC (+/- 350 V)
- Carrier wave frequency: 18 kHz

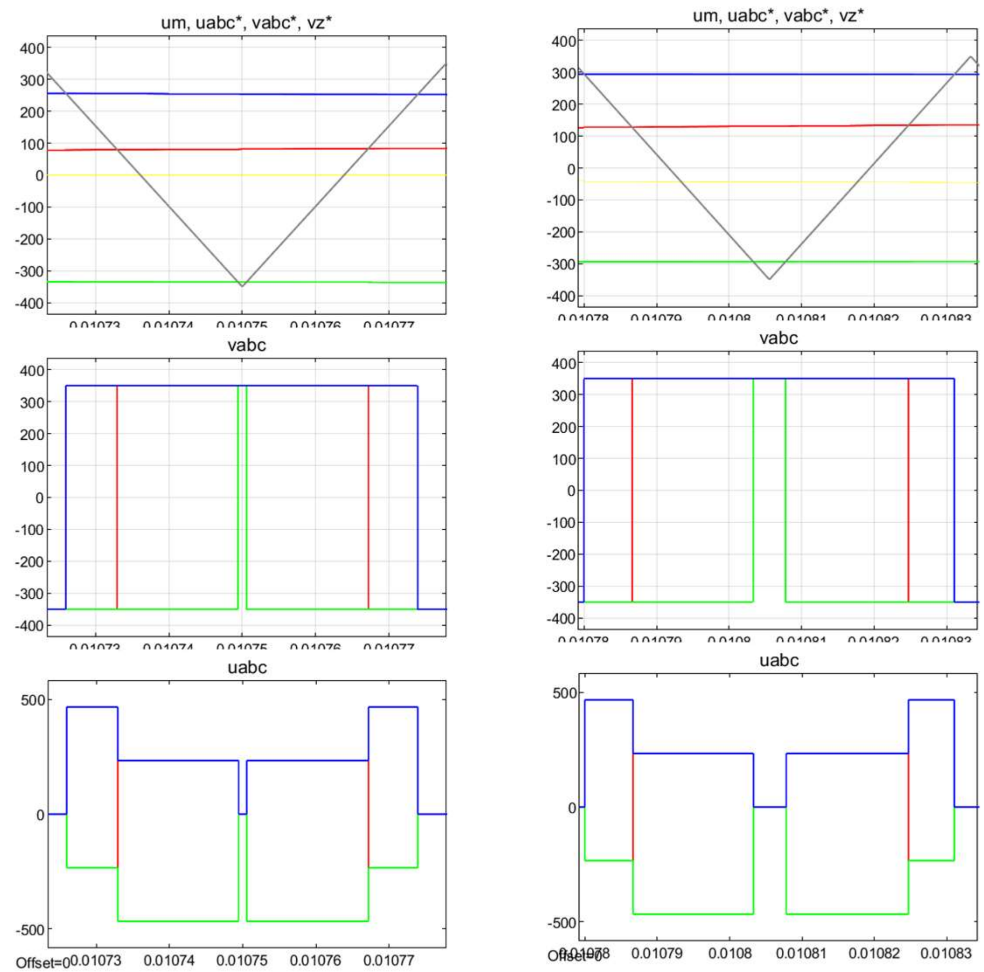
Calculate and draw the phase potential and phase voltage references together with the modulating wave for 1 carrier wave period @ phase angle 15 degrees into the positive half period of the sinusoidal phase *a*.

Solution 1.10

$u_a^* = 350*sin(15/180*pi) = 90\ V$
$u_b^* = 350*sin(15/180*pi-2*pi/3) = -338\ V$
$u_c^* = 350*sin(15/180*pi-4*pi/3) = 247\ V$

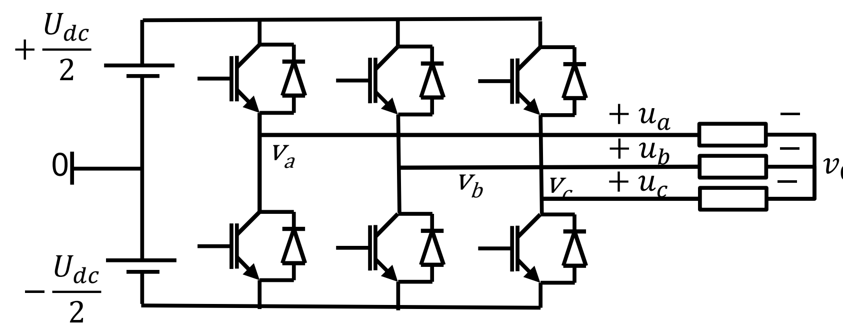
$v_z^* = (max+min)/2 = (247-338)/2 = -45\ V$

$v_a^* = ua^*-vz^* = 135\ V$
$v_b^* = ub^*-vz^* = -293\ V$
$v_c^* = uc^*-vz^* = 292\ V$

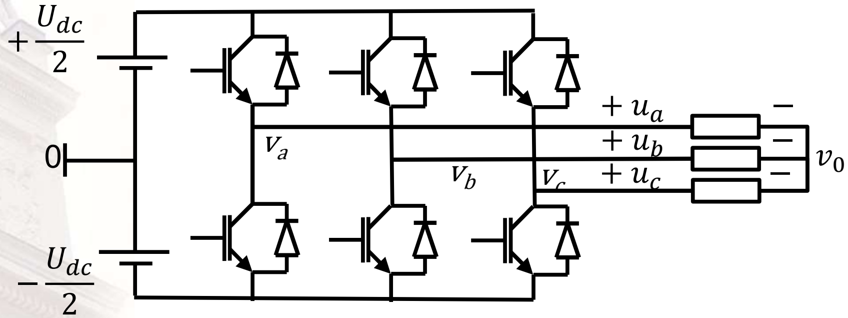


Exercise 1.11 Voltage vectors

Deduce the 8 voltage vectors that are created in a converter fed by a constant voltage!



Solution 1.11

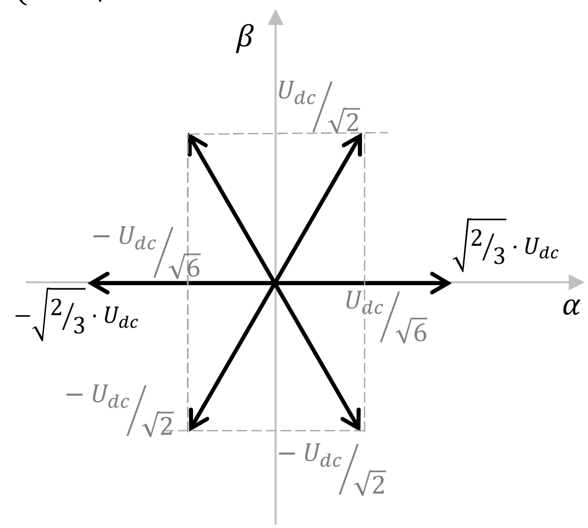


$$v_0 = \frac{v_a + v_b + v_c}{3}$$

$$\begin{cases} u_a = v_a - v_0 \\ u_b = v_b - v_0 \\ u_c = v_c - v_0 \end{cases}$$

$$\begin{cases} u_\alpha = \sqrt{3/2} \cdot u_a \\ u_\beta = \frac{1}{\sqrt{2}} \cdot (u_B - u_C) \end{cases}$$

Va	Vb	Vc	Vo	Ua	Ub	Uc	Ualfa	Ubeta
$-\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	0	0	0	0	0
$\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{6}$	$\frac{2\ U_{dc}}{3}$	$-\frac{U_{dc}}{3}$	$-\frac{U_{dc}}{3}$	$\sqrt{\frac{2}{3}}U_{dc}$	0
$\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$\frac{U_{dc}}{6}$	$\frac{U_{dc}}{3}$	$\frac{U_{dc}}{3}$	$-\frac{2\ U_{dc}}{3}$	$\frac{1}{\sqrt{6}}U_{dc}$	$\frac{1}{\sqrt{2}}U_{dc}$
$-\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{6}$	$-\frac{U_{dc}}{3}$	$\frac{2\ U_{dc}}{3}$	$-\frac{U_{dc}}{3}$	$-\frac{1}{\sqrt{6}}U_{dc}$	$\frac{1}{\sqrt{2}}U_{dc}$
$-\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$\frac{U_{dc}}{6}$	$-\frac{2\ U_{dc}}{3}$	$\frac{U_{dc}}{3}$	$\frac{U_{dc}}{3}$	$-\sqrt{\frac{2}{3}}U_{dc}$	0
$-\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{6}$	$-\frac{U_{dc}}{3}$	$-\frac{U_{dc}}{3}$	$\frac{2\ U_{dc}}{3}$	$-\frac{1}{\sqrt{6}}U_{dc}$	$-\frac{1}{\sqrt{2}}U_{dc}$
$\frac{U_{dc}}{2}$	$-\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$\frac{U_{dc}}{6}$	$\frac{U_{dc}}{3}$	$-\frac{2\ U_{dc}}{3}$	$\frac{U_{dc}}{3}$	$\frac{1}{\sqrt{6}}U_{dc}$	$-\frac{1}{\sqrt{2}}U_{dc}$
$\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	$\frac{U_{dc}}{2}$	0	0	0	0	0



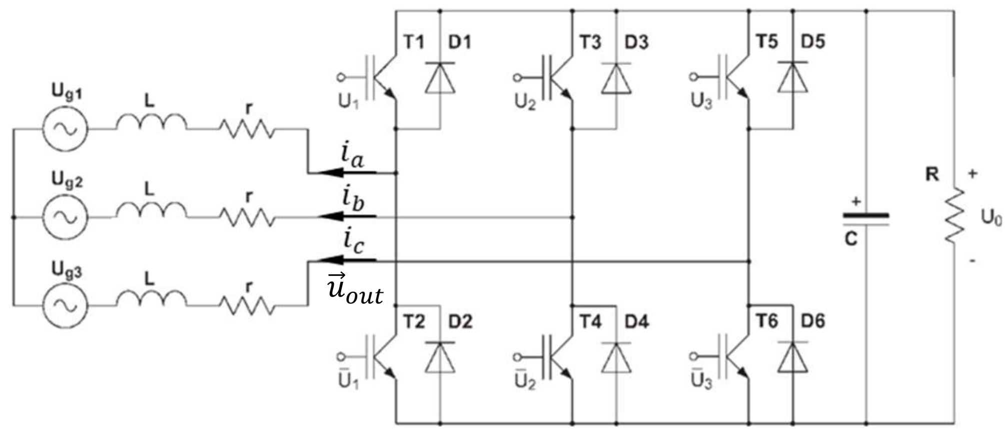
Exercise 1.12 i_D and i_Q in symmetric three phase

A three phase 2 level self commutated converter is connected to the three phase grid with net reactors. The fundamental current of from the converter to the grid is a symmetric 3-phase system:

$$i_a = \hat{\imath} \cdot cos(wt)$$

$$i_b = \hat{\imath} \cdot cos(wt\text{-}2*pi/3)$$

$$i_c = \hat{\imath} \cdot cos(wt\text{-}4*pi/3)$$



a) *Deduce the expressions for i_D and i_Q!*

b) *Determine the active and reactive power!*

c) *The DC voltage is U_{dc}. Determine the highest possible grid voltage relative to U_{dc}!*

d) *The same as c) BUT with zero current*

e) *The DC voltage is U_{dc}. Determine the highest possible grid voltage relative to U_{dc} that can be sustained at ANY grid phase angle.*

Solution 1.12a

$a) Symmetric\ 3-phase\ (a,b,c)-frame$

$$\begin{cases} i_{a(1)} = \hat{\imath}\cdot\cos(\omega\cdot t-\phi) \\ i_{b(1)} = \hat{\imath}\cdot\cos\left(\omega\cdot t-\frac{2\pi}{3}-\phi\right) \\ i_{c(1)} = \hat{\imath}\cdot\cos\left(\omega\cdot t-\frac{4\pi}{3}-\phi\right) \end{cases}$$

$Transformfrom\ (a,b,c)-frameto(\alpha,\beta)-frame$

$$i_{\alpha\beta} = \sqrt{\frac{2}{3}}\cdot\left(\hat{\imath}\cdot\cos(\omega\cdot t-\phi)\cdot e^{j0}+\hat{\imath}\cdot\cos\left(\omega\cdot t-\frac{2\pi}{3}-\phi\right)\cdot e^{j\frac{2\pi}{3}}+\hat{\imath}\cdot\cos\left(\omega\cdot t-\frac{4\pi}{3}-\phi\right)\cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \left\{\cos(x)=\frac{e^{jx}+e^{-jx}}{2}\right\} = \sqrt{\frac{2}{3}}\cdot\hat{\imath}\cdot\left(\frac{e^{j(\omega t-\phi)}+e^{-j(\omega t-j\phi)}}{2}+\frac{e^{j\left(\omega t\ \ \frac{2\pi}{3}-\phi\right)}+e^{-j\left(\omega t+\frac{2\pi}{3}-\phi\right)}}{2}\cdot e^{j\frac{2\pi}{3}}+\frac{e^{j\left(\omega t\ \ \frac{4\pi}{3}-\phi\right)}+e^{-j\left(\omega t+\frac{4\pi}{3}-\phi\right)}}{2}\cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\frac{\hat{\imath}}{2}\cdot\left(e^{j\omega t-j}\quad+e^{-j\omega t+}\quad+\left(e^{j\omega t-j\frac{2\pi}{3}-j\phi}+e^{-j\omega t+\ \frac{2\pi}{3}+j\phi}\right)\cdot e^{j\frac{2\pi}{3}}+\left(e^{j\omega t-j\frac{4\pi}{3}-j}\quad+e^{-j\omega t+j\frac{4\pi}{3}+j\phi}\right)\cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\frac{\hat{\imath}}{2}\cdot\left(e^{j\omega t}\quad+e^{-j\omega t+}\quad+e^{j\omega t-j\frac{2\pi}{3}-j\phi+j\frac{2\pi}{3}}+e^{-j\omega t+j\frac{2\pi}{3}+j\phi+j\frac{2\pi}{3}}+e^{j\omega t-j\frac{4\pi}{3}-j\phi+j\frac{4\pi}{3}}+e^{-j\omega t+j\frac{4\pi}{3}+j\phi+j\frac{4\pi}{3}}\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\frac{\hat{\imath}}{2}\cdot\left(e^{j\omega t-j\phi}+e^{-j\omega t+}\quad+e^{j\omega t-j\phi}+e^{-j\omega t+j\phi+j\frac{4\pi}{3}}+e^{j\omega}\quad+e^{-j\omega t+j\phi+j\frac{8\pi}{3}}\right) = \sqrt{\frac{2}{3}}\cdot\frac{\hat{x}}{2}\cdot\left(3\cdot e^{j\omega t-}\quad+e^{-j\omega t+j\phi}\cdot\left(1-\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\right) =$$

$$= \frac{3\sqrt{2}}{2\sqrt{3}}\cdot\hat{\imath}\cdot e^{j\omega}\quad = \sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot e^{j(\omega t-\phi)} = \sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot(\cos(\omega t-\phi)+j\cdot\sin(\omega t-\phi))$$

$$Flux\ coordinates\ i_{dq} = i_{\alpha\beta}\cdot e^{-j\left(\omega t\ \ \frac{\pi}{2}\right)} = \sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot e^{j\omega t-j\phi-j\omega t+j\frac{\pi}{2}} = \sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot e^{j\left(\frac{\pi}{2}-\phi\right)} = \sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot\left(\cos\left(\frac{\pi}{2}-\phi\right)+j\cdot\sin\left(\frac{\pi}{2}-\phi\right)\right)$$

$$\begin{cases} i_d = \sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot\cos\left(\frac{\pi}{2}-\phi\right) \\ i_q = \sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot\sin\left(\frac{\pi}{2}-\phi\right) \end{cases}$$

Solution 1.12 b

$$\begin{cases} i_d = \sqrt{\frac{3}{2}} \cdot \hat{\imath} \cdot \cos\left(\frac{\pi}{2} - \phi\right) \\ i_q = \sqrt{\frac{3}{2}} \cdot \hat{\imath} \cdot \sin\left(\frac{\pi}{2} - \phi\right) \end{cases}$$

$$The\ active\ power\ \ P = e_q \cdot i_q = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot \sqrt{\frac{3}{2}} \cdot \hat{\imath} \cdot \sin\left(\frac{\pi}{2} - \phi\right) = \frac{3}{2} \cdot \hat{e} \cdot \hat{\imath} \cdot \cos(\phi) = \sqrt{3} \cdot e_{p\_p\_rms} \cdot i_{p\_rms} \cdot \cos(\phi)$$

$$The\ reactive\ power\ Q = \sqrt{3} \cdot e_{p\_p\_rms} \cdot i_{p\_rms} \cdot \sin(\phi)$$

Note that index “p_p_rms” means Phase-To_Phase_RMS” = the Phase to phase RMS value

Solution 1.12 c

- **The highest output voltage vector length is:**

$$|\vec{u}_{out}| = \sqrt{\frac{2}{3}}U_{dc}$$

- **The grid voltage at the converter terminals, expressed in the dq-reference frame, is:**

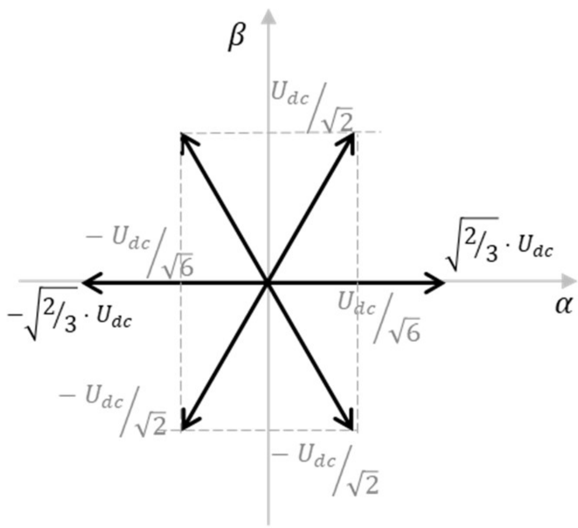
$$\vec{u}_{grid}^{dq} = R \cdot \vec{i}^{dq} + L \cdot \frac{d\vec{i}^{dq}}{dt} + j \cdot \omega \cdot L \cdot \vec{i}^{dq} + \vec{e}^{dq}$$

- **Which in stationarity is:**

$$\vec{u}_{grid}^{dq} = R \cdot \vec{i}^{dq} + j \cdot \omega \cdot L \cdot \vec{i}^{dq} + \vec{e}^{dq}$$

- **With the vector length, assuming zero resistance, is:**

$$\left|\vec{u}_{grid}^{dq}\right| = \sqrt{\left(j \cdot \omega \cdot L \cdot i_q\right)^2 + \left(j \cdot \omega \cdot L \cdot i_d + e_q\right)^2}$$

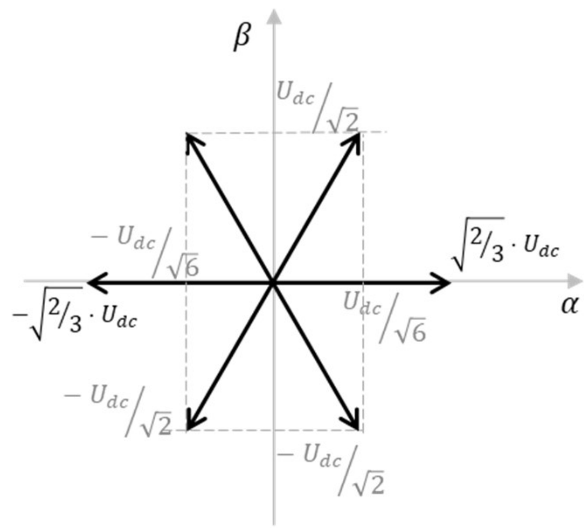


Solution 1.12 d

- **With zero grid current, the grid voltage at the converter terminal voltage is**

$$\left|\vec{u}_{grid}^{dq}\right| = \sqrt{\left(j \cdot \omega \cdot L \cdot i_q\right)^2 + \left(j \cdot \omega \cdot L \cdot i_d + e_q\right)^2} = e_q = E_{p\_p\_rms}$$

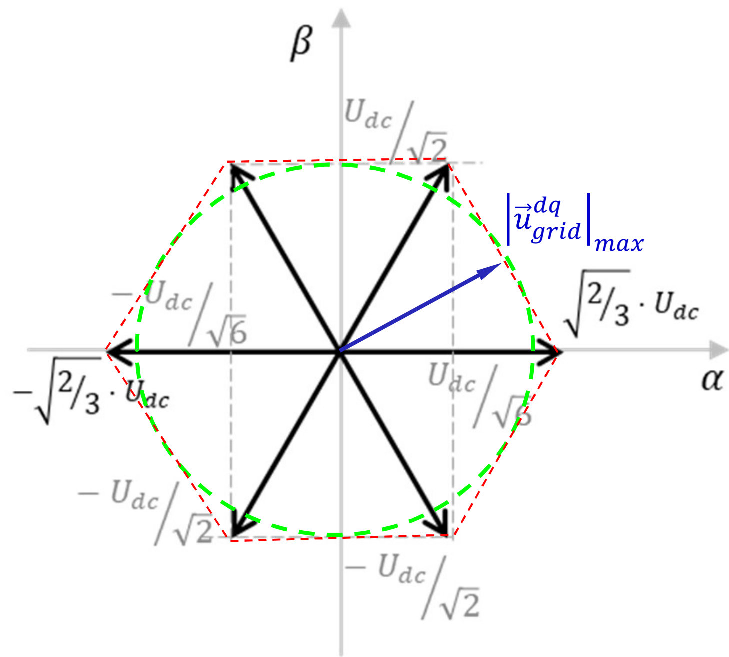
- **If there is a current flowing, the grid voltage at the converter terminal voltage must be higher than $\boldsymbol{E_{p\_p\_rms}}$, see solution to 1.11 c.**



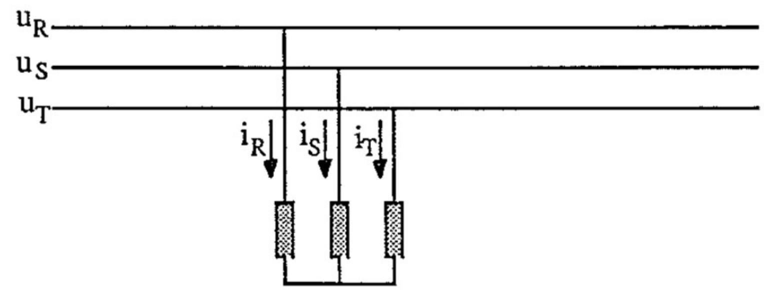
Solution 1.12 e

- **The longest vector that the converter can supply is $\sqrt{\frac{2}{3}}U_{dc}$, see the figure to the right, BUT …**
  - ***… that vector can ONLY be supplied in the corners of the hexagon.***
- **The maximum length of a voltage vector ($\left|\vec{u}_{grid}^{dq}\right|_{max}$) at the terminals of the converter, that can be supplied at ANY angle, must be shorter that a circle inscribed in the hexagon, see the figure to the right.**

$$\left|\vec{u}_{grid}^{dq}\right|_{max} = \sqrt{\frac{2}{3}}\boldsymbol{U_{dc}} \cdot \cos\left(\frac{\pi}{6}\right) = \sqrt{\frac{2}{3}}\boldsymbol{U_{dc}} \cdot \frac{\sqrt{3}}{2} = \frac{U_{dc}}{\sqrt{2}}$$



Exercise 1.13 3 phase line



Symmetric three phase

A three phase grid with the voltages u_R, u_S, u_T is loaded by sinusoidal currents i_R, i_S, i_T and the load angle φ.

a) Derive the expression for the voltage vector

b) Derive the expression for the current vector

c) Determine the active power p(t)!

Do the same derivations as above with the vectors expressed in the grid flux reference frame!

Solution 1.13a

$a)\ Equ\ 2.39, 2.40\ and\ exercise\ 4.26a$

$$\vec{u}_N = \sqrt{\frac{2}{3}} \cdot \left(u_R + u_S \cdot e^{j\frac{2\pi}{3}} + u_T \cdot e^{j\frac{4\pi}{3}}\right) = \sqrt{\frac{3}{2}} \cdot u_R + \frac{1}{\sqrt{2}} \cdot (u_S - u_T)$$

$$\left\{\begin{matrix} u_R = \hat{u} \cdot \cos(\omega \cdot t) \\ u_S = \hat{u} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \\ u_T = \hat{u} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \end{matrix}\right\} \Rightarrow \vec{u}_N = \sqrt{\frac{3}{2}} \cdot \hat{u} \cdot e^{j\omega t}$$

Solution 1.13b

$b)In\ the\ same\ way$

$$\vec{i}_N = \sqrt{\frac{2}{3}} \cdot \left(i_R + i_S \cdot e^{j\frac{2\pi}{3}} + i_T \cdot e^{j\frac{4\pi}{3}}\right) = \sqrt{\frac{3}{2}} \cdot i_R + \frac{1}{\sqrt{2}} \cdot (i_S - i_T)$$

$$\left.\begin{cases} i_R = \hat{\imath} \cdot \cos(\omega \cdot t - \phi) \\ i_S = \hat{\imath} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3} - \phi\right) \\ i_T = \hat{\imath} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3} - \phi\right) \end{cases}\right\} \Rightarrow \vec{i}_N = \sqrt{\frac{3}{2}} \cdot \hat{\imath} \cdot e^{j(\omega t - \phi)}$$

$The\ power\ in\ the\ dq-frame, the\ dot-product:$

$$P = e^{dq} \cdot i^{dq} = e_d \cdot i_d + e_q \cdot i_q$$

$$\left.\begin{cases} e_a = \hat{e} \cdot \cos(\omega \cdot t) \\ e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \\ e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \end{cases}\right\} \Rightarrow \vec{e}^{\alpha\beta} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t}$$

$Transform\ from \alpha\ \beta - frame\ to\ \ the\ dq-(flux)frame:$

$$\vec{e}^{dq} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t} \cdot e^{-j\left(\omega t\ \ \frac{\pi}{2}\right)} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t - j\omega t + \frac{\pi}{2}} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j \cdot \frac{\pi}{2}} = j \cdot \sqrt{\frac{3}{2}} \cdot \hat{e} = \vec{e}^{q}, \left(\vec{e}^{d} = 0\right)$$

$$\begin{cases} The\ active\ power\ \ P = e_q \cdot i_q = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot \sqrt{\frac{3}{2}} \cdot \hat{\imath} \cdot \sin\left(\frac{\pi}{2} - \phi\right) = \frac{3}{2} \cdot \hat{e} \cdot \hat{\imath} \cdot \cos(\phi) = \sqrt{3} \cdot e_{Heff} \cdot i_{eff} \cdot \cos(\phi) \\ The\ reactive\ power\ Q = \sqrt{3} \cdot e_{Heff} \cdot i_{eff} \cdot \sin(\phi) \end{cases}$$

Solution 1.13c

$$c)\begin{cases} Sinusoidal\ \text{mod}\,u\ lation U_{LNrms} = \dfrac{U_{dc}}{2\cdot\sqrt{2}} = \dfrac{U_{dc}}{\sqrt{8}} \approx 0.35\cdot U_{dc} \\ Sinusoidal\ \text{mod}\,u\ lation U_{LLrms} = \dfrac{\sqrt{3}}{\sqrt{2}}\cdot\dfrac{U_{dc}}{2} = \sqrt{\dfrac{3}{8}}\cdot U_{dc} \approx 0.61\cdot U_{dc} \\ Symmetriced\ \text{mod}\,u\ lation U_{LNrms} = \dfrac{U_{dc}}{\sqrt{3}\cdot\sqrt{2}} = \dfrac{U_{dc}}{\sqrt{6}} \approx 0.41\cdot U_{dc} \\ Symmetriced\ \text{mod}\,u\ lation U_{LLrms} = \dfrac{U_{dc}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{2}} = \dfrac{U_{dc}}{\sqrt{2}} \approx 0.71\cdot U_{dc} \end{cases}$$

$$Power\ in\ \alpha\beta - frame\ P(t) = \{P = \text{Re}(\vec{u}_N\cdot\vec{\imath}_N{}^*)\} = \text{Re}\left(\sqrt{\frac{3}{2}}\cdot\hat{u}\cdot e^{j\omega t}\cdot\sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot \underset{R_p}{e^{-j(\omega t-\phi)}}\right) = \frac{3}{2}\cdot\text{Re}\left(\hat{u}\cdot\hat{\imath}\cdot e^{j\omega t - j\omega t + j}\ \right) =$$

$$= \frac{3}{2}\cdot\text{Re}\left(\hat{u}\cdot\hat{\imath}\cdot e^{j\phi}\right) = \frac{3}{2}\cdot\hat{u}\cdot\hat{\imath}\cdot\cos(\phi) = \sqrt{3}\cdot U_{Heff}\cdot I_{eff}\cdot\cos(\phi)$$

$Influx \rightleftarrows\rightleftarrows$ órientation, flux is $\frac{\pi}{2}$ after voltage

$$\begin{cases} u^{dq} = u^{\alpha\beta}\cdot e^{-j\omega t+\ \frac{\pi}{2}} = \sqrt{\dfrac{3}{2}}\cdot\hat{u}\cdot e^{j\omega t}\cdot e^{-j\omega t+j\frac{\pi}{2}} = \sqrt{\dfrac{3}{2}}\cdot\hat{u}\cdot e^{j\omega t-j\omega t+j\frac{\pi}{2}} = \sqrt{\dfrac{3}{2}}\cdot\hat{u}\cdot e^{j\frac{\pi}{2}} \\ i^{dq} = i^{\alpha\beta}\cdot e^{-j\omega t+j\frac{\pi}{2}} = \sqrt{\dfrac{3}{2}}\cdot\hat{\imath}\cdot e^{j(\omega t-\phi)}\cdot e^{-j\omega t\quad\frac{\pi}{2}} = \sqrt{\dfrac{3}{2}}\cdot\hat{\imath}\cdot e^{j\omega t-j\phi-j\omega t+j\frac{\pi}{2}} = \sqrt{\dfrac{3}{2}}\cdot\hat{\imath}\cdot e^{j\frac{\pi}{2}-j} \end{cases}$$

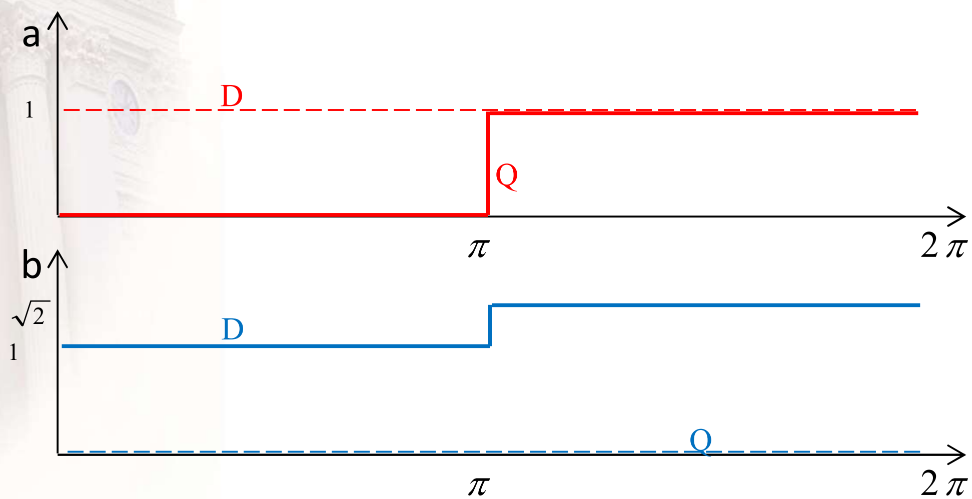
$$P(t) = \text{Re}\left(u^{dq}\cdot i^{dq^*}\right) = \text{Re}\left(\sqrt{\frac{3}{2}}\cdot\hat{u}\cdot e^{j\frac{\pi}{2}}\cdot\sqrt{\frac{3}{2}}\cdot\hat{\imath}\cdot e^{-\left(j\frac{\pi}{2}-j\phi\right)}\right) = \frac{3}{2}\cdot\text{Re}\left(\hat{u}\cdot\hat{\imath}\cdot e^{j\frac{\pi}{2}-j\frac{\pi}{2}+j\phi}\right) =$$

$$= \frac{3}{2}\cdot\text{Re}\left(\hat{u}\cdot\hat{\imath}\cdot e^{j\phi}\right) = \frac{3}{2}\cdot\hat{u}\cdot\hat{\imath}\cdot\cos(\phi) = \sqrt{3}\cdot U_{Heff}\cdot I_{eff}\cdot\cos(\phi)$$

Exercise 1.14 Symmetric 3-phase transformation

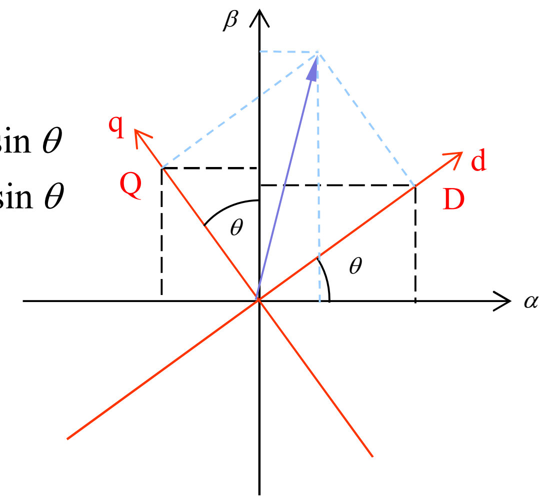
Symmetric three phase

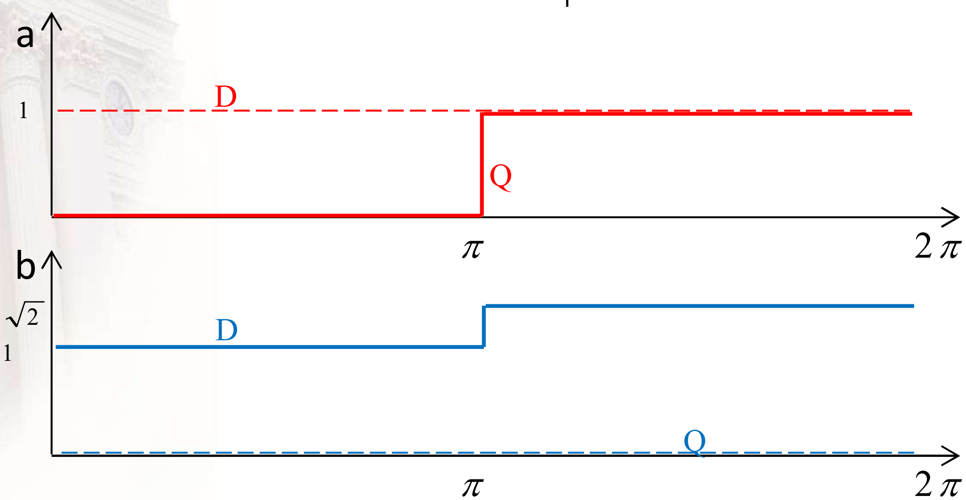
Do the inverse coordinate transformation from the (d,q) reference frame to (α, β) reference frame and the two phase to three phase transformation as well. Express the equations in component form.
Apply the coordinate transform on the following signals.



Solution 1.14

$$\begin{cases} \alpha = D \cdot \cos\theta - Q \cdot \sin\theta \\ \beta = Q \cdot \cos\theta + D \cdot \sin\theta \end{cases}$$





$0 < \theta < \pi$: $\alpha = \cos\theta$, $\beta = \sin\theta$

$\pi < \theta < 2\pi$: $\alpha = \cos\theta - \sin\theta$, $\beta = \cos\theta + \sin\theta$

$$\begin{cases} 0 < \theta < \pi\alpha = \cos\theta, \beta = \sin\theta \\ \pi < \theta < 2\pi\alpha = \sqrt{2}\cos\theta, \beta = \sqrt{2}\sin\theta \end{cases}$$

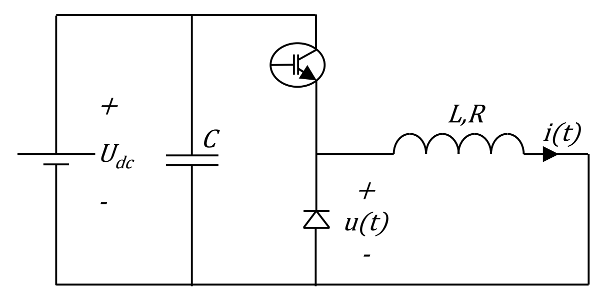
2

Exercises on Current Control

Exercise 2.1 Current increase

a. **The coil in the figure to the right has the inductance L and negligible resistance. It has no current when $t<0$. The current shall be increased to the value $i_1=0,1\ U_{dc}/L$ in the shortest possible time.**
Determine the voltage $u(t)$ and the current $i(t)$ for t>0!

b. **The switch s is operated with the period time $T=1ms$. The time constant of the coil is $L/R=10T$. The average of the current is $0,1\ U_{dc}/R$. Determine the voltage $u(t)$ and the current *i(t)*!**

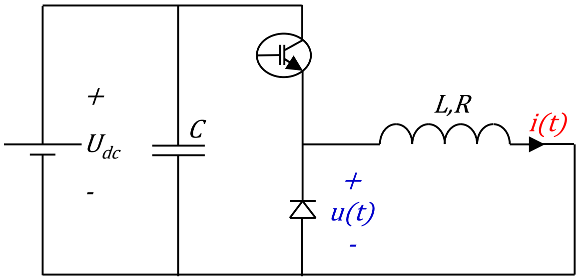


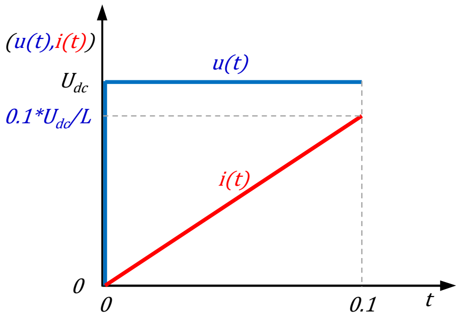
Solution 2.1

a) R=0

Calculate the shortest time for the current i to reach $0.1 \cdot U_{dc}/L$

$$\Delta t = \frac{L \cdot \Delta i}{U_{dc}} = \frac{L \cdot 0.1 \cdot U_{dc}}{U_{dc} \cdot L} = 0.1 \text{ sec}$$





Solution 2.1 Continued

b)

Average current $i_{avg} = 0.1 \cdot \frac{U_{dc}}{R}$

Average voltage $u_{avg} = R \cdot i_{avg} = R \cdot 0.1 \cdot \frac{U_{dc}}{R} = 0.1 \cdot U_{dc}$

Duty cycle $D = \frac{u_{avg}}{U_d} = \frac{0.1 \cdot U_{dc}}{U_{dc}} = 0.1$

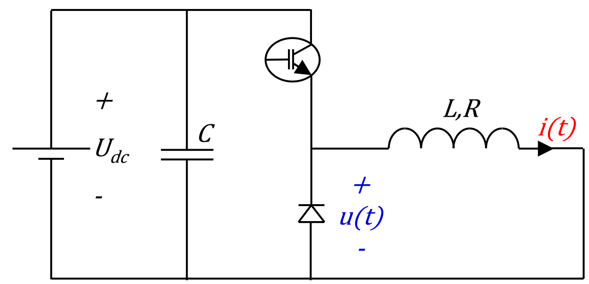
Period time $T = 1\ ms$

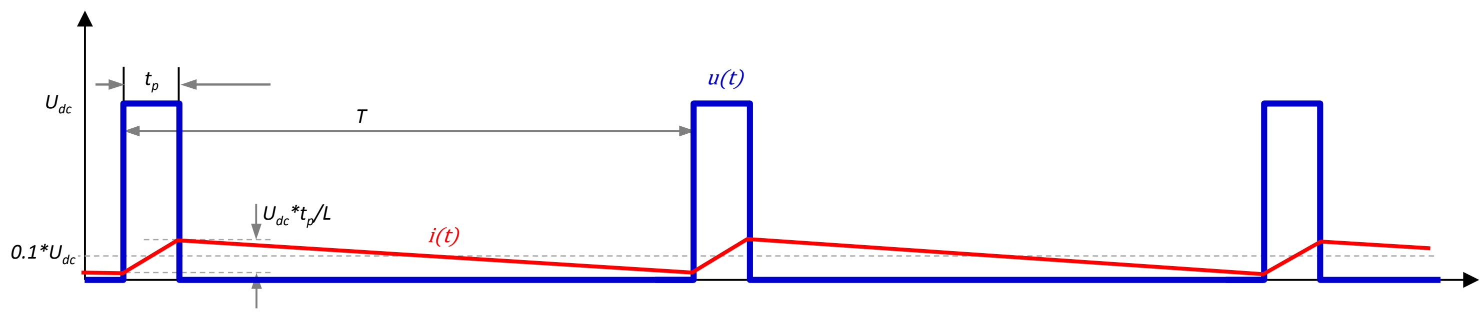
Time constant $\tau = \frac{L}{R} = 10 \cdot T = 10\ ms$

Voltage pulse time $t_p = D \cdot T = 0.1\ ms$

$$R \neq 0 \rightarrow i(t) = \frac{U_{dc}}{R} \cdot \left(1 - e^{-\frac{t}{\tau}}\right) = \{t << \tau\} \approx \frac{U_{dc}}{R} \cdot \left(1 - 1 + \frac{t}{\tau}\right) = \frac{U_{dc} \cdot D \cdot T}{R \cdot \tau} = \frac{U_{dc} \cdot t_p}{R \cdot \frac{L}{R}} = \frac{U_{dc} \cdot t_p}{L}$$

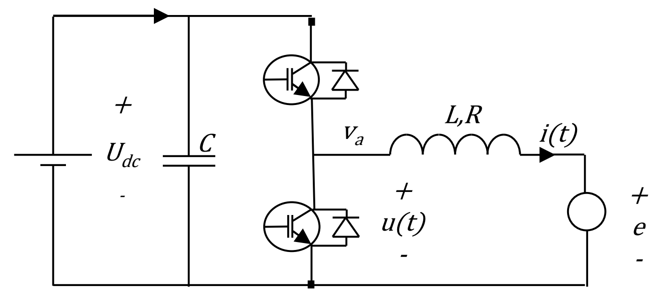
$$\begin{cases} i(t)_{start} = 0.1 \cdot \frac{U_{dc}}{R} - \frac{U_{dc} \cdot t_p}{2 \cdot L} \\ i(t)_{end} = 0.1 \cdot \frac{U_{dc}}{R} + \frac{U_{dc} \cdot t_p}{2 \cdot L} \end{cases}$$





2.2 2Q Current Control without load resistance

- **A 2 quadrant DC converter with a constant voltage load has the following data:**
  - U_{dc} = 600 V
  - L = 1 mH
  - R = 0
  - T_s = 0.1 ms
  - E = 200 V
- **Calculate and draw the output voltage patterns before, during and after a current step from 0 to 50 A and then back to 0 A again a few modulation periods after the positive step.**



2.2 Solution

- **Calculation steps:**
  1. *Calculate the voltage reference before the positive step, between the steps and after the negative step*
  2. *Calculate how many sampling periods that are needed for the positive and negative steps*
  3. *Calculate the current derivative and ripple*
  4. *Draw the waveform*

2.2 Solution, continued

- **Step 1**

$$u^*(k) = \frac{L}{T_s} \cdot (i^*(k) - i(k)) + e = \begin{cases} e = 200\text{ V } \mathit{before} \text{ the positive step} \\ \frac{1 \cdot 10^{-3}}{0.1 \cdot 10^{-3}} \cdot (50 - 0) + 200 = 700\text{ V } \mathit{during} \text{ the positive step} \\ e = 200\text{ V } \mathit{between} \text{ the steps} \\ \frac{1 \cdot 10^{-3}}{0.1 \cdot 10^{-3}} \cdot (0 - 50) + 200 = -300\text{ V } \mathit{during} \text{ the negative step} \\ e = 200\text{ V } \mathit{after} \text{ the negative step} \end{cases}$$

- **Step 2**
  - *The positive step requires 200+500V = 700V (back-emf+current increase), but the DC link only provides 600 V, i.e two sampling periods are needed, one with 200+400V and one with 200+100 V.*
  - *The negative step requires 200-500V = -300V (back-emf+current decrease), but the DC link only provides 0 V, i.e three sampling periods are needed, two with 200-200V and one with 200-100 V.*

2.2 Solution, continued

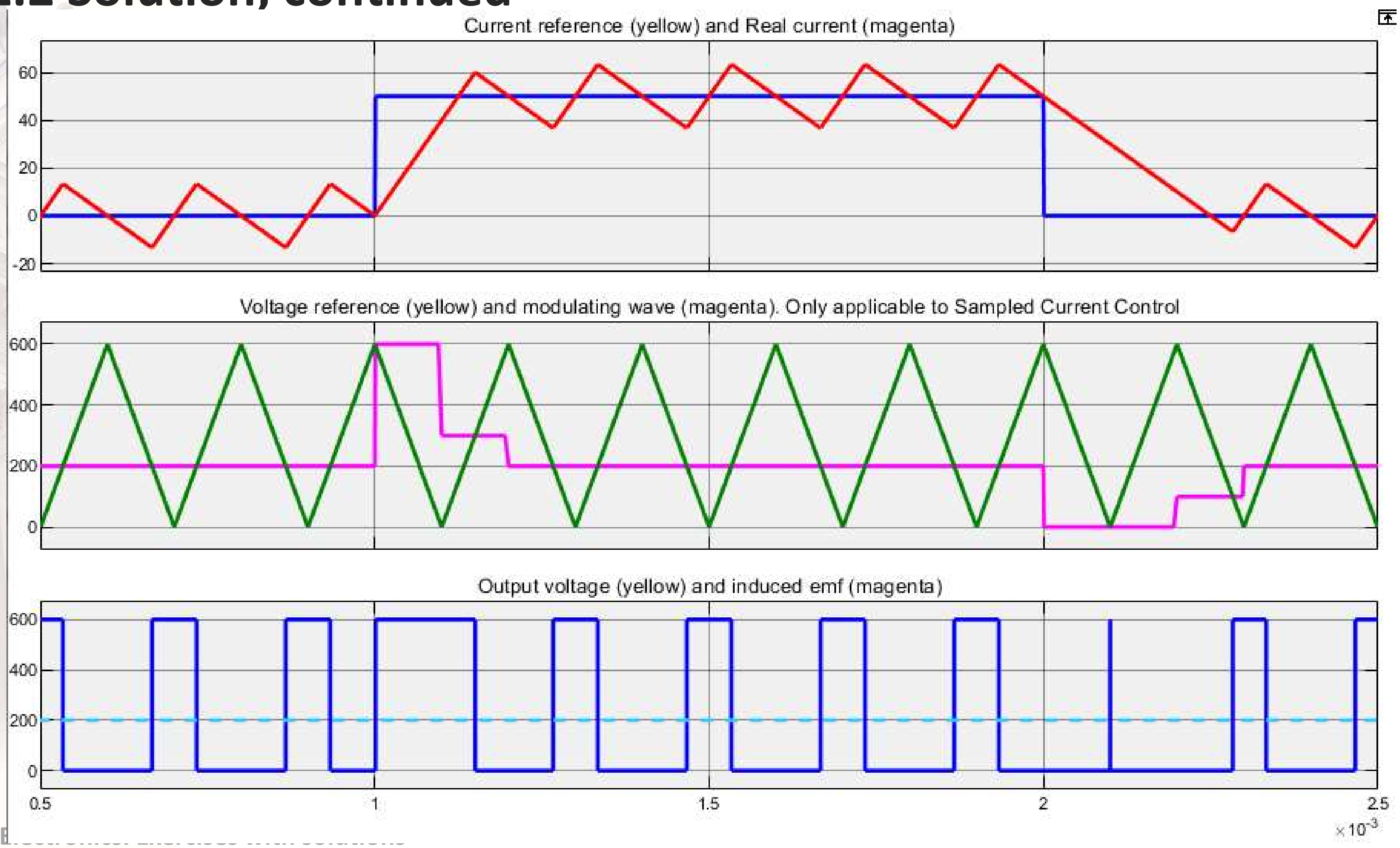
- **Step 3**

$$\left.\begin{aligned} \frac{di}{dt} &= \frac{u_L}{L} \\ t_{pulse} &= \frac{e}{U_d} \cdot T_s \end{aligned}\right\} \rightarrow \Delta i = \frac{u_L}{L} \cdot t_{pulse} = \frac{(U_d - e)}{L} \cdot \frac{e}{U_d} \cdot T_s = \frac{(600 - 200)}{1 \cdot 10^{-3}} \cdot \frac{200}{600} \cdot 0.1 \cdot 10^{-3}$$

$$= 13.3\ \text{A}$$

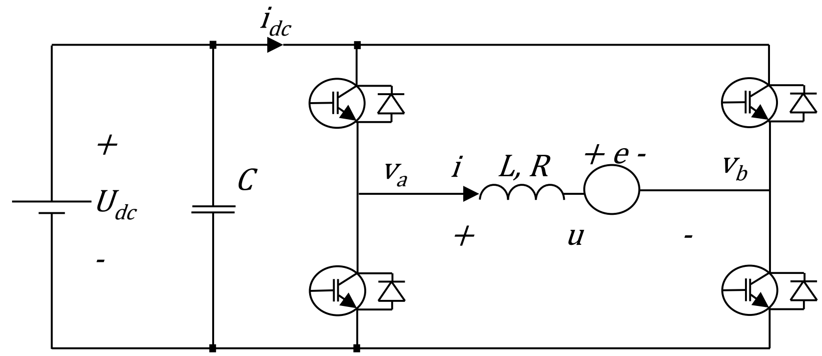
- **Step 4**
  - *Draw the carrier wave and the voltage reference wave as calculated. This gives the switching times*
  - *Note the time instants when the current will pass its reference values = when the carrier wave turns*
  - *See next page*

2.2 Solution, continued



2.3 4Q Current Control without load resistance

- **A 4 quadrant DC converter with a constant voltage load has the following data:**
  - $U_{dc} = 600\ V$
  - $L = 1\ mH$
  - $R = 0$
  - $T_s = 0.1\ ms$
  - $E = 200\ V$
- **Calculate and draw the output voltage patterns before, during and after a current step from 0 to 50 A and then back to 0 A again a few modulation periods after the positive step.**



2.3 Solution

- **Calculation steps:**
  1. *Calculate the voltage reference before the positive step, between the steps and after the negative step*
  2. *Calculate how many sampling periods that are needed for the positive and negative steps*
  3. *Calculate the current derivative and ripple*
  4. *Draw the waveform*

2.3 Solution, continued

- **Step 1**

$$u^*(k) = \frac{L}{T_s} \cdot (i^*(k) - i(k)) + e = \begin{cases} e = 200\text{ V } \mathit{before} \text{ the positive step} \\ \frac{1 \cdot 10^{-3}}{0.1 \cdot 10^{-3}} \cdot (50 - 0) + 200 = 700\text{ V } \mathit{during} \text{ the positive step} \\ e = 200\text{ V } \mathit{between} \text{ the steps} \\ \frac{1 \cdot 10^{-3}}{0.1 \cdot 10^{-3}} \cdot (0 - 50) + 200 = -300\text{ V } \mathit{during} \text{ the negative step} \\ e = 200\text{ V } \mathit{after} \text{ the negative step} \end{cases}$$

- **Step 2**
  - *The positive step requires 200+500V=700V (back-emf+current increase) = +/- 350 V, but the DC link only provides +/-300 V, i.e two sampling periods are needed, one with 200+400V = +/-300 V and one with 200+100 V=+/-150V.*
  - *The negative step requires 200-500V = -300V (back-emf+current decrease) = -/+150. The DC link provides down to -300 V, i.e 1 sampling periods is enough*

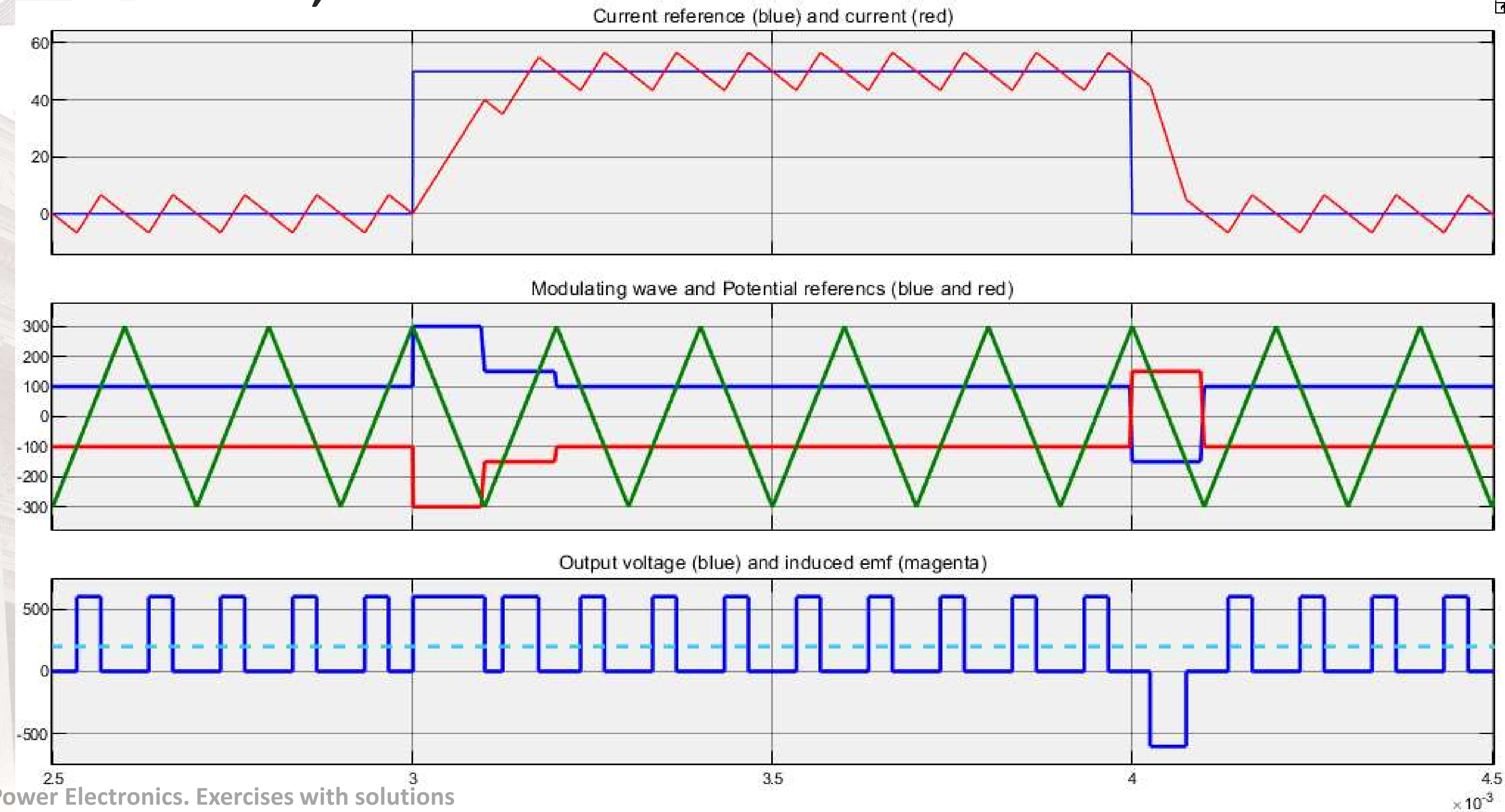
2.3 Solution, continued

- **Step 3**

$$\left.\begin{aligned}\frac{di}{dt} &= \frac{u_L}{L}\\ t_{pulse} &= \frac{e}{U_d}\cdot T_s\end{aligned}\right\} \rightarrow \Delta i = \frac{u_L}{L}\cdot t_{pulse} = \frac{(U_d - e)}{L}\cdot\frac{e}{U_d}\cdot T_s = \frac{(600-200)}{1\cdot 10^{-3}}\cdot\frac{200}{600}\cdot 0.1\cdot 10^{-3} = 13.3\ \text{A}$$

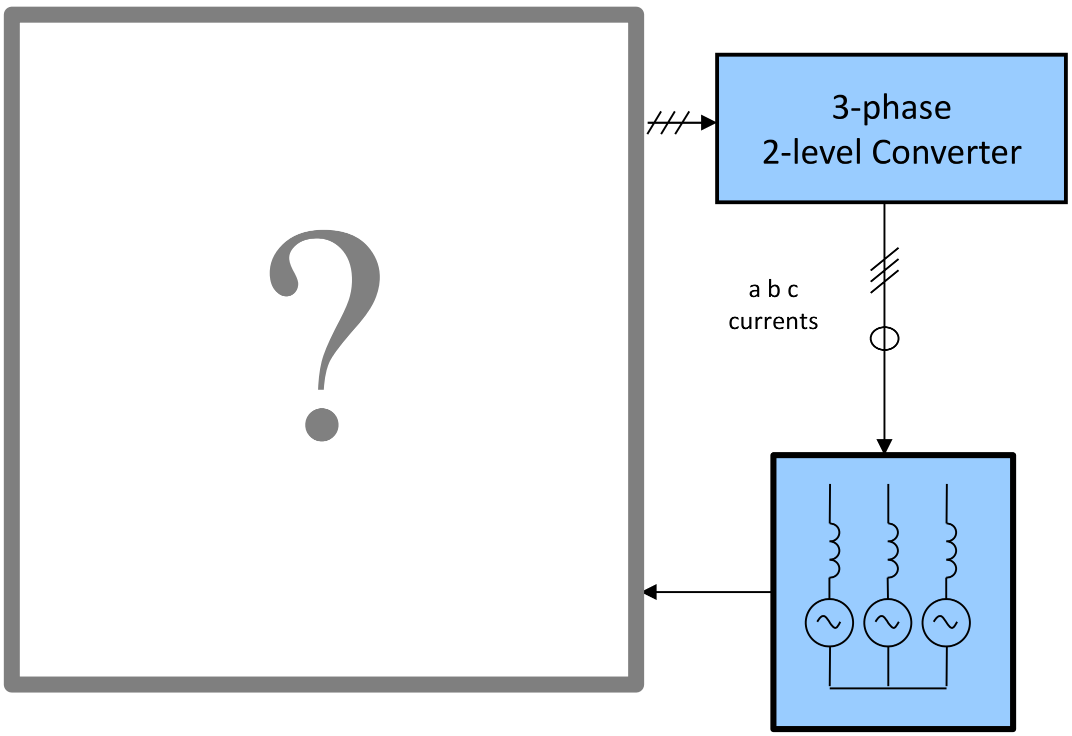
- **Step 4**
  - *Draw the carrier wave and the voltage reference wave as calculated. This gives the switching times*
  - *Note the time instants when the current will pass its reference values = when the carrier wave turns*
  - *See next page*

2.3 Solution, continued

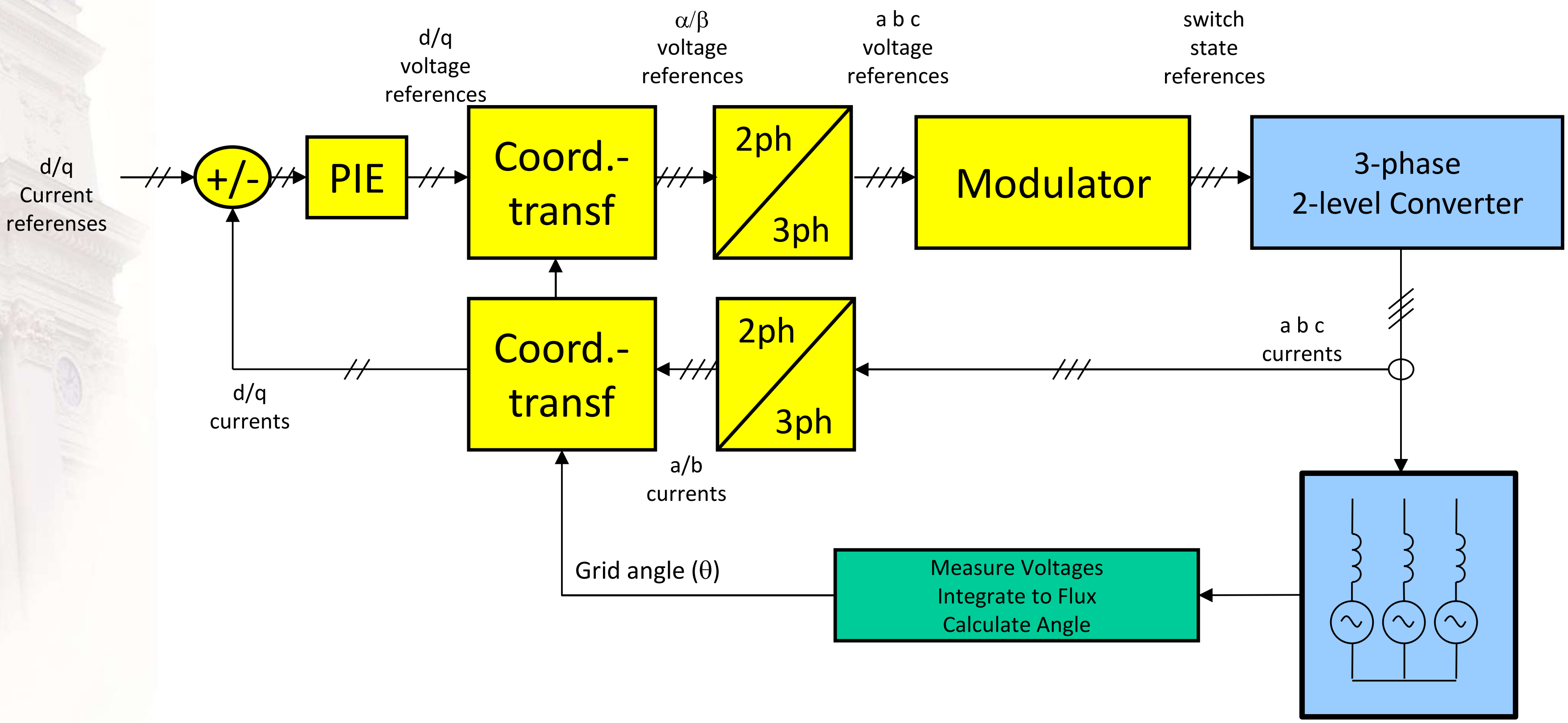


2.4. Three Phase Current Control

- Draw a block diagram with the controller structure for a 3 phase vector current controller with PI control and modulation. All transformations between 3-phase and 2 phase as well as coordinate transformations must be included.



2.4 Solution


d/q
Current
referenses
+/-
PIE
d/q
voltage
references
Coord.-
transf
α/β
voltage
references
2ph
3ph
a b c
voltage
references
Modulator
switch
state
references
3-phase
2-level Converter
d/q
currents
Coord.-
transf
2ph
3ph
a/b
currents
a b c
currents
Grid angle (θ)
Measure Voltages
Integrate to Flux
Calculate Angle

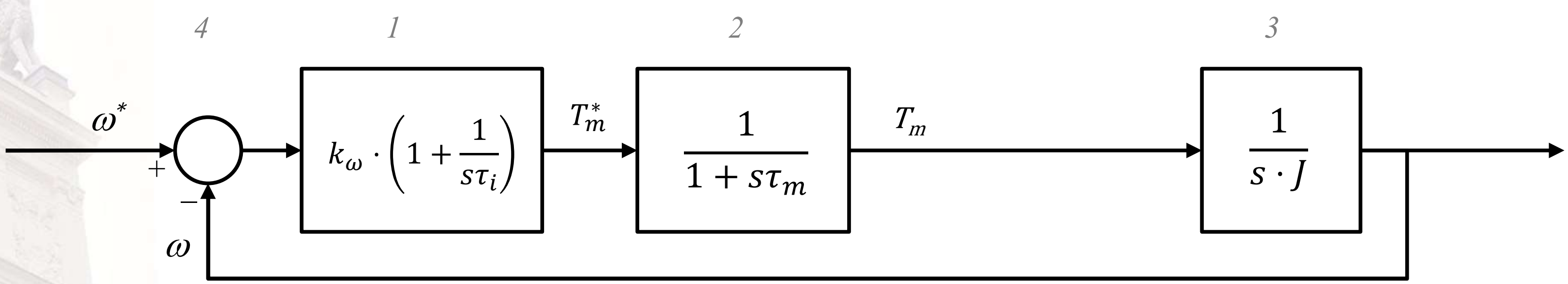
3

Exercises on Speed Control

Exercise 3.1 Cascade control

- **The speed of a motor shaft shall be controlled by so called cascade control. The torque source is modelled by a first order time constant.**
  - *Draw a block diagram of the system with speed control, torque source model and inertia*
  - *Include the load torque in the block diagram.*
  - *How large is the stationary error with a P-controller and constant load torque?*
  - *Show two different ways to eliminate the stationary fault.*

Solution 3.1a



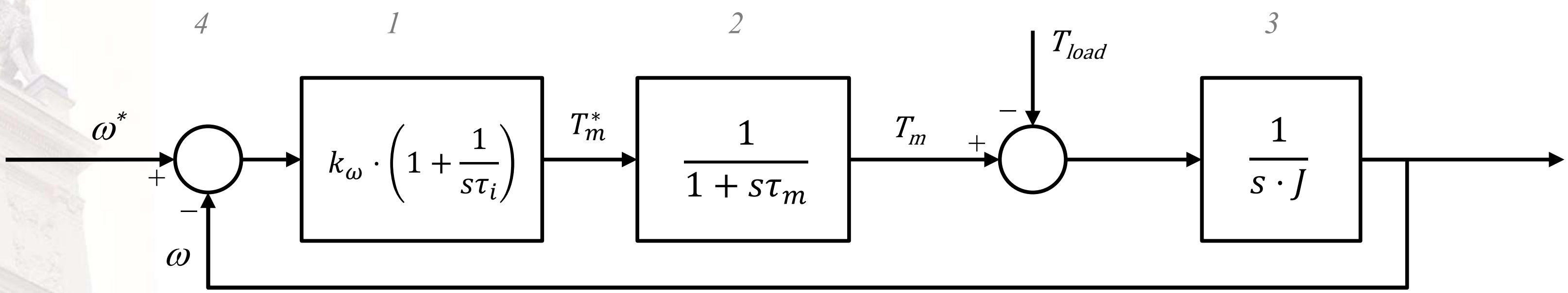
1 *The PI-controller with the gain k_ω*

2 *The torque controller is modelled as a first order low pass filter*

3 *By dividing the torque with the inertia J the angular acceleration is achieved. By integration, in the LaPlace plane, the angular speed is achieved.*

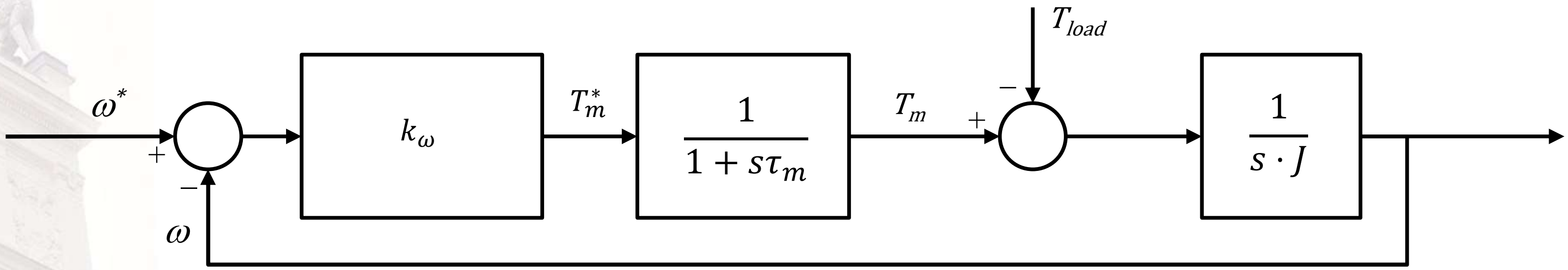
4 *By subtracting the angular speed from its reference the control error is achieved*

Solution 3.1b



See equation 9.1. The load torque is subtracted from the achieved electric torque at the output of the torque controller.

Solution 3.1c



$$\omega = (\omega^* - \omega)\cdot k_\omega \cdot \frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} - \frac{T_{load}}{s\cdot J}$$

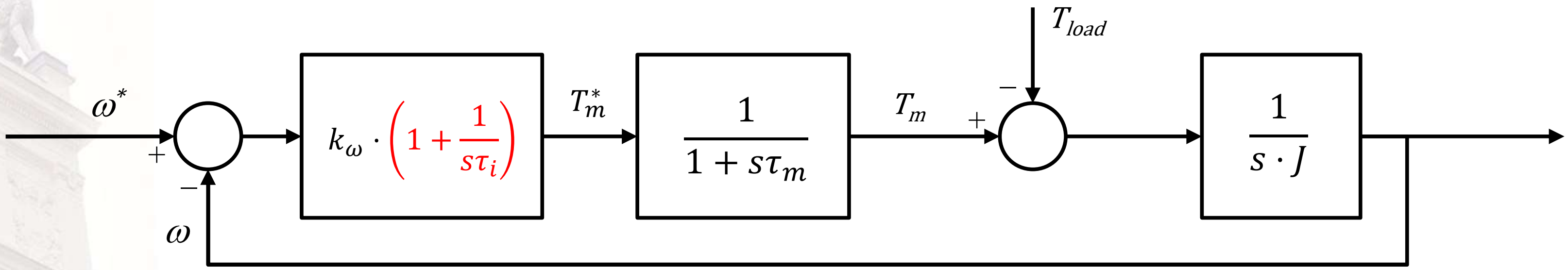
$$\omega\cdot\left(1 + k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J}\right) = \omega^*\cdot k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} - \frac{T_{load}}{s\cdot J}$$

$$\frac{\omega}{\omega^*} = \frac{k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} - \frac{T_{load}}{\omega^*\cdot s\cdot J}}{\left(1+k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J}\right)} = \frac{k_\omega}{J\cdot\tau_m}\frac{1-\frac{(1+s\tau_m)\cdot T_{load}}{k_\omega\cdot\omega^*}}{s^2+s\cdot\frac{1}{\tau_m}+k_\omega\cdot\frac{1}{J\cdot\tau_m}}$$

Stationary error:

$$\lim_{s\to 0}(\omega^* - \omega) = \lim_{s\to 0}\left(\omega^* - \frac{k_\omega}{J\cdot\tau_m}\frac{\omega^*-\frac{(1+s\tau_m)\cdot T_{load}}{k_\omega}}{s^2+s\cdot\frac{1}{\tau_m}+k_\omega\cdot\frac{1}{J\cdot\tau_m}}\right) = \frac{T_{load}}{k_\omega}$$

Solution 3.1d (PI control)



$$\omega = (\omega^* - \omega) \cdot k_\omega \cdot \frac{(1+s\tau_i)}{s\tau_i} \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J} - \frac{T_{load}}{s \cdot J}$$

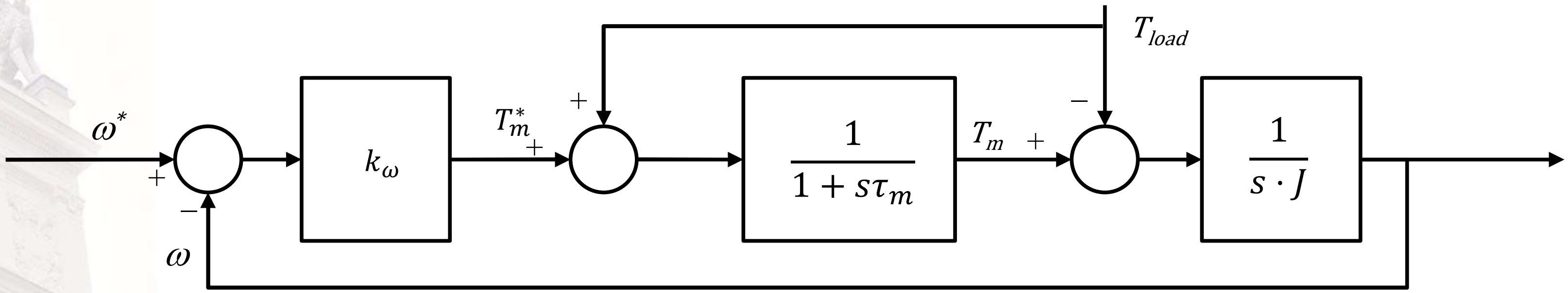
$$\omega \cdot \left(1 + k_\omega \cdot \frac{(1+s\tau_i)}{s\tau_i} \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J}\right) = \omega^* \cdot k_\omega \cdot \frac{(1+s\tau_i)}{s\tau_i} \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J} - \frac{T_{load}}{s \cdot J}$$

$$\frac{\omega}{\omega^*} = \frac{k_\omega \cdot \frac{(1+s\tau_i)}{s\tau_i} \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J} - \frac{T_{load}}{\omega^* \cdot s \cdot J}}{\left(1 + k_\omega \cdot \frac{(1+s\tau_i)}{s\tau_i} \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J}\right)} = \frac{k_\omega}{J \cdot \tau_i \cdot \tau_m} \cdot \frac{(1+s\tau_i) - \frac{s\tau_i \cdot (1+s\tau_m) \cdot T_{load}}{k_\omega \cdot \omega^*}}{s^3 + s^2 \cdot \frac{1}{\tau_m} + s \cdot \frac{k_\omega}{J \cdot \tau_m} + k_\omega \cdot \frac{k_\omega}{J \cdot \tau_i \cdot \tau_m}}$$

Stationary error:

$$\lim_{s \to 0}(\omega^* - \omega) = \lim_{s \to 0}\left(\omega^* - \omega^* \cdot \frac{k_\omega}{J \cdot \tau_i \cdot \tau_m} \cdot \frac{(1+s\tau_i) - \frac{s\tau_i \cdot (1+s\tau_m) \cdot T_{load}}{k_\omega \cdot \omega^*}}{s^3 + s^2 \cdot \frac{1}{\tau_m} + s \cdot \frac{k_\omega}{J \cdot \tau_m} + k_\omega \cdot \frac{k_\omega}{J \cdot \tau_i \cdot \tau_m}}\right) = 0$$

Solution 3.1d (Feed forward of known load torque)



$$\omega = (\omega^* - \omega)\cdot k_\omega \cdot \frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} + \frac{T_{load}}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} - \frac{T_{load}}{s\cdot J}$$

$$\omega\cdot\left(1 + k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J}\right) = \omega^*\cdot k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} + \frac{T_{load}}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} - \frac{T_{load}}{s\cdot J}$$

$$\frac{\omega}{\omega^*} = \frac{k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} + \frac{T_{load}}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J} - \frac{T_{load}}{\omega^*\cdot s\cdot J}}{\left(1+k_\omega\cdot\frac{1}{(1+s\tau_m)}\cdot\frac{1}{s\cdot J}\right)} = \frac{k_\omega}{J\cdot\tau_m}\frac{1+\frac{T_{load}}{k_\omega\cdot\omega^*} - \frac{(1+s\tau_m)\cdot T_{load}}{k_\omega\cdot\omega^*}}{s^2+s\cdot\frac{1}{\tau_m}+k_\omega\cdot\frac{1}{J\cdot\tau_m}}$$

Stationary error:

$$\lim_{s\to 0}(\omega^*-\omega) = \lim_{s\to 0}\left(\omega^* - \frac{k_\omega}{J\cdot\tau_m}\frac{\omega^* + \frac{T_{load}}{k_\omega} - \frac{(1+s\tau_m)\cdot T_{load}}{k_\omega}}{s^2+s\cdot\frac{1}{\tau_m}+k_\omega\cdot\frac{1}{J\cdot\tau_m}}\right) = 0$$

Exercise 3.2 DC motor control

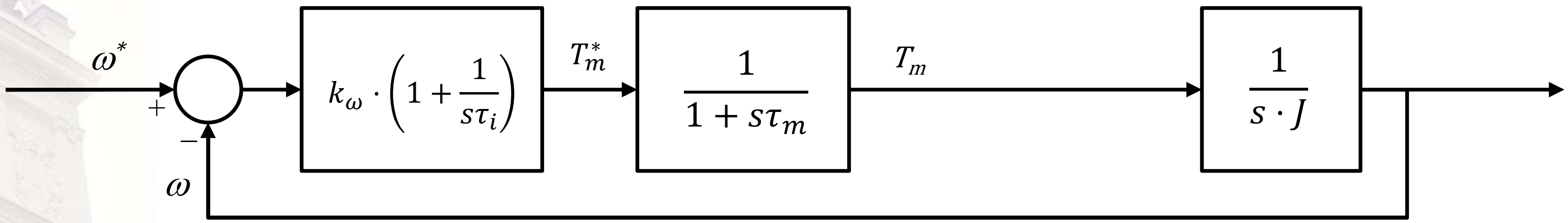
A DC motor with the inertia J=0,033 kgm^2 is driven by a converter with current control set for dead-beat current control at 3.33 ms sampling time. The speed of the DC motor is controlled by a P-regulator. The current loop is modelled with a first order time constant that equals the pulse interval of the converter.

a) *Draw a block diagram of the system with speed control with the models of the current loop and the motor. Calculate* k_ω *= the gain of the speed control for maximum speed without oscillatory poles.*

b) *The motor is loaded with the torque* T_l*. How large is the speed stationary error?*

c) *If the speed is measured with a tachometer and lowpass filtered, what does that mean for* k_ω*?*

Solution 3.2a



$$Opencircuit G = k_\omega \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J}$$

$$Closedloop = \frac{G}{1+G} = \frac{K_\omega \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J}}{1 + k_\omega \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s \cdot J}} = \frac{k_\omega}{s \cdot J \cdot (1+s\tau_m) + k_\omega}$$

$$= \frac{k_\omega}{J \cdot \tau_m \cdot s^2 + s \cdot J + k_\omega}$$

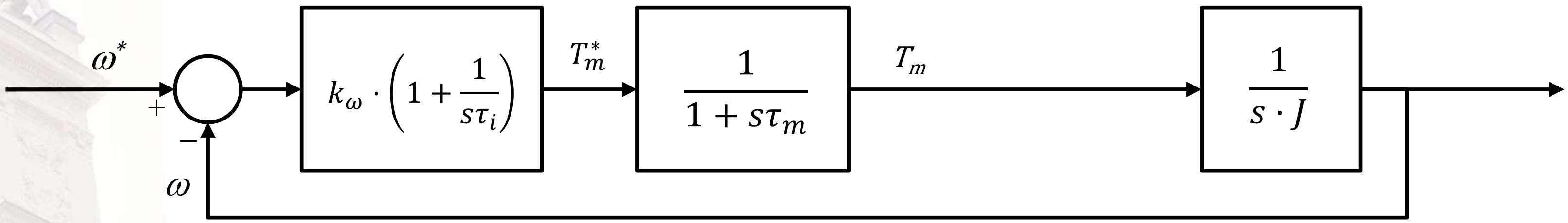
$$Characteristic\ equation:\ s^2 + \frac{s}{\tau_m} + \frac{k_\omega}{J \cdot \tau_m} = 0\ with\ the\ roots\ s = -\frac{1}{2 \cdot \tau_m} \pm \sqrt{\frac{1}{4 \cdot \tau_m^2} - \frac{k_\omega}{J \cdot \tau_m}}$$

$$Fastest\ operation\ is\ achived\ when\ theroots\ are\ the\ same \frac{1}{4 \cdot \tau_m^2} = \frac{k_\omega}{J \cdot \tau_m} \Rightarrow k_\omega = \frac{J}{4 \cdot \tau_m}$$

$$Dead\ Beat\ control\ @\ T_s = 3.3ms$$

$$k_\omega = \frac{0.033}{4 \cdot 3.3 \cdot 10^{-3}} = 2.5$$

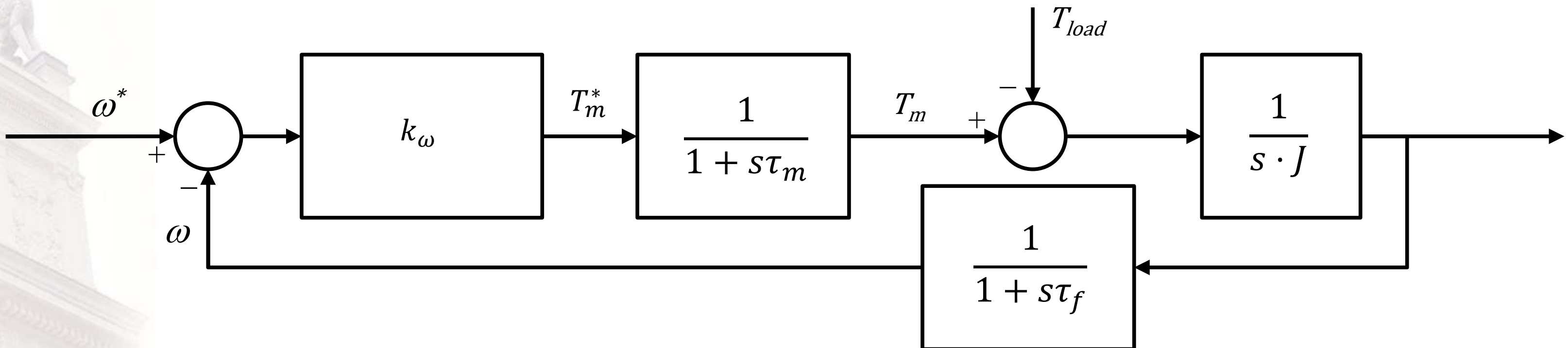
Solution 3.2b



Stationary Error

$$\lim_{s\to 0}(\omega^* - \omega) = \lim_{s\to 0}\left(\omega^* - \frac{k_\omega}{J\cdot\tau_m}\frac{\omega^* - \frac{(1+s\tau_m)\cdot T_{load}}{k_\omega}}{s^2 + s\cdot\frac{1}{\tau_m} + k_\omega\cdot\frac{1}{J\cdot\tau_m}}\right) = \frac{T_{load}}{k_\omega}$$

Solution 3.2c



It is now the measured speed (ω_t) that is controlled and the system becomes of 3rd order.

IF we assume that the filter time constant (τ_ω) is much longer that the torque control time constant (τ_m), then the system (including the filter) is now slower than the system without a filter, implying a need for a lower gain.

The solution to 3.2a can be applied, but with the torque control time constant replaced by the filter time constant, thus giving a lower speed controller gain.

Exercise 3.3 Pump control

- **A pump is driven at variable speed by an electric machine with a PI speed controller. The total inertia for both pump and electric machine is J=0,11. The power converter is a current controlled switched amplifier where the current control has an average response time of 100 µs, which is considered very fast if the integration time of the PI control is not of the same magnitude.**

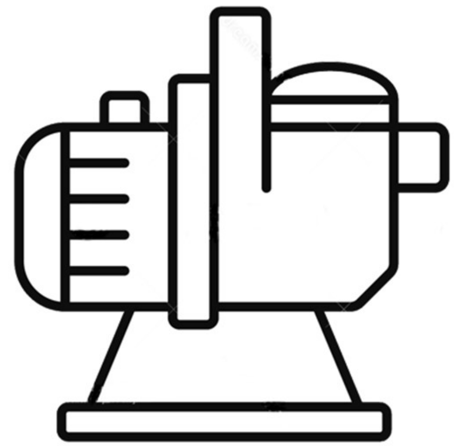
a) Draw the speed control system as a block diagram with the PI control and your selection of models for torque source, load torque and inertia.

b) Dimension the PI control so that the system has a double pole along the negative real axis.

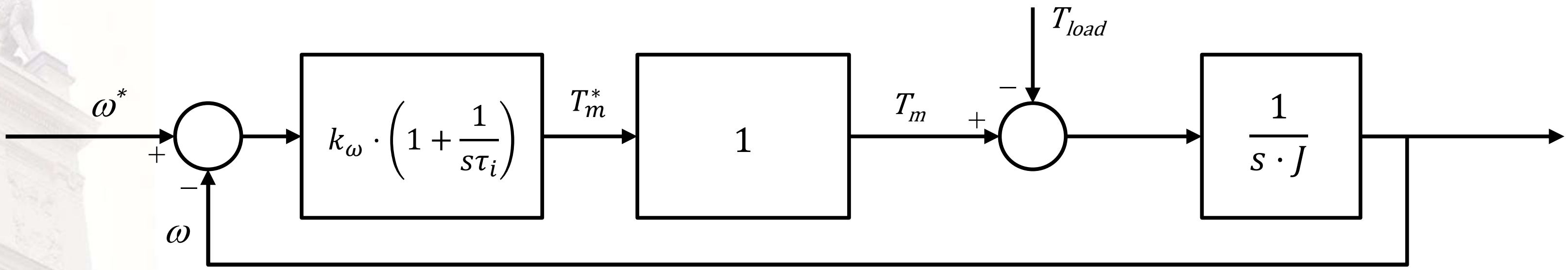
c) If the integration part for some reason is excluded (Ti=∞), how large is the speed error then?

d) If the current loop can not be considered as very fast, how is the it modelled in the block diagram?

e) There is a standard method for dimensioning the speed control in d). What is it called?

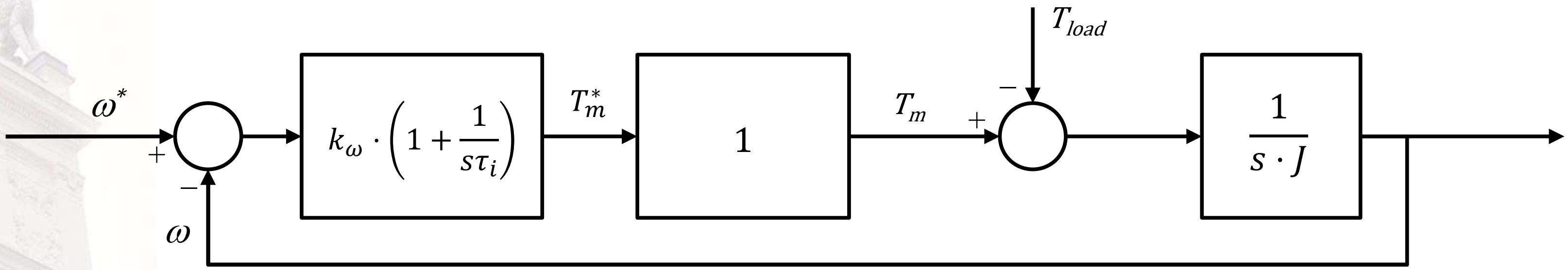


Solution 3.3a



The torque source is modelled as a unity gain, as the torque response time is very short (100µs) compared to the expected dynamics of the pump drive.
No measurement filter is modelled for the same reason.
The PI-controller is used to eliminate stationary errors.

Solution 3.3b



$Opencircuit(The\ torque\ source\ is\ 100\mu s, very\ fast)\ G = K_\omega \cdot \left(1+\frac{1}{s\tau_i}\right)\cdot 1 \cdot \frac{1}{s\cdot J}$

$$Closed\ loop = \frac{G}{1+G} = \frac{k_\omega \cdot \left(1+\frac{1}{s\tau_i}\right)\cdot \frac{1}{s\cdot J}}{1+k_\omega \cdot \left(1+\frac{1}{s\tau_i}\right)\cdot \frac{1}{s\cdot J}} = \frac{k_\omega \cdot (s\tau_i + 1)}{s\cdot J\cdot s\tau_i + k_\omega \cdot (s\tau_i+1)}$$

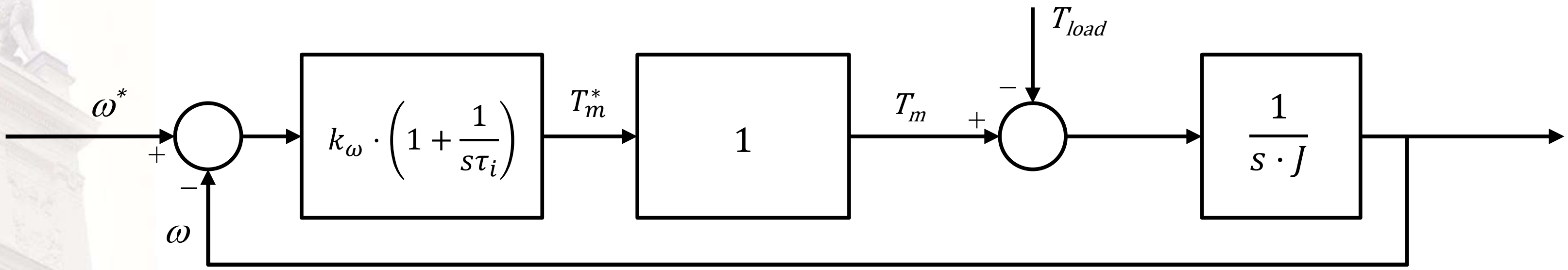
$$= \frac{k_\omega \cdot (s\tau_i + 1)}{s^2\cdot J\cdot \tau_i + s\cdot K_\omega \cdot \tau_i + k_\omega}$$

$$Characteristic\ equation:\ s^2 + s\cdot\frac{k_\omega}{J} + \frac{k_\omega}{J\cdot\tau_i} = 0\ with \rightleftarrows therootss = -\frac{k_\omega}{2\cdot J} \pm \sqrt{\frac{{k_\omega}^2}{4\cdot J^2} - \frac{k_\omega}{J\cdot\tau_i}}$$

$$Double\ root:\ s \Rightarrow \frac{{k_\omega}^2}{4\cdot J^2} - \frac{k_\omega}{J\cdot\tau_i} = 0 \Rightarrow \frac{{k_\omega}^2}{4\cdot J^2} = \frac{k_\omega}{J\cdot\tau_i} \Rightarrow k_\omega = \frac{4\cdot J}{\tau_i}$$

$$Assume\ \tau_i = 100\ ms(as\ \tau_i >> \tau_m, 100\mu s), J = 0.11 K_\omega = \frac{4\cdot 0.11}{0.1} = 4.4$$

Solution 3.3c



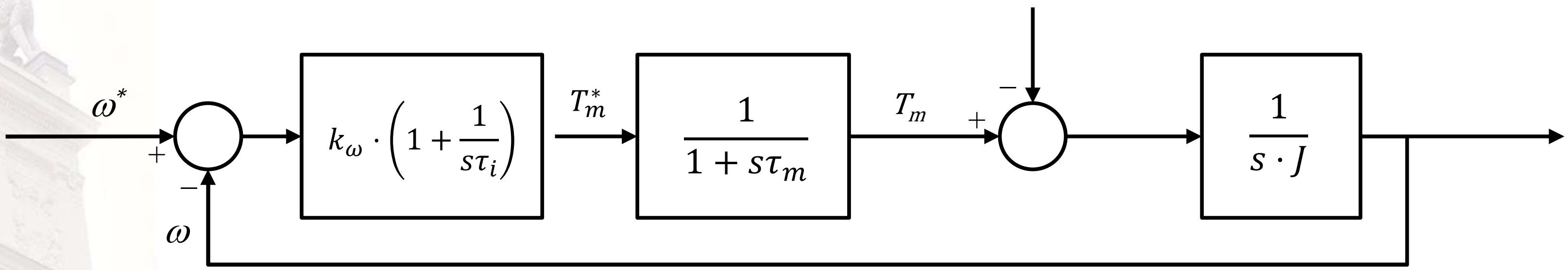
$$T = (\omega^* - \omega) \cdot k_\omega \cdot \left(1 + \frac{1}{s\tau_i}\right) = \{\tau_i \to \infty\} = (\omega^* - \omega) \cdot k_\omega$$

$$\begin{cases} T = (\omega^* - \omega) \cdot k_\omega \\ Stationary, T = T_{load} \end{cases}$$

$$T_{load} = (\omega^* - \omega) \cdot k_\omega$$

$$(\omega^* - \omega) = \frac{T_{load}}{k_\omega}$$

Solution 3.3d



Solution 3.3e

- **With new closed loop system, there will a another third pole to place. This is more complicated, but a recommended method is the Symmetric optimum, see chapter 9.5**

$$\omega_0 = \frac{1}{\sqrt{\tau_i \cdot \tau_m}} \quad (eq \rightleftarrows 9.19)$$

$$\tau_i = a^2 \cdot \tau_m \; \tau_m < \tau_i, a > 1 (eq \rightleftarrows 9.20)$$

$$No\ complex\ poles, set\ a = 3 (chapter 9.5)$$

$$Set\ all\ three\ poles\ the\ same\ at\ \omega_0$$

$$\tau_i = a^2 \cdot \tau_m = 3^2 \cdot 100 \cdot 10^{-6} = 0.9ms$$

$$K_p = \frac{a \cdot J}{T_i} = \frac{3 \cdot 0.11}{0.9 \cdot 10^{-3}} = 367(!)$$

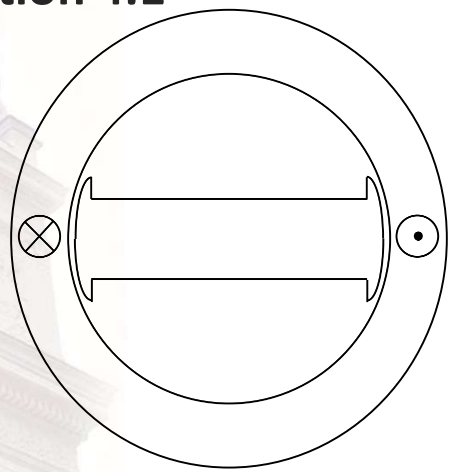
4

Exercises on MMF distribution

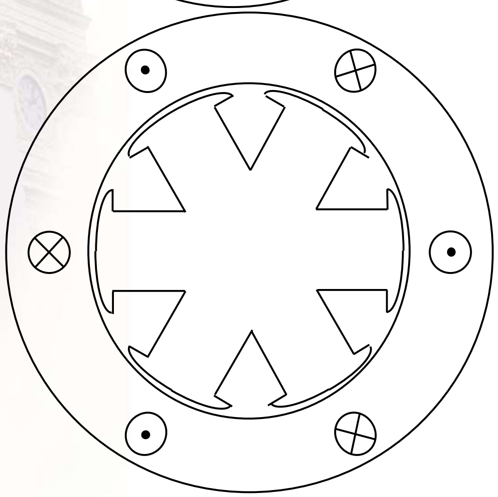
Exercise 4.1 2- and 6-pole motor

- **Draw a cross section of one two pole and one six pole synchronous machine with salient poles. Draw also a diameter harness (Swedish “diameterhärva”) which covers all poles.**

Solution 4.1



2-pole synchrounous machine

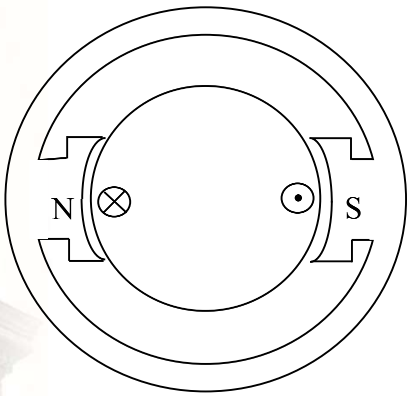


6-pole synchrounous machine

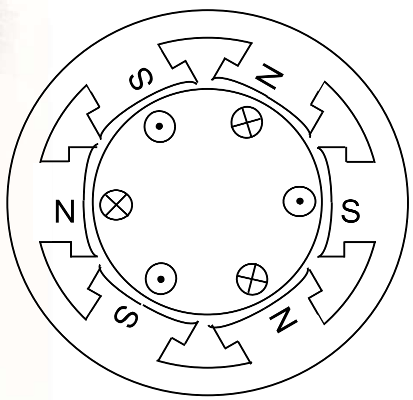
Exercise 4.2 DC machine

- **Draw a cross section of a DC machine with salient poles. Draw also a diameter harness which covers all poles.**

Solution 3.2



2-pole DC machine



6-pole DC machine

Exercise 4.3 mmf

"In electrical engineering, an armature is the power producing component of an electric machine. The armature can be on either the rotor (the rotating part) or the stator (stationary part) of the electric machine". [Wikipedia].

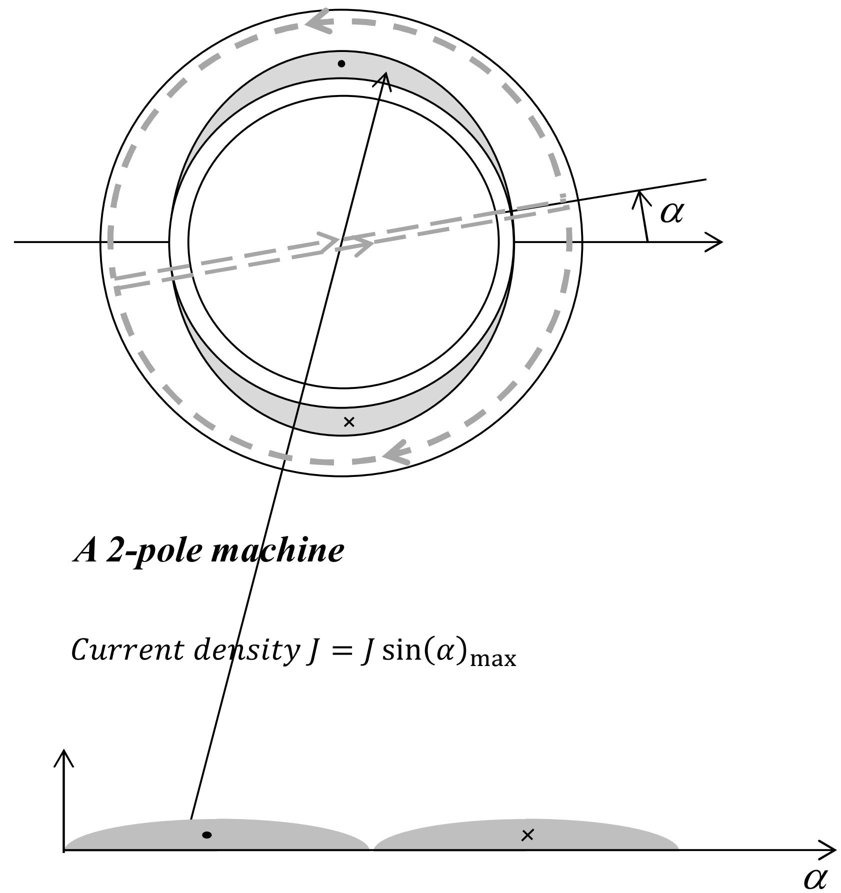
In the other part the field is produced.

A two pole armature winding in the stator of an alternating current machine is approximately sinusoidally distributed according to the figure below.

The current density (current per angle unit) is J=Jmax*sin(α) [A/radian]. The airgap is constant $\delta=\delta_0$.

Note that the outspread figure is done by spreading the windings from α=0 and that the machine is seen from the back,that is why the current directions change.

How large is the magnetomotive force F(α)?
Where will the magnetomotive force be found?



Solution 4.3

The magnetic field in the lower closed loop is clock wise, while the magnetic field in the upper closed loop has the opposite direction. However, along the center line, the direction from both loops has the same direction, and the contribution from both loops add.
Use ampèr's law in one loop. The magnetomotoric force in the air gap has contribution from both the lower and the upper loop.

$$F = 2 \cdot \int_{\alpha}^{\alpha+18} J \cdot d\alpha = J_{max} \cdot \int_{\alpha}^{\alpha+180} \sin(\alpha) \cdot d\alpha = 2 \cdot J_{max} \cdot (\cos(\alpha) - \cos(180 + \alpha)) =$$
$$= 2 \cdot J_{max} \cdot (\cos(\alpha) - \cos(180) \cdot \cos(\alpha) + \sin(180) \cdot \sin(\alpha)) = 2 \cdot J_{max} \cdot \cos(\alpha)$$

Solution 4.3 cont'd

- *Where will the magnetomotive force be found in the magnetic circuit ?*

$mmf \rightleftarrows equals\ the\ total\ current\ inside\ in\ one\ loop.$

$$N \cdot I = \oint \bar{H} \cdot d\bar{s} = H_{Fe} \cdot s_{Fe} + H_{air} \cdot \delta = \frac{B_{Fe}}{\mu\mu_0} \cdot s_{Fe} + \frac{B_{air}}{\mu_0} \cdot \delta =$$

$$= \frac{s_{Fe}}{\mu\mu_0 \cdot A_{Fe}} \cdot \psi + \frac{\delta}{\mu_0 \cdot A_\delta} \cdot \psi = R_{Fe} \cdot \psi + R_\delta \cdot \psi$$

$$R_{Fe} = \frac{s_{Fe}}{\mu\mu_0 \cdot A_{Fe}}, R_\delta = \frac{\delta}{\mu_0 \cdot A_\delta}$$

As the reluctance is proportionel to $1/\mu$, the reluctance in the iron can be neglected compared to the air gap reluctance.

I.e. the magnetomotive force will be concentrated in the two air gaps

The magnetomotive force in one air gap will be

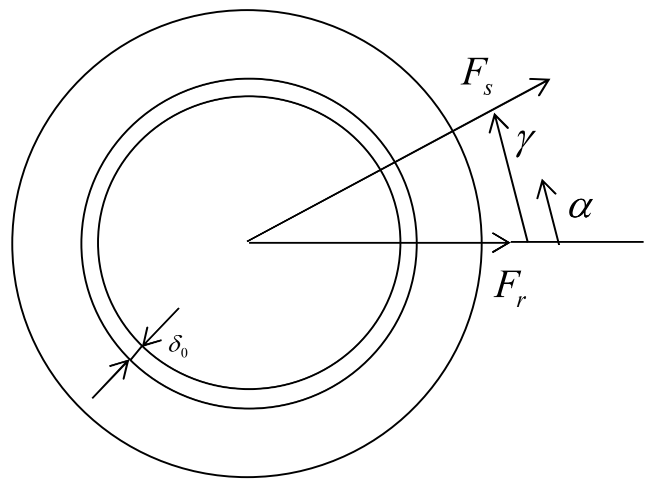
$$F = J\cos(\alpha)_{\max}$$

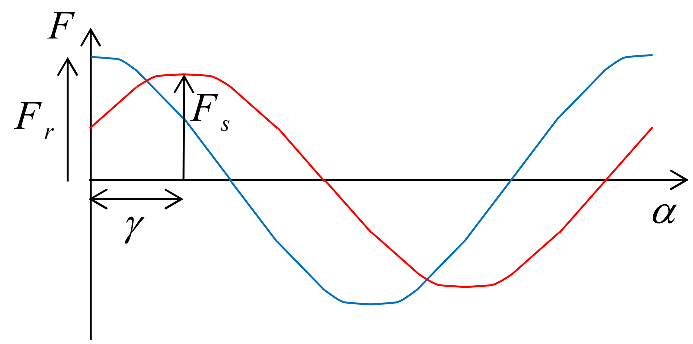
Exercise 4.9 2 wave mmf

- An electrical machine has two waves of magnetomotive force. One is caused by the current distribution in the rotor and the other by the current distribution in the stator, see figure to the right.

  The machine has a constant airgap, $\delta=\delta_0$ and the iron in the stator and the rotor has infinite magnetic conductivity. The peak amplitude of the waves of the magnetomotive force are for the stator and for the rotor.

  Calculate the energy in the airgap.





Solution 4.9

- **Both the stator and the rotor are cylindrical, thus the airgap reluctance R is the same in all directions**

$See\ equ(8.8)\ \vec{F}_\delta = \vec{F}_r + \vec{F}_s = F_x + j \cdot F_y$

$See\ equ(8.9)\ F_x = \hat{F}_s \cdot \cos(\gamma) + \hat{F}_r$

$See\ equ(8.9)\ F_y = \hat{F}_s \cdot \sin(\gamma)$

$$Seeequ(8.10)\ W_{magn} = \frac{1}{2} \cdot \frac{\hat{F}_x^2}{R} + \frac{1}{2} \cdot \frac{F_y^2}{R} = \frac{\hat{F}_s^2 \cdot \cos^2(\gamma) + 2 \cdot \hat{F}_s \cdot \hat{F}_r \cdot \cos(\gamma) + \hat{F}_r^2 + \hat{F}_s^2 \cdot \sin^2(\gamma)}{2R} =$$

$$= \frac{\hat{F}_s^2 + 2 \cdot \hat{F}_s \cdot \hat{F}_r \cdot \cos(\gamma) + \hat{F}_r^2}{2R}$$

Exercise 4.10 Torque

- Same as 3.9. Assume that no electric energy can be fed to or from the machine and that the system is lossless.

- How large is the mechanical torque as a function of the angle γ?

Solution 4.10

See exercise 3.9
The airgap reluctance R is the same in all directions

No energy supplied to the system W_{magn} *+* W_{magn} *=constant*

$$thus \quad \frac{dW_{magn}}{d\gamma} + \frac{dW_{mec}}{d\gamma} = 0 \Rightarrow \frac{dW_{mec}}{d\gamma} = -\frac{dW_{magn}}{d\gamma}$$

$$See\ equ(8.11)\ T = \frac{dW_{mec}}{d\gamma}$$

$$See\ equ(8.13)\ T = -\frac{dW_{magn}}{d\gamma} = -\frac{1}{2} \cdot \frac{d\left(\frac{F_s^2 + 2 \cdot F_s \cdot F_r \cdot \cos(\gamma) + F_r^2}{R}\right)}{d\gamma} = \frac{F_s \cdot F_r \cdot \sin(\gamma)}{R} = \frac{F_{sy} \cdot F_r}{R}$$

There is no reluctance torque

Exercise 4.11 Flux

A machine with salient poles in the rotor and cylindrical stator has its armature winding in the stator. The effective number of winding turns is $N_{a,\,eff}$ and the magnetized rotor contributes to the air gap flux with Φ_m The main inductances are L_{mx} and L_{my} in the x- and y directions.

a) How large is the flux contribution from the rotor that is linked to the armature winding?

b) How large is the resulting flux that is linked with the armature winding?

c) Draw a figure of how the armature current vector is positioned in the x-y plane to be perpendicular to the resulting air gap flux !

Solution 4.11a

- **The magnetized rotor contribution to the airgap flux**

ϕ_{m}

- **The effective number of winding turns in the stator**

$\mathrm{N}_{\mathrm{a,eff}}$

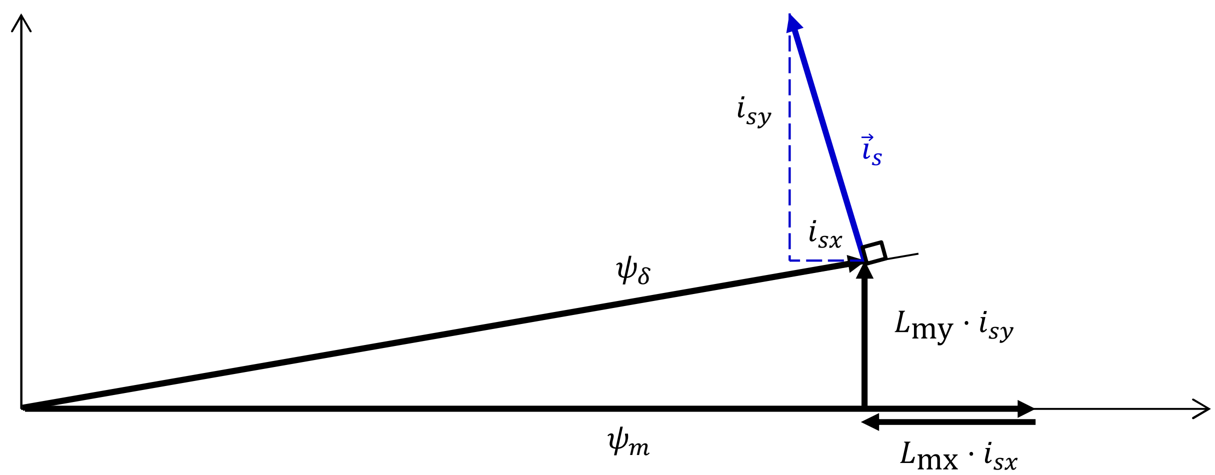
- **The linked flux contribution from the rotor to the armature winding**

$$\psi_m = \mathrm{N}_{\mathrm{a,eff}} \cdot \phi_{\mathrm{m}}$$

Solution 4.11b

The stator main inductance in the x-direction	L_{mx}
The stator main inductance in the y-direction	L_{my}
The armature winding current in the x-direction	i_{sx}
The armature winding current in the y-direction	i_{sy}
The magnetizing flux in x-direction	ψ_m
The resulting flux, linked with the armature winding	$\psi_a = (\psi_m + L_{\mathrm{mx}} \cdot i_{sx}) + j \cdot L_{\mathrm{my}} \cdot i_{sy}$

Solution 4.11c



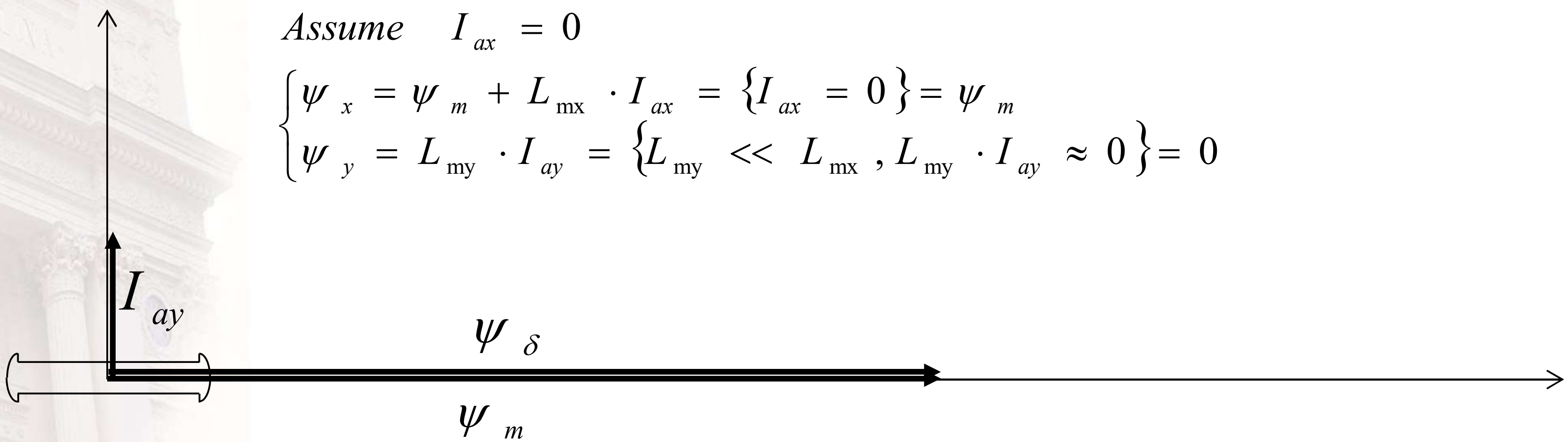
Exercise 4.12 Flux vector

Same as 4.11 but L_{my}<< L_{mx}. Draw a stylized picture of a cross section of the machine and draw a figure of how the armature current vector is positioned in the x-y plane to be perpendicular to the resulting air gap flux !

Solution 4.12

$$Assume \quad I_{ax} = 0$$

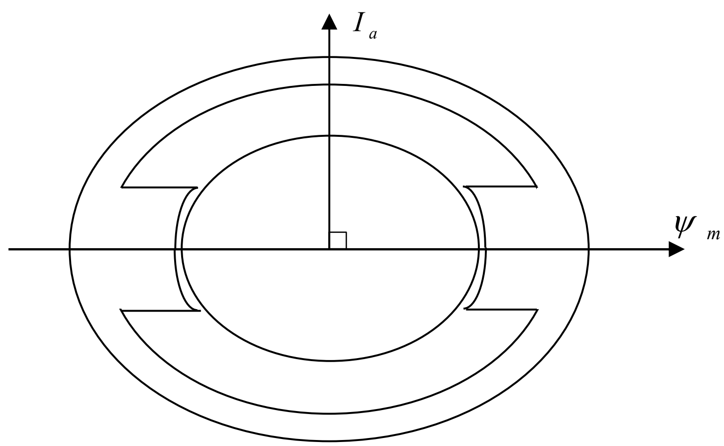
$$\begin{cases} \psi_x = \psi_m + L_{\mathrm{mx}} \cdot I_{ax} = \{I_{ax} = 0\} = \psi_m \\ \psi_y = L_{\mathrm{my}} \cdot I_{ay} = \{L_{\mathrm{my}} << L_{\mathrm{mx}}, L_{\mathrm{my}} \cdot I_{ay} \approx 0\} = 0 \end{cases}$$



Exercise 4.13 Armature current vector

Same as 4.12 but now the armature winding is in the rotor, which is cylindrical, and the stator has salient poles. Draw a stylized picture of a cross section of the machine and draw a figure of how the armature current vector is positioned in the x-y plane to be perpendicular to the resulting air gap flux !

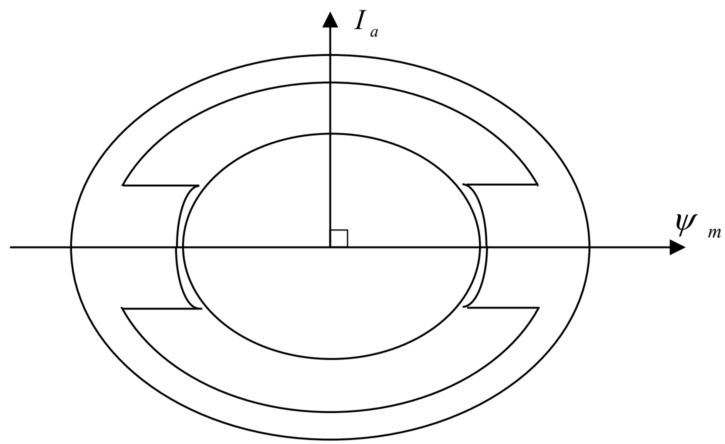
Solution 4.13



Exercise 4.14 Rotation problem

Suggest two ways of solving tha rotation problem, i. e. how the angle of the armature current vector to the air gap flux vector can be maintained during rotation for the cases in 4.12 and 4.13!

Solution 4.14



See chapter 8.8 and 10.1. According to chapter 8.8 the armature DC-winding must not be fixed to the stator.

Alt 1 This can be achieved be means of two or three phase AC-windings, see figure 8.9.

Exercise 4.15 Voltage equation

A three phase armature winding with the resistances R_a, the leakage inductances $L_{a\lambda}$ and the fluxes ψ_1, ψ_2, and ψ_3 that are linked to the respective armature windings.

a) Form the voltage equations first for each phase and then jointly in vector form!

b) Express all vectors in rotor coordinates instead of stator coordinates and separate the equation into real and imaginary parts.

Solution 4.15a

$$Equation\quad 8.28$$

$$\begin{cases} U_a = R_a \cdot i_a + \dfrac{d\psi_1}{dt} = R_a \cdot i_a + \dfrac{d\left(\psi_{\delta 1} + L_{a\lambda} \cdot i_a\right)}{dt} \\ U_b = R_a \cdot i_b + \dfrac{d\psi_2}{dt} = R_a \cdot i_b + \dfrac{d\left(\psi_{\delta 2} + L_{a\lambda} \cdot i_b\right)}{dt} \\ U_c = R_a \cdot i_c + \dfrac{d\psi_3}{dt} = R_a \cdot i_c + \dfrac{d\left(\psi_{\delta 3} + L_{a\lambda} \cdot i_c\right)}{dt} \end{cases}$$

$$Equation\quad 8.29$$

$$\vec{U}_s^{\alpha\beta} = R_a \cdot \vec{i}_s^{\alpha\beta} + \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = R_a \cdot \vec{i}_s^{\alpha\beta} + \frac{d\left(\vec{\psi}_\delta^{\alpha\beta} + L_{a\lambda} \cdot \vec{i}_s^{\alpha\beta}\right)}{dt}$$

Solution 4.15b

Perform a transformation from the $\alpha\beta-$ frame to $xy-$ frame.
Assume you are "sitting" on the $xy-$ frame, which is rotating in positive direction,
then you will see the $\alpha\beta-$ frame rotating in negative direction
Equation 8.30

$$u_s^{\alpha\beta} = R_a \cdot \vec{i}_s^{\alpha\beta} + \frac{d\left(\vec{\psi}_\delta^{\alpha\beta} + L_{a\lambda} \cdot \vec{i}_s^{\alpha\beta}\right)}{dt}$$

Transform by multiply by $e^{-j\omega t}$ (*negative direction*)

$$\begin{cases} \vec{u}_s^{xy} = \vec{u}_s^{\alpha\beta} \cdot e^{-j\omega t} \Rightarrow \vec{u}_s^{\alpha\beta} = \vec{u}_s^{xy} \cdot e^{j\omega t} \\ \vec{i}_s^{xy} = \vec{i}_s^{\alpha\beta} \cdot e^{-j\omega t} \Rightarrow \vec{i}_s^{\alpha\beta} = \vec{i}_s^{xy} \cdot e^{j\omega t} \\ \vec{\psi}_s^{xy} = \vec{\psi}_s^{\alpha\beta} \cdot e^{-j\omega t} \Rightarrow \vec{\psi}_s^{\alpha\beta} = \vec{\psi}_s^{xy} \cdot e^{j\omega t} \end{cases}$$

Insert

$$\vec{u}_s^{xy} \cdot e^{j\omega t} = R_a \cdot \vec{i}_s^{xy} \cdot e^{j\omega t} + \frac{d\left(\vec{\psi}_s^{xy} \cdot e^{j\omega t} + L_{a\lambda} \cdot \vec{i}_s^{xy} \cdot e^{j\omega t}\right)}{dt} = R_a \cdot \vec{i}_s^{xy} \cdot e^{j\omega t} + \frac{d\left(\left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right) \cdot e^{j\omega t}\right)}{dt} \Rightarrow$$

$$\vec{u}_s^{xy} \cdot e^{j\omega t} = R_a \cdot \vec{i}_s^{xy} \cdot e^{j\omega t} + \frac{d\left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)}{dt} \cdot e^{j\omega t} + j\omega \cdot e^{j\omega t} \cdot \left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)$$

$$\vec{u}_s^{xy} = R_a \cdot \vec{i}_s^{xy} + \frac{d\left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)}{dt} + j\omega \cdot \left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)$$

Solution 4.15b cont'd

$$\vec{U}_s^{xy} = R_s \cdot \vec{i}_s^{xy} + \frac{d\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)}{dt} + j \cdot \omega \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)$$

Separate the equation in a real and in a imaginary part

Equation 8.31

$$\begin{cases} L_{sx} = \left(L_{mx} + L_{s\lambda}\right) \\ L_{sy} = \left(L_{my} + L_{s\lambda}\right) \end{cases}$$

$$U_{sx} = R_s \cdot i_{sx} + \frac{d\left(\psi_m + \left(L_{mx} + L_{s\lambda}\right) \cdot i_{sx}\right)}{dt} - \omega_r \cdot \left(L_{my} + L_{s\lambda}\right) \cdot i_{sy} =$$

$$= R_s \cdot i_{sx} + \frac{d\left(\psi_m + L_{sx} \cdot i_{sx}\right)}{dt} - \omega_r \cdot L_{sy} \cdot i_{sy}$$

$$U_{sy} = R_s \cdot i_{sy} + \frac{d\left(\left(L_{my} + L_{s\lambda}\right) \cdot i_{sy}\right)}{dt} + \omega_r \cdot \left(\psi_m + \left(L_{mx} + L_{s\lambda}\right) \cdot i_{sx}\right) =$$

$$= R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot \left(\psi_m + L_{sx} \cdot i_{sx}\right)$$

Solution 4.15b cont'd

Separate the equation below in real and imaginary parts, see equation (8.31)

$$\vec{u}_s^{xy} = R_a \cdot \vec{i}_s^{xy} + \frac{d\left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)}{dt} + j\omega \cdot \left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)$$

$$\begin{cases} u_{sx} = R_a \cdot i_{sx} + \dfrac{d}{dt}\left(\psi_m + L_{mx} \cdot i_{sx} + L_{a\lambda} \cdot i_{sx}\right) - \omega_r \cdot \left(L_{my} \cdot i_{sy} + L_{a\lambda} \cdot i_{sy}\right) = R_a \cdot i_{sx} + \dfrac{d}{dt}\left(\psi_m + L_{sx} \cdot i_{sx}\right) - \omega_r \cdot L_{sy} \cdot i_{sy} \\ u_{sy} = R_a \cdot i_{sy} + \dfrac{d}{dt}\left(L_{my} \cdot i_{sy} + L_{a\lambda} \cdot i_{sy}\right) + \omega_r \cdot \left(\psi_m + L_{mx} \cdot i_{sx} + L_{a\lambda} \cdot i_{sx}\right) = R_a \cdot i_{sy} + L_{sy} \cdot \dfrac{di_{sy}}{dt} + \omega_r \cdot \left(\psi_m + L_{sx} \cdot i_{sx}\right) \end{cases}$$

Exercise 4.16 DC machine voltage equation

An armature winding is designed as a commutator winding, positioned in the rotor.

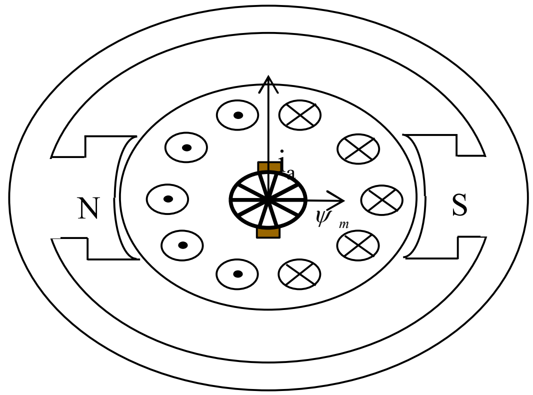
a. Draw a stylized picture of a cross section of the machine and show the resulting current distribution in the armature circuit that gives maximum torque if L_{my}=0.

b. Given the position of the commutator as in a), form an expression of the torque!

c. Give the voltage equation for the aramature circuit as it is known via the sliding contacts positioned as in b)

Solution 4.16a,b

a) See figure 10.2



b) Torque (*Equation* 10.1) $T = \psi_m \cdot i_a$

Solution 4.16c

$c)\ Voltage\quad (Equation\ 8.31)$

$See\ paragraph\ 10.2,\ the\ x-axis\ windings\ are\ never\ used,$
$the\ x-axis\ current\ is\ always\ zero,\ see\ equation\ 10.1$

$$u_{ax} = R_s \cdot \underbrace{i_{ax}}_{=0} + \underbrace{\frac{d}{dt}\underbrace{\psi_m}_{constant}}_{=0} + \frac{d}{dt}\left(L_{mx} \cdot \underbrace{i_{ax}}_{=0} + L_{a\lambda} \cdot \underbrace{i_{ax}}_{=0}\right) - \omega_r \cdot \underbrace{L_{sy}}_{=0} \cdot i_{ay} = 0$$

$$u_{ay} = u_a = R_a \cdot i_a + \frac{d}{dt}\left(\underbrace{L_{my}}_{=0} \cdot i_a + L_a \cdot i_a\right) + \omega_r \cdot \left(\psi_m + L_a \cdot \underbrace{i_{ax}}_{=0}\right) =$$

$$= R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + \omega_r \cdot \psi_m$$

Exercise 4.17 DC machine torque, power and flux

A DC machine has the following ratings:

U_{an}=300V

I_{an}=30A

R_a=1Ω

L_a=5mH

n_n=1500 rpm

Determine the rated torque T_n , the rated power P_n and the rated magnetization ψ_{mn}.

Solution 4.17

U_{an}= 300 V
I_{an}= 30 A
R_a= 1 ohm
L_a= 5 mH
n_a= 1500 rpm
At the nominal point, all values are constant

$$U_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_a = \left\{ \frac{di_a}{dt} = 0 \right\} = R_a \cdot i_a + e_a \Rightarrow$$

SOLUTION

$$\begin{cases} Power \quad P_{motor} = e_a \cdot i_a = \left[U_a - R_a \cdot i_a\right] \cdot i_a = (300 - 30 \cdot 1) \cdot 30 = 8100 \quad W \\ Torque \quad T_n = \dfrac{P_{motor}}{\omega_n} = \dfrac{8100}{\dfrac{1500}{60} \cdot 2\pi} = 51.6 \quad Nm \\ Flux \quad \psi_{\omega n} = \dfrac{e_a}{\omega_n} = \dfrac{U_a - R_a \cdot i_a}{\omega_n} = \dfrac{300 - 30 \cdot 1}{\dfrac{1500}{60} \cdot 2\pi} = 1.72 \quad Vs \end{cases}$$

Exercise 4.18 DC machine controller

Same data as in 4.17. The machine is fed from a switched converter with the sampling interval T_s =1ms, and the DC voltage U_{d0} =300V.

Derive a suitable controller for torque control at constant magnetization. The current is measured with sensors that give a maximum signal for i_a = I_0 =30A.

A DC machine has the following ratings:

R_a=1Ω

L_a=5mH

n_n=1500 rpm

Solution 4.18

$$U_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_a$$

$$u_a^*(k) = R_a \cdot \frac{i_a(k+1) + i_a(k)}{2} + L_a \cdot \frac{i_a(k+1) - i_a(k)}{T_s} + e_a(k) = R_a \cdot \frac{i_a^*(k) - i_a(k)}{2} + R_a \cdot i_a(k) + \frac{L_a}{T_s} \cdot \left(i_a^*(k) - i_a(k)\right) + e_a(k)$$

$$u_a^*(k) = \frac{R_a}{2} \cdot \left(i_a^*(k) - i_a(k)\right) + R_a \cdot i_a(k) + \frac{L_a}{T_s \cdot} \cdot \left(i_a^*(k) - i_a(k)\right) + e_a(k) = \left(\frac{R_a}{2} + \frac{L_a}{T_s}\right) \cdot \left(i_a^*(k) - i_a(k)\right) + R_a \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) + e_a(k)$$

$$u_a^*(k) = \left(\frac{R_a}{2} + \frac{L_a}{T_s}\right) \cdot \left(\left(i_a^*(k) - i_a(k)\right) + \frac{R_a}{\left(\frac{R_a}{2} + \frac{L_a}{T_s}\right)} \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + e_a(k)$$

$$u_a^*(k) = \left(\frac{R_a}{2} + \frac{L_a}{T_s}\right) \cdot \left(\left(i_a^*(k) - i_a(k)\right) + \frac{T_s}{\left(\frac{T_s}{2} + \frac{L_a}{R_a}\right)} \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + e_a(k)$$

$$u_a^*(k) = \left(\frac{1}{2} + \frac{0.005}{0.001}\right) \cdot \left(\left(i_a^*(k) - i_a(k)\right) + \frac{0.001}{\left(\frac{0.001}{2} + \frac{0.005}{1}\right)} \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + u_a(k) - 1 \cdot i_a(k) = \{R_a = 1, L_a = 0.005, T_s = 0.001,\} =$$

$$u_a^*(k) = 5.5 \cdot \left(\left(i_a^*(k) - i_a(k)\right) + 0.182 \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + \left(u_a(k) - i_a(k)\right)$$

Exercise 4.20 PMSM controller

A permanently magnetized synchronous machine has the following ratings:

$U_{line\text{-}to\text{-}line}=220V$

$I_{sn}=13A$

$n_n=3000$ rpm

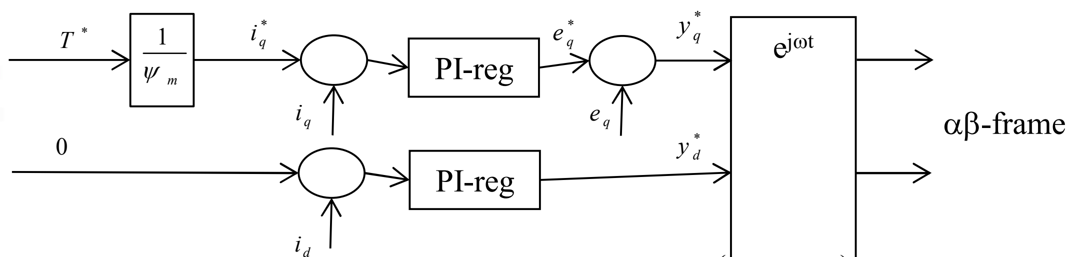
$R_a=0,5\Omega$

$L_d= L_q=7mH$

The machine is driven by a switched amplifier with the DC voltage U_{d0} =350V. The frequency of the modulating triangular wave f_{tri} is 1000 Hz . The current sensor measures currents up to a maximum of I_0 =25A.

Suggest a structure for the control of the torque of the machine together with a set of relevant equations.

Solution 4.20



Equation (11.3)(assume stationarity) $u_{sq} = R_s \cdot i_{sq} + \underbrace{L_{sq} \cdot \frac{di_{sq}}{dt}}_{=0} + \omega \cdot \left(\psi + L_s \cdot \underbrace{i_{sd}}_{=0} \right) = R_s \cdot i_{sq} + \omega \cdot \psi$

Angular frequency $\omega = 2\pi \cdot \frac{3000}{60} = 314.2$

Assume $i_{sq} = i_{sn}, i_{sd} = 0$ $\quad i_{sq} = \sqrt{\frac{3}{2}} \cdot 13 = 15.9\ A$

Voltage drop over res & ind $\quad \Delta e_R = 0.5 \cdot 15.9 = 8\,V$

Max symmetrized voltage $\quad u_{LL\_eff} = \frac{350}{\sqrt{2}} = 247.5\ V$

Back − emf voltage $\quad e = u_{LL\_eff} - \Delta e_R = 247.5\ V - 8\,V = 239.5\,V = \omega \cdot \psi$

Exercise 4.25 Electric car

You are to design an electric car. You have a chassis with space for batteries and an electric motor. The battery weight is 265 kg, the storing capacity is
32 kWh and can be charged with 5 kW. The battery no load voltage e_0 ranges from 170V to 200V and its inner resistance is R_b =0,14Ω. The motor is a two-pole three phase alternating current motor with the rating 50 kW at the rated speed n_{nm} =3000 rpm. The car has two gears, which give the speed 120 km/h at the rated speed of the motor, corresponding to the net gear 1/2,83. The weight of the car is 1500 kg including the battery weight. The requirement is to manage a 30% uphill.

Data		
Motor, 2-pole, 3 phase AC		
Rated power	50 kW	
Rated motor speed		3000 rpm
Battery		
Voltage		170-200 V
Charge capacity		32 kWh
Max charging power	5 kW	
Internal resistance		0.14 ohm
Weight		265 kg
Vehicle		
Weight		1500 kg (incl battery)
Vehicle speed at rated motor speed	120 km/h	
Gear		1/2.83 at 120 km/h
Rated uphill	30%	

Exercise 4.25 cont'd

a) What is the rated torque of the motor?

b) What rated stator voltage would you choose when you order the motor?

c) Which is the minimum rated current for the transistors of the main circuit?

d) What gearing ratio holds for the low gear?

e) When driving in 120 km/h, the power consumption is 370 Wh/km. How far can you drive if the batteries are fully loaded when you start? For a certain drive cycle in city traffic, the average consumption is 190 Wh/km. How far can the car be driven in the city?

f) What is the cost/10 km with an energy price of 2 SEK/kWh?

Solution 4.25a,b

$$a)\ Angular\ speed\ at\ rated\ speed\ \omega = \frac{3000}{60}\cdot 2\cdot\pi = 314$$

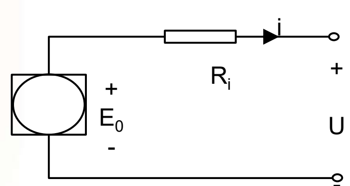
$$Torque\ at\ rated\ speed\ T = \{Power\ P = T\cdot\omega\} = \frac{P}{\omega} = \frac{50000}{314} = 159\ Nm$$

$$b)\ P = 50\ kW = u\cdot i = (e_0 - R_i\cdot i)\cdot i = \left\{\begin{matrix} e_0 = 170 - 200\ V \\ use\ 170\ V\end{matrix}\right\} = (170 - 0.14\cdot i)\cdot i = 170\cdot i - 0.14\cdot i^2$$

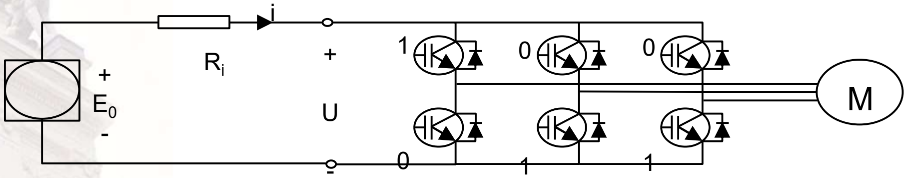
$$i^2 - \frac{170\cdot i}{0.14} + \frac{50000}{0.14} = 0 \Rightarrow i = \frac{85}{0.14} \pm \sqrt{\left(\frac{85}{0.14}\right)^2 - \frac{50000}{0.14}} = 500\ A$$

$$u = 170 - 0.14\cdot 500 = 100\ V_{dc}$$

$$With\ symmetrized\ 3-phase\ ac\ voltage\ \hat{u}_{LL} = 100\ V_{dc} \Rightarrow u_{LL} = \frac{u_{dc}}{\sqrt{2}} \approx 71\ V$$



Solution 4.25c



$$c)\ \text{Assume power factor} = 0.9$$

$$P_{ac} = \sqrt{3} \cdot u_{LL} \cdot I_{phase} \cdot 0.9 \Rightarrow 50000 = \sqrt{3} \cdot 71 \cdot I_{phase} \cdot 0.9$$

$$I_{phase\_eff} = \frac{50000}{\sqrt{3} \cdot 71 \cdot 0.9} = 452\ A$$

$$\hat{I}_{phase} = 639\ A$$

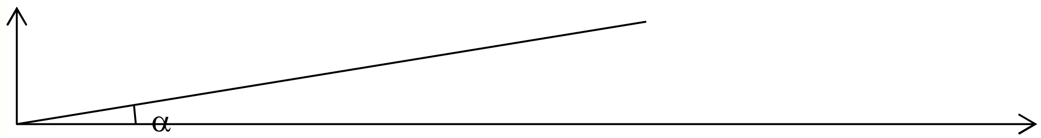
See figure above, "1" means the transistor is conducting, "0" the transistor is not conducting

E.g. the top left transistor is the only transistor in upper position which is conducting,

thus the full dc – current is flowing through this transistor,

Rated transistor current is 639 A

Solution 4.25d



$$d)\ \text{The uphill slope is } 30\%.\ \arctan(\alpha) = 0.3 \Rightarrow \alpha = 17^{o}$$

$$\text{The requested force } F = 1500 \cdot 9.81 \cdot \sin(17^{o}) = 4228\ N$$

$$\text{Assume wheel radius } r = 0.3\ m$$

$$\text{Torque } T = F \cdot r = 4228 \cdot 0.3 = 1268\ Nm$$

$$\text{The motor torque at rated power } T_{motor} = 159\ Nm$$

$$\text{Assume the low gear, the gear ratio} = \frac{1268}{159} = 8.0$$

Solution 4.25 e,f

$$e)\ Maximum\ battery\ charge = 32\ kWh$$

$$Battery\ consumption\ at\ 120\ km/h = 370\ Wh/km$$

$$How\ far\ with\ fully\ loaded\ battery\ at\ 120\ km/h = \frac{32}{0.37} \approx 86\ km$$

$$Battery\ consumption\ in\ average\ city\ traffic = 190\ Wh/km$$

$$How\ far\ in\ average\ city\ traffic = \frac{32}{0.19} \approx 168\ km$$

$$f)\ Cost/10\ km\ at\ 120\ km/h = 10 \cdot 0.37 \cdot 2\ SEK/kWh = 7.40\ SEK$$

$$Cost/10\ km\ in\ average\ city\ traffic = 10 \cdot 0.19 \cdot 2\ SEK/kWh = 3.80\ SEK$$

5

Exercises on PMSM

Exercise 5.1 Flux and noload voltage

A permanent magnetized synchronous machine is magnetized with at the most 0,7 Vs linkage flux in one phase. It is not connected.

a. *How large is the flux vector as a function of the rotor position?*
b. *How large is the induced voltage vector as a function of rotor position and speed?*
c. *At which speed is the voltage too large for a frequency converter with a dc voltage of 600V?*

Solution 5.1a

Given:
$\hat{\psi}_{phase} = 0.7\, Vs, No\ load, open\ stator$

Sought:
$\vec{\psi} = f(\Theta_r)$

Solution:
From equation (3.4) it is learned that the magnitude of the vector equals the "phase-to-phase" RMS-value of the same quantity:

$$|\vec{\psi}_m| = \frac{\sqrt{3}}{\sqrt{2}} \cdot \hat{\psi}_{phase} = 0.86Vs$$

The flux vector is oriented along the PMSM rotor magnet pole

Solution 5.1b,c

b)

From equation (3.5):

$$E \cdot e^{j\omega t} = \vec{e} = \omega \cdot \vec{\psi}_m \cdot e^{j\frac{\pi}{2}}$$

The induced voltage is "flux x speed" and $\frac{\pi}{2}$ radians ahead.

c)

According to figure 2.24 the voltage vector is $\sqrt{\frac{2}{3}} \cdot U_{dc}$.

The longest vectro that can be sustained at any angle is the radius of a circle inscribed in a hexagon defined by the active voltage vectors (i.e. not the zero vectors),

$$|\vec{u}|_{max} = \omega_{max} \cdot \vec{\psi}_m = \sqrt{\frac{2}{3}} \cdot U_{dc} \cdot \frac{\sqrt{3}}{4} = \frac{U_{dc}}{\sqrt{2}} \rightarrow \omega_{max} = \frac{U_{dc}}{0.86 \cdot \sqrt{2}}$$

Exercise 5.2 Inductance and torque generation

A permanent magnetized synchronous machine has a cylindrical rotor with $L_{mx} = L_{my} = L_m = 2$ mH. The magnetization is the same as in 5.1, i.e. 0,7 Vs linkage flux in one phase. The machine is controlled so that the stator current along the *x* axle is zero ($i_{sx}=0$).

a) How large torque can the machine develop if the phase current is limited to 15 A RMS?

b) Draw the flux linkage from the permanent magnets and from the stator current in (*x, y*) coordinates together with induced voltage and voltage for the frequency 25 Hz and the stator resistance 0,2Ω!

c) How large stator current is required to reduce the flux to zero?

Solution 5.2a

Given:
Flux linkage (in vector form), $\psi_m = 0.86\,Vs$
Frequency 25 Hz $\rightarrow$ $\omega_{el} = 2 \cdot \pi \cdot 25 = 50 \cdot \pi \frac{rad}{s}$
Max phase current: $i_{phase,RMS,max} = 15\,A$
Inductances: $L_{mx} = L_{my} = L_m = 2\,mH$

Sought:
a) Max Torque

Solution:

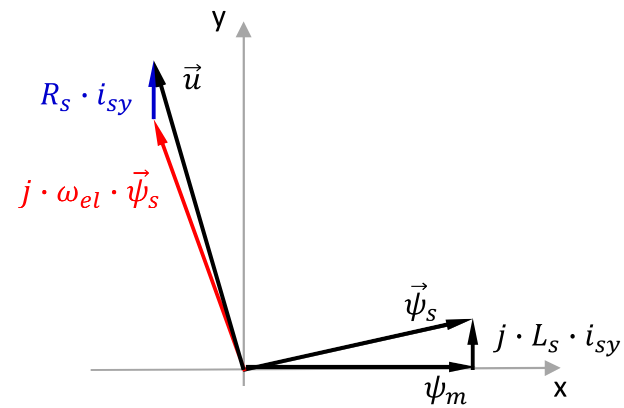
General torque equation: $T = \psi_m \cdot i_{sy} + (L_{sx} - L_{sy}) \cdot i_{sx} \cdot i_{sy}$
$i_{sx} = 0 \rightarrow T = \psi_m \cdot i_{sy}$
$i_{sy,max} = |\vec{i}_s| = \sqrt{3} \cdot i_{phase,RMS,max} = 26\,A$
$T_{max} = \psi_m \cdot i_{sy,max} = 0.86 \cdot 26 = 22\,Nm$

Solution 5.2b

Given:
Flux linkage (in vector form), $\psi_m = 0.86\, Vs$
Frequency 25 Hz $\rightarrow \omega_{el} = 2 \cdot \pi \cdot 25 = 50 \cdot \pi \frac{rad}{s}$
Max phase current: $i_{phase,RMS,max} = 15\, A$
Phase resistance = $R_s = 0.2\, \Omega$
Inductances: $L_{sx} = L_{sy} = L_s = 2\, mH$

Sought:
a) Flux linkage and Voltage components in the (x,y) frame

Solution:
$\psi_m = 0.86\, Vs$
$\vec{\psi}_s = \psi_m + j \cdot L_s \cdot i_{sy} = 0.86 + j \cdot 0.002 \cdot 26 = 0.86 + j \cdot 0.052$
$\vec{u} = R_s \cdot i_{sy} + j \cdot \omega_{el} \cdot \vec{\psi}_s = R_s \cdot i_{sy} + j \cdot \omega_{el} \cdot \left(\psi_m + j \cdot L_s \cdot i_{sy}\right) =$
$= R_s \cdot j \cdot 26 + j \cdot 50 \cdot \pi \cdot (0.86 + j \cdot 0.052) = j \cdot 5.2 - 8 + j \cdot 132$ V

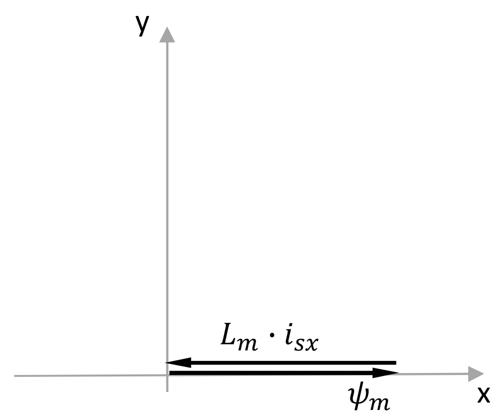


Solution 5.2c

Given:
Flux linkage (in vector form), $\psi_m = 0.86\ Vs$
Frequency 25 Hz $\rightarrow \omega_{el} = 2 \cdot \pi \cdot 25 = 50 \cdot \pi \frac{rad}{s}$
Max phase current: $i_{phase,RMS,max} = 15\ A$
Phase resistance = $R_s = 0.2\ \Omega$
Inductances: $L_{mx} = L_{my} = L_m = 2\ mH$

Sought:
a) Stator current for zero stator flux linkage

Solution:
$\psi_m = 0.86\ Vs$
$\vec{\psi}_s = \psi_m + (L_m \cdot i_{sx} + j \cdot L_m \cdot i_{sy}) = 0 \rightarrow i_{sx} = \frac{\psi_m}{i_{sx}} = \frac{0.86}{0.002} = 430\ A$



Exercise 5.3 PMSM Control

The machine in example 5.2 is vector controlled. The voltage is updated every 100 µs, i. e. the sampling interval is T_s=100 µs. The machine shall make a torque step from 0 to maximum torque when the rotor is at standstill. The DC voltage is 600V.

a. Determine the voltage that is required to increase the current i_{sy} from zero to a current that corresponds to maximum torque in one sample interval!

b. Is the DC voltage sufficient?

Solution 5.3a

Data

Sampling time	$T_s = 100\ \mu s$
Torque, see execisen 5.2a	$22.3\ Nm$
Dclink voltage	$U_{dc} = 600\ V$
Start from standstill	$\omega = 0$

a) It is a 3 − phase load, see the theory in chapter 3.7, particularly equ (3.17) and (3.18)
Since it will be a step in i_{sy} (called i_q in the equations) following expressions are valid

$$u_x^*(t) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot \left((0 - 0) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2}\right)} \cdot \sum_0^{k-1} (0 - 0) \right) - \underbrace{\omega}_{=0} \cdot L_s \cdot i_q = 0$$

$$u_y^*(t) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot \left((26 - 0) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2}\right)} \cdot \sum_0^{k-1} (0 - 0) \right) + \underbrace{\omega}_{=0} \cdot \left(\psi_m + L_s \cdot \underbrace{i_d}_{=0} \right) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot 26 = \left(\frac{2 \cdot 10^{-3}}{1 \cdot 10^{-4}} + \frac{0.2}{2}\right) \cdot 26 = 522\ V$$

Solution 5.3b

b) The maximum line-to-line voltage from a dclink voltage is $= \frac{U_{dc}}{\sqrt{2}} = 424\ V$

If we are lucky and the step in $u_y^*(k)$ happens to point in the direction of one of the six voltage vectors defining the hexagon we will have the voltage $\sqrt{\frac{2}{3}} \cdot U_{dc} = 490\ V$, still too low than the requested $522\ V$.

The step will take more than one sampling interval

Exercise 5.4 Torque

A two-pole permanently magnetized synchronous machine with the parameters Lmx=Lmy=Lm=15mH is used in an airplane and is therefore driven with stator frequencies up to 400 Hz. The stator resistance is negligable. The phase current is limited to 10A rms. The motor is fed by a converter with the DC voltage 600V.

a. Determine the magnetization from the permanent magnets considering the case when all voltage is needed and the machine is developing full torque (all the current along the q-axle) and 200 Hz stator frequency!
b. Determine the torque!

c. Determine the torque at 400 Hz stator frequency provided a part of the current is needed for demagnetization!

Solution 5.4a,b

Data

PMSM	$2-pole$
$L_{sx} = L_{sy}$	$15\ mH$
f_{max}	$400\ Hz$
R_s	$0\,\Omega$
I_{phase}	$10\ A$
Dclink voltage $U_{dc} =$	$600\ V$

a) Sought ψ_m at max voltage and full torque, and no need for field weakening at this low speed

$L_{sx} = L_{sy}$, thus no reluctance torque

$f = 200\ Hz$

Start with equation (11.2)

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d}{dt}\left(\psi_m + L_s \cdot \vec{i}_s\right) + j \cdot \omega_r \cdot \left(\psi_m + L_s \cdot \vec{i}_s\right) = \left\{ R_s = 0.\ Assume\ stationarity \Rightarrow \frac{d}{dt} = 0 \right\} =$$

$$= j \cdot \omega_r \cdot \psi_m + j \cdot \omega_r \cdot L_s \cdot \vec{i}_s \Rightarrow |\vec{u}| = \left| j \cdot \omega_r \cdot \psi_m + j \cdot \omega_r \cdot L_s \cdot \vec{i}_s \right|$$

$$\left|\frac{U_{dc}}{\sqrt{2}}\right| = \sqrt{\left(2\pi \cdot 200 \cdot \psi_m\right)^2 + \left(2\pi \cdot 200 \cdot L_s \cdot \vec{i}_s\right)^2} \Rightarrow \psi_m = \frac{\sqrt{\frac{600^2}{2} - \left(2\pi \cdot 200 \cdot 0.015 \cdot 10 \cdot \sqrt{2}\right)^2}}{2\pi \cdot 200} = 0.26\ Vs$$

b) $T = \psi_m \cdot i_{sy} = 0.26 \cdot 10 \cdot \sqrt{3} = 4.5\ Nm$

Solution 5.4c

$c)$ *In this case* $i_{sx} <> 0$ *since field weakening is used*

$L_{sx} = L_{sy}$, *thus no reluctance torque*

$f = 400\ Hz$

Assume stationarity $\Rightarrow \frac{d}{dt} = 0$

$$\begin{cases} |\vec{u}| = \left|R_s \cdot \vec{i}_s + j\omega_r \cdot \psi_m + j\omega_r \cdot L_s \cdot \vec{i}_s\right| = \{R_s = 0\} = \sqrt{(\omega_r \cdot L_s \cdot i_{sy})^2 + (\omega_r \cdot \psi_m + \omega_r \cdot L_s \cdot i_{sx})^2} = 424\ V \\ |i_s| = \sqrt{i_{sx}^2 + i_{sy}^2} = 10\sqrt{3} \end{cases}$$

$$|u|^2 = \omega_r^2 \cdot L_s^2 \cdot i_{sy}^2 + \omega_r^2 \cdot \psi_m^2 + \omega_r^2 \cdot L_s^2 \cdot i_{sx}^2 + 2 \cdot \omega_r \cdot \psi_m \cdot \omega_r \cdot L_s \cdot i_{sx} =$$

$$= \underbrace{\omega_r}_{2513.3}{}^2 \cdot \left(\underbrace{L_s}_{0.015}{}^2 \cdot \underbrace{\left(i_{sx}^2 + i_{sy}^2\right)}_{10\sqrt{3}^2} + \underbrace{\psi_m}_{0.26}{}^2 + 2 \cdot \underbrace{\psi_m}_{0.26} \cdot \underbrace{L_s}_{0.015} \cdot i_{sx} \right) = 424^2$$

$$\begin{cases} i_{sx} = \dfrac{\left(\dfrac{424}{2513.3}\right)^2 - 0.015^2 \cdot 10^2 \cdot \sqrt{3}^2 - 0.26^2}{2 \cdot 0.26 \cdot 0.015} = 13.7\ A \\ i_{sy} = \sqrt{\left(10 \cdot \sqrt{3}\right)^2 - 13.7^2} = 10.6\ A \end{cases}$$

$T = 10.6 \cdot 0.26 = 2.76\ Nm$

I.e. the torque has dropped to about half of the torque at $200\ Hz$, *which is not a surprise*

(*a bit more than half since we use* $10.6\ A$ *instead of* $10\sqrt{3} = 17.3\ A$)

Exercise 5.5 Drive system

You are designing an electric bicycle with a synchronous machine as a motor, coupled to the chain by a planetary gear. The power of the motor is 200W and it has 10 poles. The speed of the motor is 1000rpm at full power. The motor is fed from a three phase converter with batteries of 20V. The stator resistance and inductance can be neglected.

a. Determine the magnetization expressed as a flux vector at rated operational with full voltage from the frequency converter!
b. Determine the phase current at full torque!
c. Determine the moment of inertia if the bicycle and its driver weigh 100kg, the gear ratio is 1:10 and the rated speed is 25 km/h!Om cykel med förare väger 100 kg, hur stort tröghetsmoment upplever drivmotorn om utväxlingen är 1:10 och märkhastigheten är 25 km/h?
d. How long is the time for accelaration?

Solution 5.5a

Data

PMSM	$p = 10 - pole$
Power	$200\,W$
Dclink voltage $U_{dc} =$	$20\,V$
speed at full power	$1000\,rpm$
$L_{sx} = L_{sy}$	$0\,mH$, no reluct. torque
R_s	$0\,\Omega$

$a)$ *Magnetization flux vector* $|\psi_s|$ *at rated, nom speed. Assume stationarity,* $\Rightarrow \frac{d}{dt} = 0$

$|\vec{u}^{xy}| = |j\omega_r \cdot \psi_s|$, *see equ* (11.2)

$$|\vec{\psi}_s| = \frac{\frac{U_{dc}}{\sqrt{2}}}{\omega_{r,el}} = \frac{U_{dc}}{\sqrt{2}\cdot\omega_{r,mech}\cdot\frac{p}{2}} = \frac{20}{\sqrt{2}\cdot 2\pi\cdot\frac{1000}{60}\cdot\frac{10}{2}} = 0.027\,Vs \approx \psi_{pm}\ , as\ L_s = 0$$

Solution 5.5b,c,d

$$b)Torque \quad T_{mech} = \frac{p}{2}\cdot T_{el} = \frac{p}{2}\cdot\psi_{pm}\cdot i_{sy} = \frac{P}{\omega_{r,mech}} = \frac{200}{2\pi\cdot\frac{1000}{60}} = 1.91\,Nm$$

$$i_{sy} = \frac{T_{mech}}{\frac{p}{2}\cdot\psi_{pm}} = \frac{1.91}{\frac{10}{2}\cdot 0.027} = 14.15\,A$$

$$I_s = \frac{14.15}{\sqrt{3}} = 8.17\,A$$

$$c)\ Inertia \quad Energy\ \frac{1}{2}\cdot J\cdot\omega_{r,mech}{}^2 = \frac{1}{2}\cdot m\cdot v^2 \Rightarrow J_{ekv} = m\cdot\left(\frac{v}{\omega_{r,mech}}\right)^2 = 100\cdot\left(\frac{\frac{25}{3.6}}{104.72}\right)^2 = 0.44\,kgm^2$$

$$d)\,Acceleration\,time \quad \omega_{r,mech} = \int\frac{T_{mech}}{J_{ekv}}dt = \frac{T_{mech}}{J_{ekv}}\cdot t_{acc} \Rightarrow t_{acc} = \frac{\omega_{r,mech}\cdot J_{ekv}}{T_{mech}} = \frac{104.72\cdot 0.44}{1.91} = 24.1\,s$$

6

Losses and temperature

Exercise 6.1 - 1Q converter, losses and temperature

A buck converter supplies a load according to the figure. The semiconductor are mounted on a heat sink . The converter is modulated with a 5 kHz carrier wave, U_{dc} *= 400V, e=100V, L=1.5 mH*. **The current to the load has an average value of 10 A. The following data is extracted from data-sheets:**

IGBT:

- *Threshold voltage = 1.0 V*
- *Differential resistance = 5.0 mOhm.*
- *Turn-on loss Eon = 1.5 mJ assuming a DC link voltage and current of 400 V DC and 50 A*
- *Turn-off loss Eoff = 0.6 mJ assuming a DC link voltage and current of 400 V DC and 50 A*

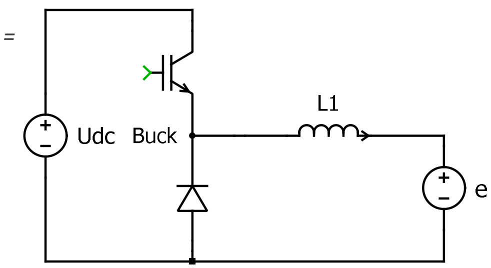
Diode:

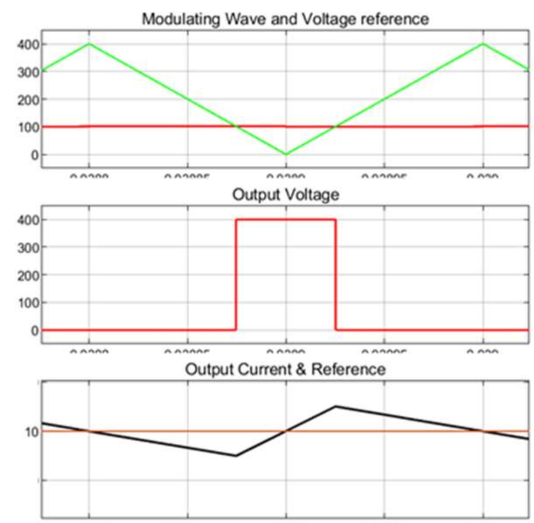
- *Threshold voltage = 0.8 V*
- *Differential resistance = 7 mohm.*
- *Reverse recovery Charge Qf = 1 µC @ 400 V DC link & 50 A*

Thermal:

- *Thermal resistance of the heat sink* $R_{th,ha}$ *= 2.6 K/W*
- *Thermal resistance of the IGBT* R_{th,jc_T} *= 0.6 K/W*
- *Thermal resistance of the Diode* R_{th,jc_D} *= 0.7 K/W*
- *Ambient temperature = 35 C*
- *Disregard the thermal resistance case-to-heatsink.*

a) Calculate the current ripple.
b) Calculate the losses of the transistor and the diode.
c) Calculate the junction temperatures of the transistor and the diode.





Exercise 6.1, datasheets

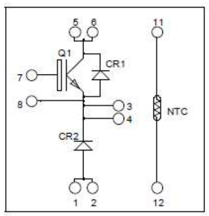
https://www.microsemi.com/existing-parts/parts/137150#resources

Microsemi Power Matters.

APTGTQ100SK65T1G

Buck chopper
High speed IGBT 5 Power Module

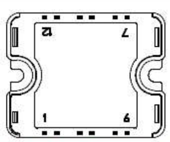
V_{CES} = 650V
I_C = 100A @ Tc = 25°C



Application
- AC and DC motor control
- Switched Mode Power Supplies

Features
- High speed IGBT 5
  - Low voltage drop
  - Low tail current
  - Switching frequency up to 100 kHz
  - Low leakage current
- Very low stray inductance
- Internal thermistor for temperature monitoring

Benefits
- Outstanding performance at high frequency operation
- Direct mounting to heatsink (isolated package)
- Low junction to case thermal resistance
- Solderable terminals both for power and signal for easy PCB mounting
- Low profile
- RoHS compliant



Pins 1/2 ; 3/4 ; 5/6 must be shorted together

All ratings @ T_j = 25°C unless otherwise specified

Absolute maximum ratings

Symbol	*Parameter*		*Max ratings*	*Unit*
V_{CES}	Collector - Emitter Voltage		650	V
I_C	Continuous Collector Current	T_C = 25°C	100	A
		T_C = 80°C	60	
I_{CM}	Pulsed Collector Current	T_C = 25°C	200	
V_{GE}	Gate – Emitter Voltage		±20	V
P_D	Power Dissipation		250	W

CAUTION: These Devices are sensitive to Electrostatic Discharge. Proper Handling Procedures Should Be Followed.

Microsemi Power Matters.

APTGTQ100SK65T1G

Electrical Characteristics

Symbol	*Characteristic*	*Test Conditions*		*Min*	*Typ*	*Max*	*Unit*
I_{CES}	Zero Gate Voltage Collector Current	V_{GE} = 0V, V_{CE} = 650V				100	µA
$V_{CE(sat)}$	Collector Emitter Saturation Voltage	V_{GE} =15V, I_C = 100A	T_j = 25°C		1.65	2.2	V
			T_j = 150°C		1.9		
$V_{GE(th)}$	Gate Threshold Voltage	V_{GE} = V_{CE}, I_C = 1mA		3.3	4.0	4.7	V
I_{GES}	Gate – Emitter Leakage Current	V_{GE} = 20V, V_{CE} = 0V				240	nA

Dynamic Characteristics

Symbol	*Characteristic*	*Test Conditions*		*Min*	*Typ*	*Max*	*Unit*
C_{ies}	Input Capacitance	V_{GE} = 0V, V_{CE} = 25V, f = 1MHz			6000		pF
C_{oes}	Output Capacitance				100		
C_{res}	Reverse Transfer Capacitance				22		
Q_G	Gate charge	V_{GE} = 15V, I_C = 100A, V_{CE} = 520V			240		nC
$T_{d(on)}$	Turn-on Delay Time	Inductive Switching (25°C), V_{GE} = 15V, V_{Bus} = 400V, I_C = 50A, R_G = 2Ω			21		ns
T_r	Rise Time				15		
$T_{d(off)}$	Turn-off Delay Time				180		
T_f	Fall Time				18		
$T_{d(on)}$	Turn-on Delay Time	Inductive Switching (150°C), V_{GE} = 15V, V_{Bus} = 400V, I_C = 50A, R_G = 2Ω			20		ns
T_r	Rise Time				15		
$T_{d(off)}$	Turn-off Delay Time				205		
T_f	Fall Time				26		
E_{on}	Turn on Energy	V_{GE} = 15V, V_{Bus} = 400V, I_C = 50A, R_G = 2Ω	T_j = 150°C		1.5		mJ
E_{off}	Turn off Energy		T_j = 150°C		0.6		
R_{Gint}	Integrated gate resistor				2.5		Ω
R_{thJC}	Junction to Case Thermal Resistance					0.6	°C/W

Diode ratings and characteristics (Per diode)

Symbol	*Characteristic*	*Test Conditions*		*Min*	*Typ*	*Max*	*Unit*
V_{RRM}	Peak Repetitive Reverse Voltage					650	V
I_{RM}	Reverse Leakage Current	V_R=650V				100	µA
I_F	DC Forward Current		Tc = 25°C		100		A
V_F	Diode Forward Voltage	I_F = 100A, V_{GE} = 0V	T_j = 25°C		1.6	2.2	V
			T_j = 150°C		1.65		
t_{rr}	Reverse Recovery Time	I_F = 50A, V_R = 400V, di/dt =3000A/µs	T_j = 25°C		46		ns
			T_j = 150°C		62		
Q_{rr}	Reverse Recovery Charge		T_j = 25°C		1		µC
			T_j = 150°C		2		
R_{thJC}	Junction to Case Thermal Resistance					0.7	°C/W

Microsemi Power Matters.

APTGTQ100SK65T1G

Temperature sensor NTC (see application note APT0406 on www.microsemi.com).

Symbol	*Characteristic*		*Min*	*Typ*	*Max*	*Unit*
R_{25}	Resistance @ 25°C			50		kΩ
$\Delta R_{25}/R_{25}$				5		%
$B_{25/85}$	T_{25} = 298.15 K			3952		K
ΔB/B		T_C=100°C		4		%

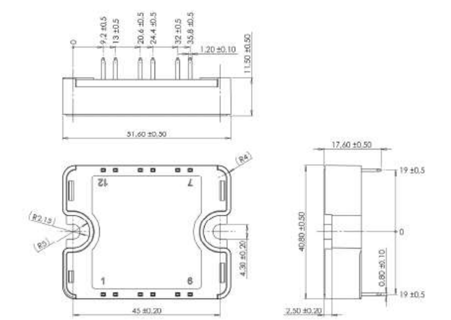
$$R_T = \frac{R_{25}}{\exp\left[B_{25/85}\left(\frac{1}{T_{25}} - \frac{1}{T}\right)\right]}$$

T: Thermistor temperature
R_T: Thermistor value at T

Thermal and package characteristics

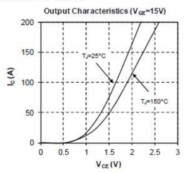
Symbol	*Characteristic*			*Min*	*Max*	*Unit*
V_{ISOL}	RMS Isolation Voltage, any terminal to case t =1 min, 50/60Hz			4000		V
T_J	Operating junction temperature range			-40	175	°C
T_{JOP}	Recommended junction temperature under switching conditions			-40	$T_J$max -25	
T_{STG}	Storage Temperature Range			-40	125	
T_C	Operating Case Temperature			-40	125	
Torque	Mounting torque	To heatsink	M4	2	3	N.m
Wt	Package Weight				80	g

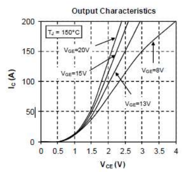
Package outline (dimensions in mm)

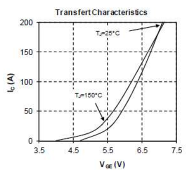


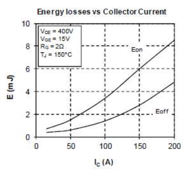
See application note 1904 - Mounting Instructions for SP1 Power Modules on www.microsemi.com

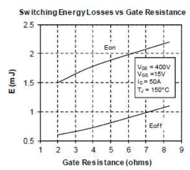
Typical performance curve

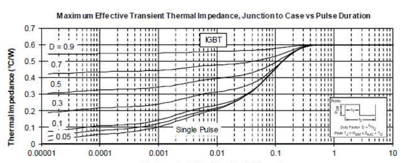




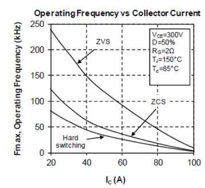


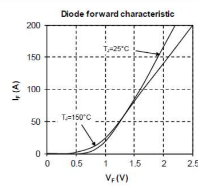


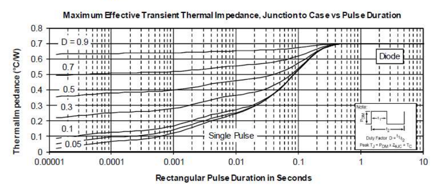




APTGTQ100SK65T1G







BGA Heat Sink - High Performance maxiFLOW/superGRIP

ATS Part#:	ATS-X50400G-C1-R0
Description:	40.00 x 40.00 x 12.50 mm BGA Heat Sink - High Performance maxiFLOW/superGRIP
Heat Sink Type:	maxiFLOW
Heat Sink Attachment:	superGRIP
Equivalent Part Number:	N/A

Image above is for illustration purpose only.

Features & Benefits

- Designed for 40 x 40 mm components
- Requires minimal space around the component's perimeter; ideal for densely populated PCBs
- Allows the heat sink to be detached and reattached without damaging the component or the PCB, an important feature in the event a PCB may need to be reworked
- Strong, uniform attachment force helps achieve maximum performance from phase-changing TIMs
- Eliminates the need to drill mounting holes in the PCB

Thermal Performance

AIR VELOCITY		@200 LFM 1.0 M/S	@300 LFM 1.5 M/S	@400 LFM 2.0 M/S	@500 LFM 2.5 M/S	@600 LFM 3.0 M/S	@700 LFM 3.5 M/S	@800 LFM 4.0 M/S
THERMAL RESISTANCE	Unducted Flow	2.6 °C/W	2 °C/W	1.8 °C/W	1.6 °C/W	1.4 °C/W	1.3 °C/W	1.2 °C/W
	Ducted Flow	2	N/A	N/A	N/A	N/A	N/A	N/A

Product Detail

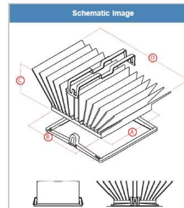
Schematic Image	Dimension A	Dimension B	Dimension C	Dimension D	TIM	Finish
	40.00 mm	40.00 mm	12.50 mm	69.2 mm	T766	BLUE-ANODIZED

Image above is for illustration purpose only.

Notes:

- Dimension A and B refer to component size.
- Dimension C is the heat sink height from the bottom of the base to the top of the fin field.
- Thermal performance data are provided for reference only. Actual performance may vary by application.
- ATS reserves the right to update or change its products without notice to improve the design or performance.
- ATS certifies that this heat sink assembly is RoHS-6 and REACH compliant.
- Contact ATS to learn about custom options available.

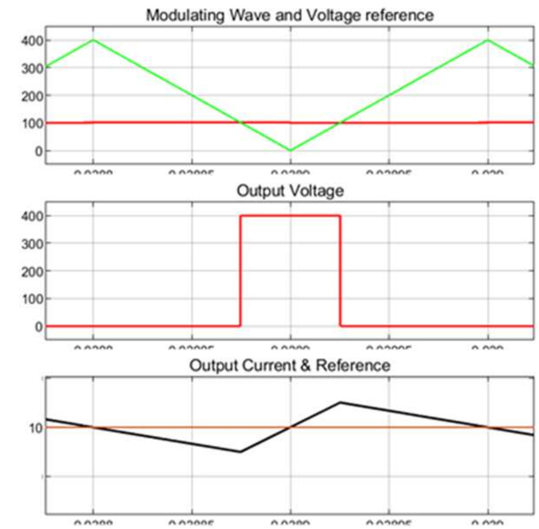
For more information, to find a distributor or to place an order, please contact us at 781-769-2800 (North America), sales@qats.com or www.qats.com.

Innovations in Thermal Management®

Rev - 040517

Solution 6.1 a

a) $\Delta i = \frac{Udc - e}{L} \cdot \Delta t = \frac{400-100}{1.5e^{-3}} \cdot \frac{1}{5000} \cdot \frac{1}{4} = 10$



Solution 6.1 b(1)

The losses of the transistor consists of conduction losses and switching losses

The conduction losses requires both average and RMS-values of the current.

The average current of the transistor for the switch period (not the transistor conducts ¼'th of the period) is:

$$i_{T,ave} = \frac{10}{4} = 2.5\ A$$

The RMS current of the transistor is calculated as:

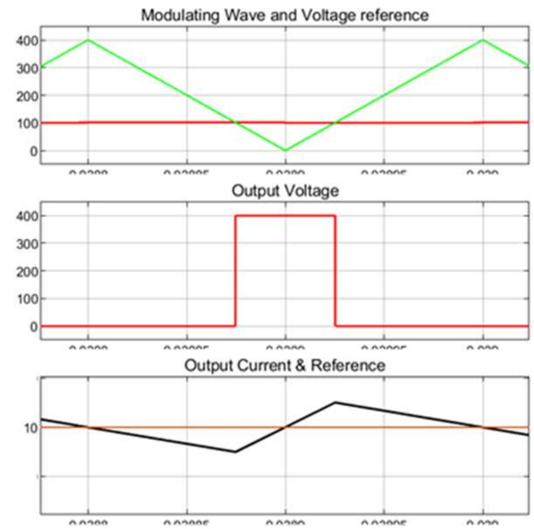
$$i_{T,RMS} = \sqrt{\frac{1}{T}\int_0^T i_T^2 dt} = \sqrt{\frac{1}{T}\int_0^{t_p}\left(i_{T,min} + \left(\frac{i_{T,max} - i_{T,min}}{t_p}\right)\cdot t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{t_p}\left(i_{T,min}\cdot\left(1-\frac{t}{t_p}\right) + i_{T,max}\cdot\frac{t}{t_p}\right)^2 dt} = \left\{t_p = \frac{\boldsymbol{T}}{4}\right\}$$

$$= \sqrt{\frac{1}{4}\left[\frac{i_{T,min}^2 + i_{T,min}\cdot i_{T,max} + i_{T,max}^2}{3}\right]} = \sqrt{\frac{1}{4}\cdot\frac{(5^2 + 15^2 + 5\cdot 15)}{3}} = 5.2\ A$$

The switching losses of the transistor can be calculated from the turn-on and the turn-off energies, scaled with the difference in voltage (no difference in this case, both 400 V) and current (50A vs 5 and 15 A respectively for turn on and turn off). Thus, the total transistor losses are:

$$P_T = 1.0\cdot 2.5 + 5e^{-3}\cdot 5.2^2 + \left(1.5e^{-3}\cdot\frac{5}{50} + 0.6e^{-3}\cdot\frac{15}{50}\right)\cdot 5000 = 4.3\ W$$



Solution 6.1 b(2)

The losses of the diode also consists of conduction losses and switching losses.

The conduction losses requires both average and RMS-values of the current.

The average current of the transistor for the switch period (not the transistor conducts ¼'th of the period) is:

$$i_{D,ave} = \frac{10 \cdot 3}{4} = 7.5\ A$$

The RMS current of the transistor is calculated as:

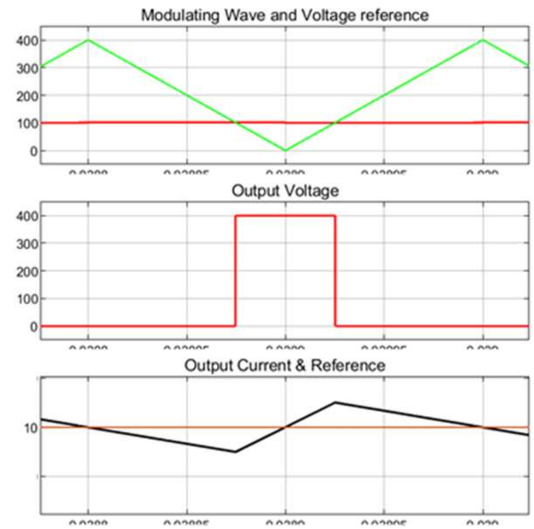
$$i_{T,RMS} = \sqrt{\frac{1}{T}\int_0^T i_T^2 dt} = \sqrt{\frac{1}{T}\int_0^{t_p}\left(i_{T,min} + \left(\frac{i_{T,max} - i_{T,min}}{t_p}\right)\cdot t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{t_p}\left(i_{T,min}\cdot\left(1 - \frac{t}{t_p}\right) + i_{T,max}\cdot\frac{t}{t_p}\right)^2 dt} = \left\{t_p = \frac{\boldsymbol{T}}{4}\right\}$$

$$= \sqrt{\frac{3}{4}\left[\frac{i_{T,min}^2 + i_{T,min}\cdot i_{T,max} + i_{T,max}^2}{3}\right]} = \sqrt{\frac{\mathbf{3}}{4}\cdot\frac{(5^2 + 15^2 + 5\cdot 15)}{3}} = \mathbf{9.0}\ A$$

The switching losses of the diode can be calculated form the "reverse recovery charge", see equation 6.17, scaled with the switching voltage (to become an Energy) and the switching frequency (to be a power = energy/second). Thus, the total diode losses are:

$$P_D = 0.8 \cdot 7.5 + \ 7e^{-3} \cdot 9.0^2 + \ 400 \cdot 1e^{-6} \cdot \frac{15}{50} \cdot 5000 = 7.2\ W$$

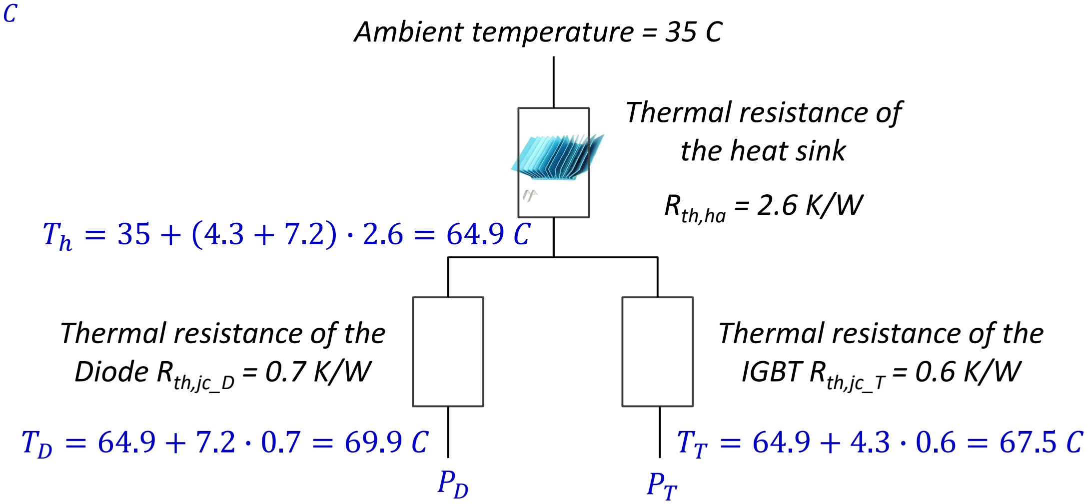


Solution 6.1 c

c) $T_h = 35 + (4.3 + 7.2) \cdot 2.6 = 64.9\ C$

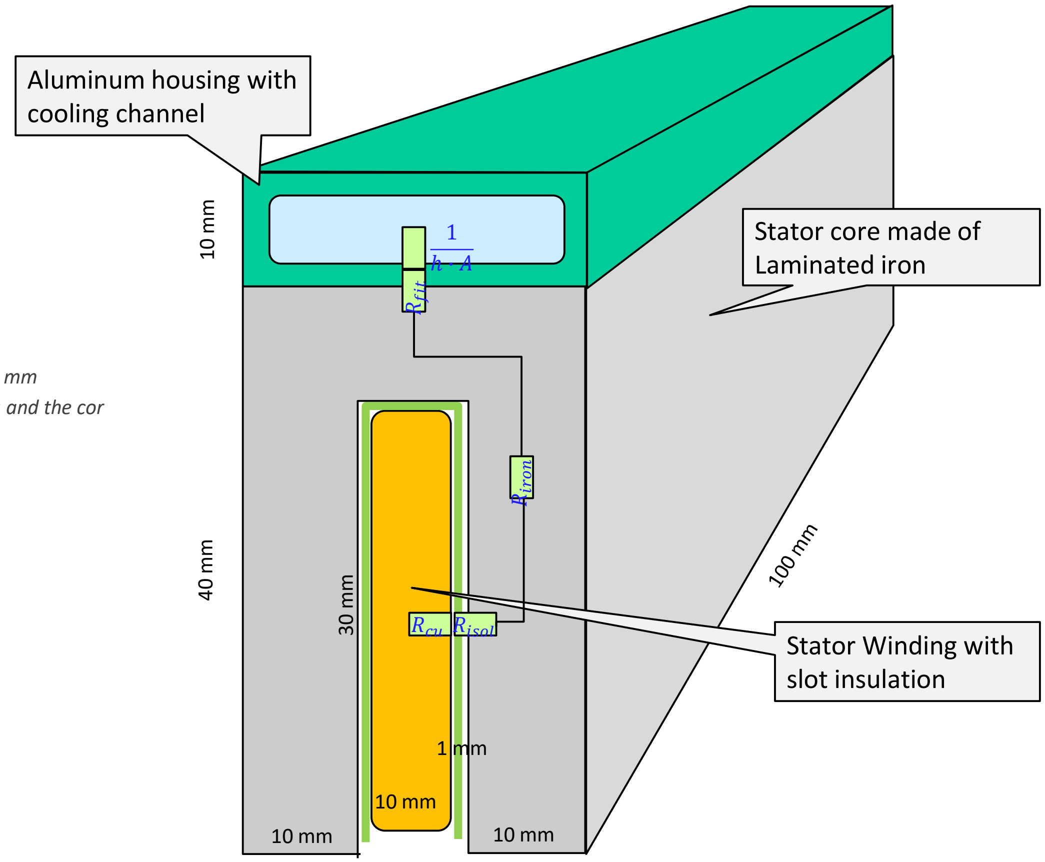
$T_T = 64.9 + 4.3 \cdot 0.6 = 67.5\ C$

$T_D = 64.9 + 7.2 \cdot 0.7 = 69.9\ C$



Exercise 6.2

- **Assume.**
  - ***For the winding***
    - *10 A/mm^2 in winding*
    - *60 C water temp*
    - *Copper resistivity: 1.7e-8 Ohm*m*
    - *Fill factor 50 %*
    - *All copper losses in one point in the middle of the winding*
    - *The slot liner is 1 mm thick*
    - *The iron path starts at half the tooth height and has tooth width (15+10) mm*
    - *The shrink fit of the core leads to a 0.05 mm airgap between the housing and the cor*
  - ***Cooling:***
    - *Heat transfer Coefficient h=1000*
- **Thermal conductivity (λ):**
  - ***Winding (Copper): 400***
  - ***Slot insulation: 1***
  - ***Stator core (Iron): 80***
  - ***Air: 0.024***
- **Estimate**
  - ***Conductor temperature***



6.2 Solution

- **Calculate the heat losses:**

$$P_{loss} = \rho_{el.cu} \cdot \frac{0.1}{0.01 \cdot 0.03 \cdot k_{fill}} \cdot (10e6 \cdot 0.01 \cdot 0.03 \cdot k_{fill})^2 = 26\ W$$

- **Calculate the thermal resitances:**

$$R_{cu} = \frac{1}{\lambda_{winding}} \cdot \frac{0.005}{0.030 \cdot 0.1} = 0.042\ [K/W]$$

$$R_{isol} = \frac{1}{\lambda_{liner}} \cdot \frac{0.001}{0.030 \cdot 0.1} = 0.33\ [K/W]$$

$$R_{iron} = \frac{1}{\lambda_{core}} \cdot \frac{0.025}{0.010 \cdot 0.1} = 0.31\ [K/W]$$

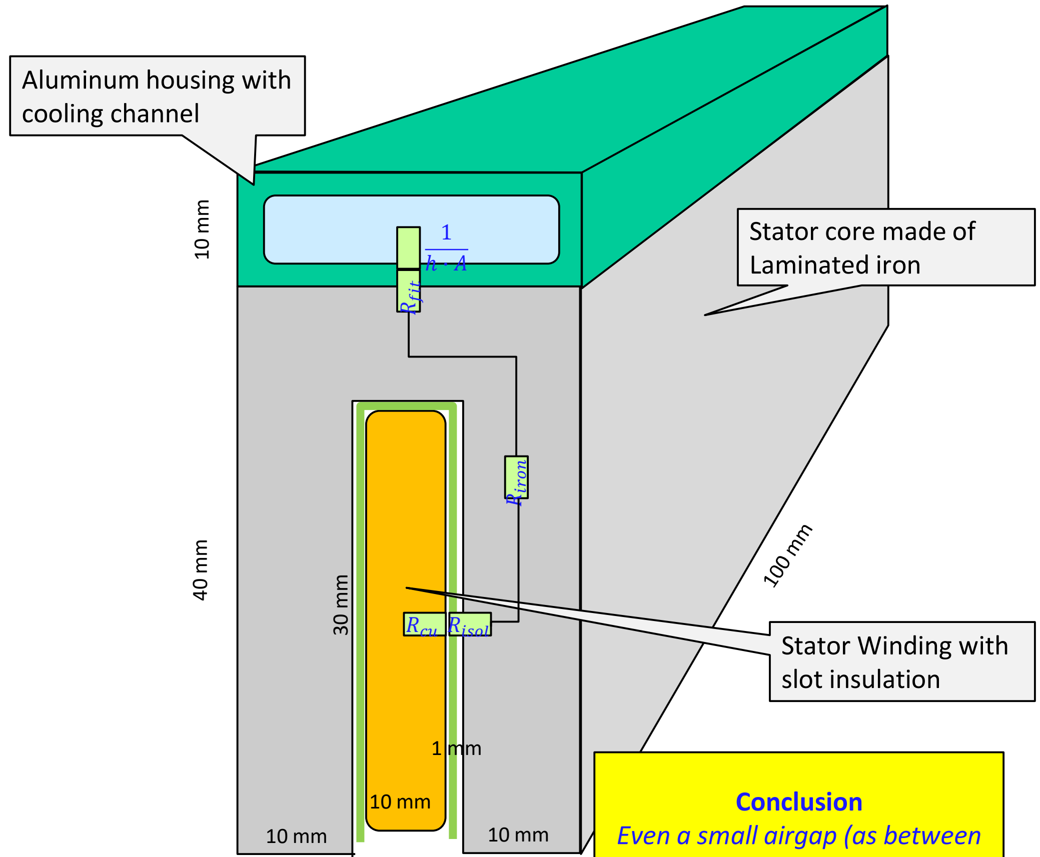
$$R_{fit} = \frac{1}{\lambda_{air}} \cdot \frac{0.00005}{0.015 \cdot 0.1} = 1.4\ [K/W]$$

- **Calculate the temperature drops**

0.67 $[K/W]$

$$T_{wind} = T_{coolant} + P_{loss} \cdot \left(R_{cu} + R_{isol} + R_{iron} + R_{fit} + \frac{1}{h \cdot A}\right) =$$

$$= 60+25.5*(0.042+0.33+0.31+1.4+1/1000/0.015/0.1) = 130\ C$$



Conclusion
Even a small airgap (as between the core and housing (0.05 mm assumed), can contribute a lot to the winding temperature!

7

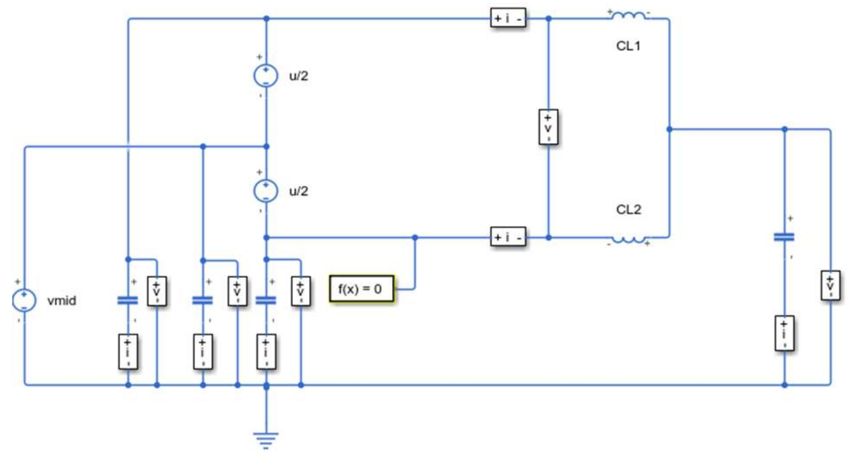
EMC

7.1 Common mode disturbance

- A symmetric 1-phase voltage source (u=320 V, 50 Hz) has its midpoint connected to a potential (vmid = 100 V, 1000 Hz) relative to ground.
- Between the 1-phase voltage source terminals there is an inductive load (L) connected, see figure.
- There are parasitic capacitors (C=10 μF) from some nodes to ground, see figure.

  a) Calculate the differential mode current in the load.

  b) Calculate the common mode current in the middle of the load.



7.1 solution

a) The differential model current

```
>> w = 2*pi*50;
>> L = 1e-3;
>> C = 1e-5;
>> udm = 320;
>> idm = udm/(w*2*L) =  509.2958
```

b) The common model current flows through the inductors in parallel

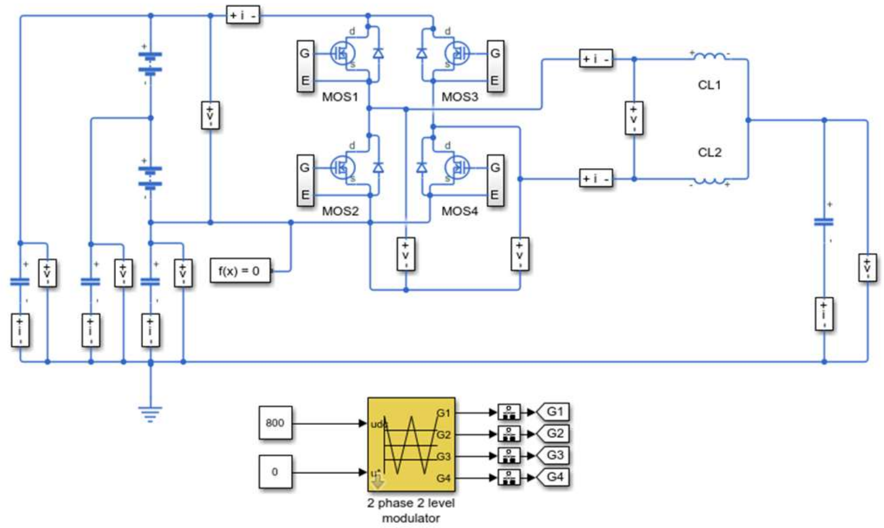
```
>> w = 2*pi*1000;
>> L = 1e-3;
>> C = 1e-5;
>> ucm = 100;
>> icm = ucm/(j*w*L + 1/j/w/C) = 0.0000 +10.3817i
```

7.2 Common model disturbance with a 4Q converter

- A 4Q converter is supplied from a series connection of 2 batteries at 400 V each.
- Between the 4Q converter output terminals there is an inductive load (L=1mH) connected, see figure.
- There are parasitic capacitors (C=10 μF) from some nodes to ground, see figure.
- The converter is modulated with carrier wave modulation at 5000 Hz. And voltage reference to the modulator u* = 0 V.

  a) Calculate the differential mode current in the load.

  b) Calculate the common mode current in the middle of the load.



Exam 2012-05-21

Exercise Exam 2012-05-21 1a - The four quadrant DC-DC converter

a) Draw a four quadrant DC/DC converter with a three phase diode rectifier connected to the power grid. Between the rectifier and the DC link capacitor is a BIG inductor connected. This inductor, the dc-link capacitor and protection against too high inrush currents should be included in the drawing. The transistors are of IGBT-type. (2 p.)

b) The three-phase grid, to which the three phase diode rectifier is connected, has the line-to-line voltage 400 V_{rms} at 50 Hz. The bridge output voltage of the four quadrant DC/DC converter is 430 V.
Calculate the average voltage at the rectifier dc output.
Calculate the duty cycle of the four quadrant dc/dc converter. (2 p.)

c) Due to the big inductor between the rectifier and the DC link capacitor the rectifier output DC-current can be regarded as constant, 172 A. The 4Q bridge load can be regarded as a constant voltage in series with a 5.1 mH inductance.

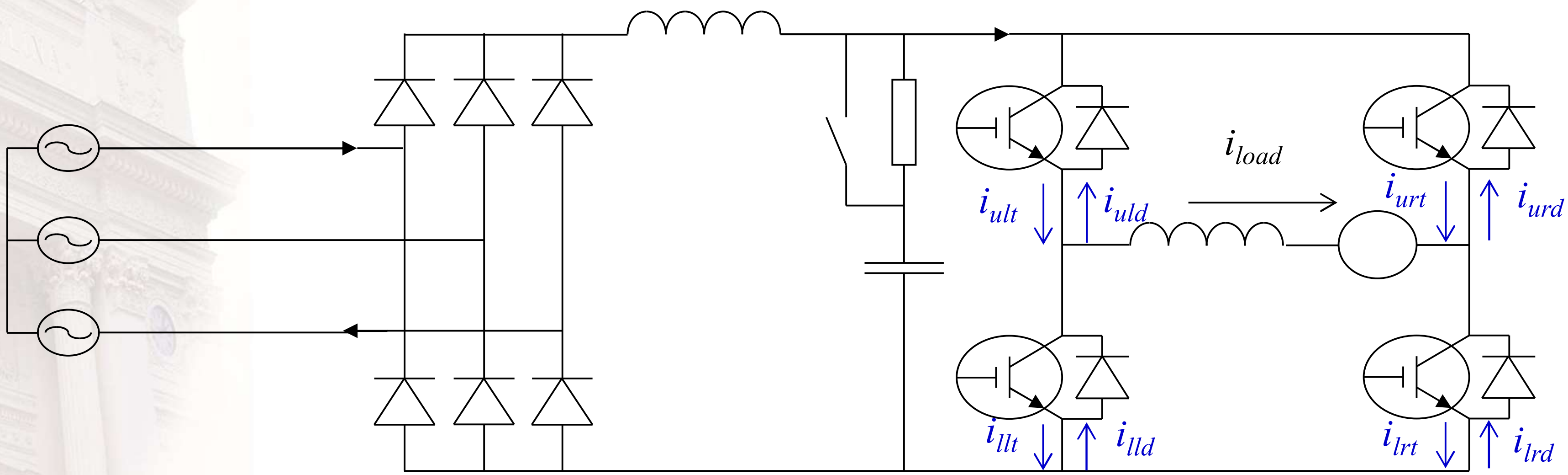
- The rectifier diode threshold voltage is 1.0 V and its differential resistance is 2.2 mohm.
- The rectifier diode turn-on and turn-off losses can be neglected
- The IGBT transistor threshold voltage is 1.4 V and its differential resistance is 12 mohm.
- The turn-on loss of the IGBT transistor is 65 mJ and its turn-off loss is 82 mJ.
- The IGBT diode threshold voltage is 1.1 V and its differential resistance is 9.5 mohm.
- The IGBT diode turn-off losses is 25 mJ, while the turn-on loss can be neglected
- Both the IGBT transistor and the IGBT diode turn-on and turn-off losses are nominal values at 900 V DC link voltage and 180 A turn-on and turn-off current.
- The switching frequency is 2 kHz.

Make a diagram of the 4Q load current
Calculate the rectifier diode losses.
Calculate the IGBT transistor losses of each IGBT in the four quadrant converter.
Calculate the IGBT diode losses of each IGBT in the four quadrant converter. (4 p.)

d) Which is the junction temperature of the IGBT transistor and of the IGBT diode, and which is the junction temperature of the rectifier diodes?

- The thermal resistance of the heatsink equals 0.025 K/W?
- The thermal resistance of the IGBT transistor equals 0.043 K/W?
- The thermal resistance of the IGBT diode equals 0.078 K/W?
- The thermal resistance of the rectifier diode equals 0.12 K/W?
- The ambient temperature is 42 ^{o}C.
- The rectifier diodes and the four quadrant converter IGBTs share the heatsink. (2 p.)

Solution Exam 2012-05-21 1a



Solution Exam 2012-05-21 1b

Average DC voltage

(Since the rectifier is loaded with a BIG inductor and in stationary state, the DC link voltage must be equal to the average of the rectified grid voltage)

$$U_{dc\_ave} = \frac{3}{\pi} \cdot 400 \cdot \sqrt{2}\ V = 540\ V$$

4Q average bridge voltage

(This is given in the question)

$$U_{dc4QC} = 430\,V$$

4Q output voltage duty cycle

(The 4Q output voltage is modulated to 430 V from 540 V DC))

$$D = \frac{430}{540} = 0.8$$

Solution Exam 2012-05-21 1c_1

Rectifier diode current	*172 A*
Rectifier diode threshold voltage	*1.0 V*
Rectifier diode diff resistance	*2.2 mohm*
Rectifer diode on state voltage	*1+172*0.0022=1.38 V*
Rectifier diode power loss	*1.38*172*0.33=78 W (conducting 33% of time)*
Rectifer diode thermal resistance	*0,12 K/W*
Continous rectifier output current	*172 A*
The continous 4Q load current	*172/0.8=215 A (to maintain the power)*

Solution Exam 2012-05-21 1c_2

4Q load current $I_{pulse,avg}$=215A

4Q load inductance 5.1 mH

Only the upper left and lower right transistors have losses and the lower left and upper right diodes have losses. The other semiconductors do not conduct since the 4Q output current is strictly positive.

The load current ripple can be calculated as:

$$\Delta i = \frac{u-e}{L}\Delta t = \frac{540-430}{0.0051}\ 0.8 * \frac{1}{2*2000} = 4.3\ A$$

The "duty cycle" of the upper left, and lower right, transistor current is:

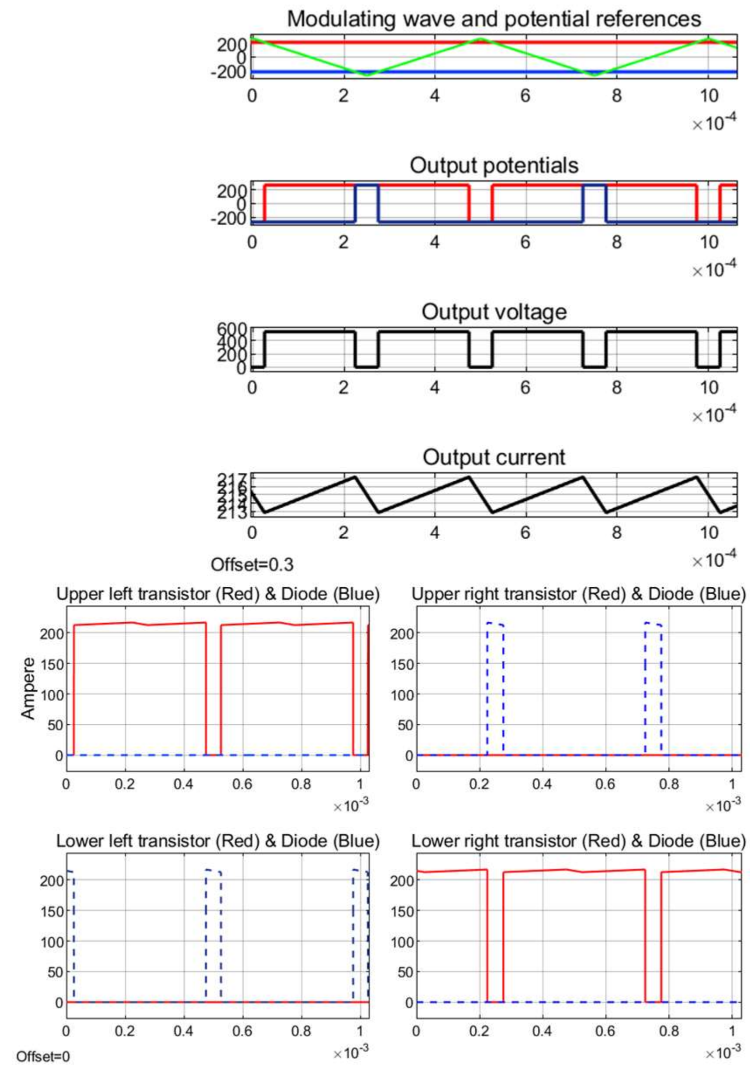
$$D_{tr} = 1 - \frac{D}{2} = 0.9$$

The average transistor current is

$$i_{T.ave} = Dtr * I_{pulse,avg} = 194\ A$$

The rms value of the transistor currents is:

$$i_{Tr,rms} = \sqrt{D_{tr} * (i_1^2 + \Delta i * i_1 + \frac{\Delta i^2}{3})} = 204\ A$$



Solution Exam 2012-05-21 1c_3

The "duty cycle" of the upper left, and lower right, diode current is:

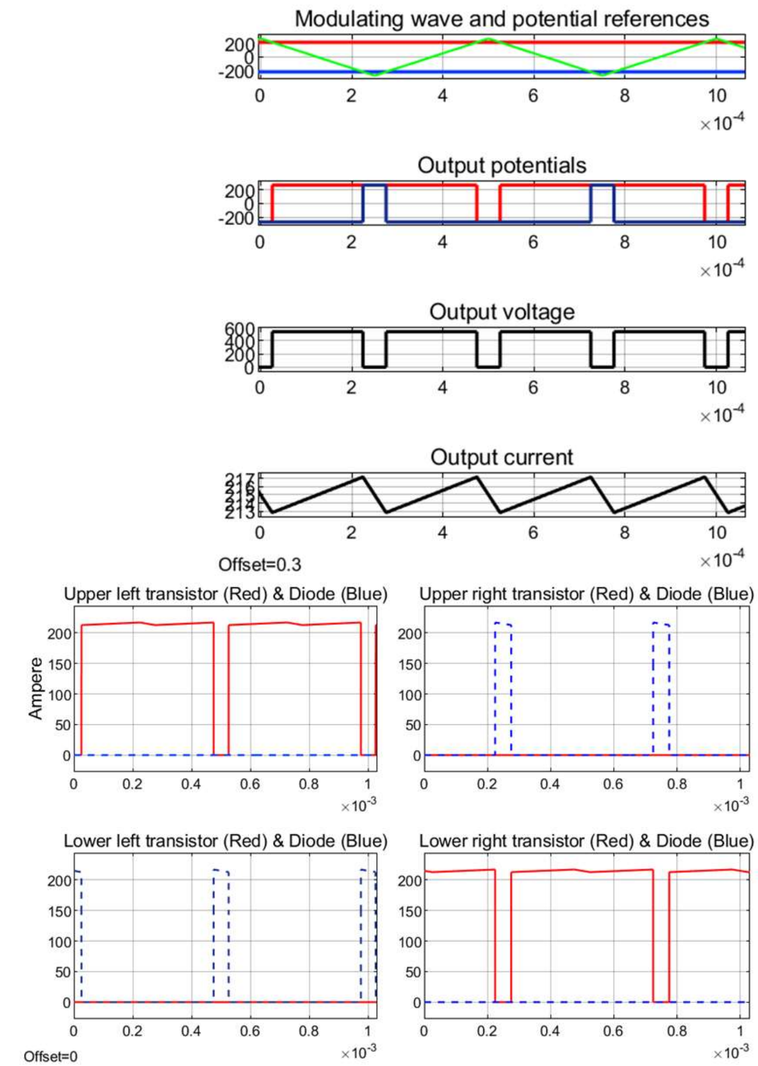
$$D_d \frac{D}{2} = 0.1$$

The average diode current is

$$i_{D,ave} = Dtr * I_{pulse,avg} = 21.5\ A$$

The rms value of the transistor currents is:

$$i_{D,rms} = \sqrt{D_d * (i_1^2 + \Delta i * i_1 + \frac{\Delta i^2}{3})} = 68\ A$$



Solution Exam 2012-05-21 1c_4

4QC transistor rms-current	*204A*
4QC transistor avg-current	*194A*
4QC transistor threshold voltage	*1.4 V*
4QC transistor diff resistance	*12 mohm*
4QC transistor turn-on loss	*65 mJ*
4QC transistor turn-off loss	*82 mJ*
4QC transistor thermal resistance	*0,043 K/W*

$$P_{onstate} = 1.4 \cdot 194 + 204^2 \cdot 0.012 = 771\,W$$

$$P_{switch} = 2000 \cdot \left(0.065 \cdot \frac{210.7}{180} + 0.082 \cdot \frac{219.3}{180} \right) \cdot \frac{540}{900} = 211\,W$$

$$P_{total} = 771 + 211 = 982\,W$$

Solution Exam 2012-05-21 1c_5

4QC diode threshold voltage	*1.1 V*
4QC diode diff resistance	*9.5 mohm*
4QC diode turn-off losses	*25 mJ*
4QC diode thermal resistance	*.078 W/K*

$$\begin{cases} I_{\max} = 219.3\,A \\ I_{\min} = 210.7\,A \end{cases}$$

$$\begin{cases} I_{rms} = \sqrt{0.1 \cdot \left(\dfrac{219.3^2 + 219.3 \cdot 210.7 + 210.7^2}{3} \right)} = 68.0\,A \\ I_{avg} = 0.1 \cdot \left(\dfrac{219.3 + 210.7}{2} \right) = 21.5\,A \end{cases}$$

$$\begin{cases} P_{onstate} = 1.1 \cdot 21.5 + 68^2 \cdot 0.0095 = 67.6W \\ P_{switch} = 2000 \cdot 0.025 \cdot \dfrac{210.7}{180} \cdot \dfrac{540}{900} = 35.1W \\ P_{total} = 67.6 + 35.1 = 103W \end{cases}$$

Solution Exam 2012-05-21 1c_6

Upper left IGBT transistor loss	*982 W*
Upper right IGBT transistor loss	*0 W*
Lower right IGBT transistor loss	*982 W*
Lower left IGBT transistor loss	*0 W*
Upper right IGBT diode loss	*103 W*
Upper left IGBT diode loss	*0 W*
Lower left IGBT diode loss	*103 W*
Lower right IGBT diode loss	*0 W*

Solution Exam 2012-05-21 1d

Rectifier diode (6) *Loss each 78 W* *Rth diode 0.12 K/W* *Temp diff 9.4 ^{o}C*	*IGBT diode (2)* *Loss each 103 W* *Rth diode 0.078K/W* *Temp diff 8.0 ^{o}C*	*IGBT transistor (2)* *Loss each 982W* *Rth trans 0.043 K/W* *Temp diff 42.2 ^{o}C*

Heatsink

Contribution fron 6 rectifier diodes and from two IGBT.

Heatsink thermal resistance	*0.025 K/W*
Ambient temperature	*42 ^{o}C*
Total loss to heatsink	*6*78+2*103+2*982=2638 W*
Heatsink sink temperature	*42+2656*0.025=108 ^{o}C*

Junction temperature

Rectifier diode	*108+9.4=117 ^{o}C*
IGBT diode	*108+8=116 ^{o}C*
IGBT transistor	*108+42.2=150 ^{o}C*

Exam 20120521 2 - Snubbers, DC/DC converter, semiconductor

a. Draw an IGBT equipped step down chopper (buck converter)
with an RCD snubber. The dclink voltage on the supply side is 250V and the load voltage is 175 V.
Give a detailed description of how the RCD charge-discharge snubber should operate. Explain why the snubbers are needed (2 p.)

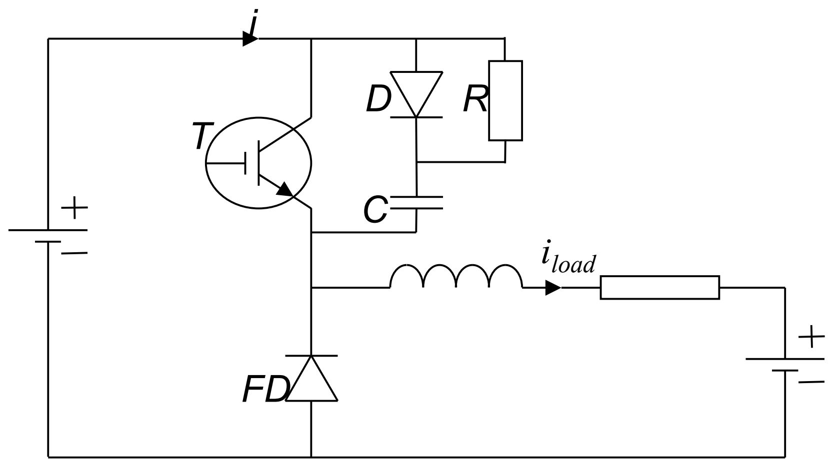
b. Calculate the snubber capacitor for the commutation time
0.012 ms.
The load current is 12 A, assumed constant during the commutation.
Calculate the snubber resistor so the discharge time (3 time constants)
of the snubber capacitor is less than the IGBT on state time.
The switch frequency is 1.5 kHz (4 p.)

Exam 20120521 2 - Snubbers, DC/DC converter, semiconductor

c. Draw the main circuit of a forward DC/DC converter.
The circuit should include DM-filter (differential mode) ,
CM (common mode)
filter, rectifier, dc link capacitors, switch transistor
and a simple output filter.
The circuit should also include snubbers. (2 p.)

d. Draw the diffusion structure of a MOSFET. In the figure the different doping areas
must be found. Draw where in the structure the unwanted
stray transistor effect can be found. What is done to avoid
this effect.
Also draw where in the structure
the anti-parallel diode effect can be found. (2 p.)

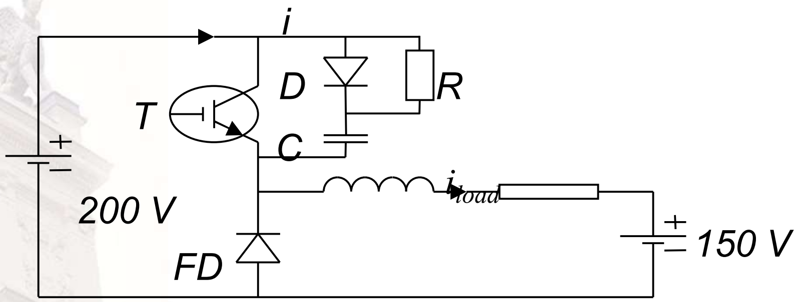
Solution Exam 2012-05-21 2a



At turn off of transistor *T*, the current *i* commtutates over to the capacitor *C* via diode *D*. The capacitor *C* charges until the potential of the transistor emitter reduces till the diode *FD* becomes forward biased and thereafter the load current i_{load} flows through diode *FD* and the current $i=0$.

A turn on of the transistor *T,* the capacitor *C* is discharged via the the transistor *T* and resistor *R*. *T*he diode *FD* becomes reverse biased and the current *i* commutates to the transistor *T*.

Solution Exam 2012-05-21 2b



Load current	*12 A*
Supply voltage	*250 V*
Load voltage	*175 V*
Commutation time	*0.012 ms*
Switching frequency	*1.5 kHz*

At turn off of transistor *T*, the capacitor *C* charges until the potential of the transistor emitter reduces till the diode *FD* becomes forward biased and thereafter the load current commutates to the freewheeling diode.

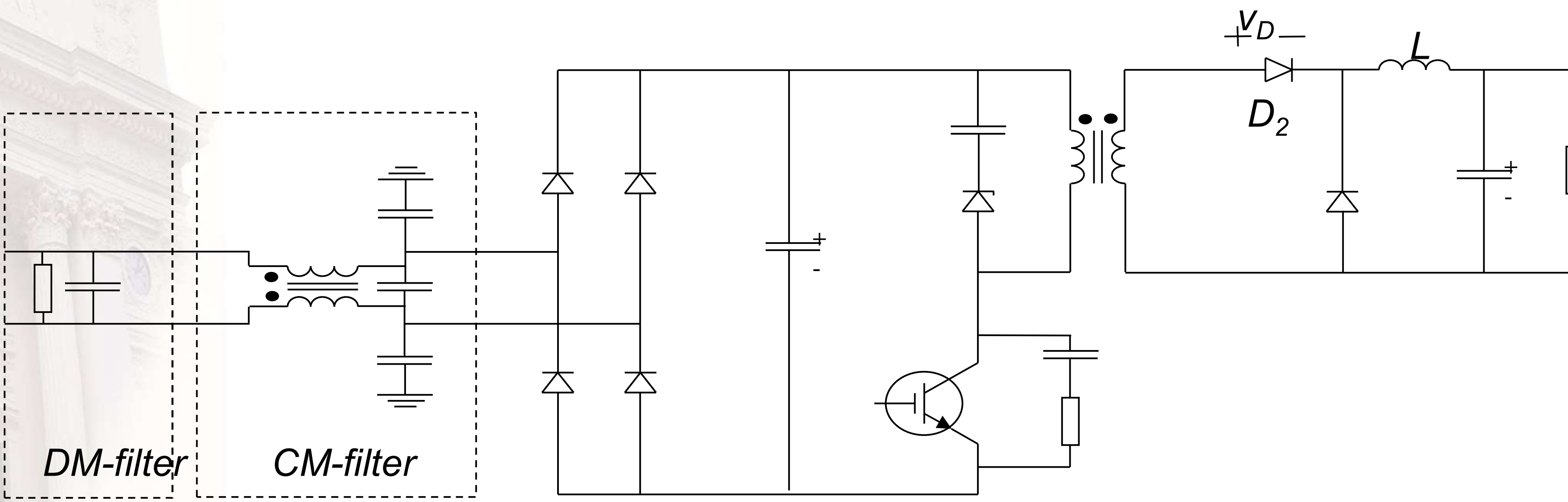
$$i = C \cdot \frac{du}{dt} \quad \Rightarrow \quad C = \frac{i \cdot dt}{du} = \frac{12 \cdot 12 \cdot 10^{-6}}{250} = 0.58 \ \mu F$$

A turn on of the transistor *T*the current *i* commutates to the transistor *T*, and the capacitor *C* is discharged via the the transistor *T* and resistor *R*. As the load voltage is 175V the duty cycle is 70%. The switching frequency is 1.5 kHz and the on state time is 0.47 ms, and thus the time constant =0.16 ms

$$\tau = C \cdot R \quad \Rightarrow \quad R = \frac{\tau}{C} = \frac{156 \cdot 10^{-6}}{0.58 \cdot 10^{-6}} = 269 \ \Omega$$

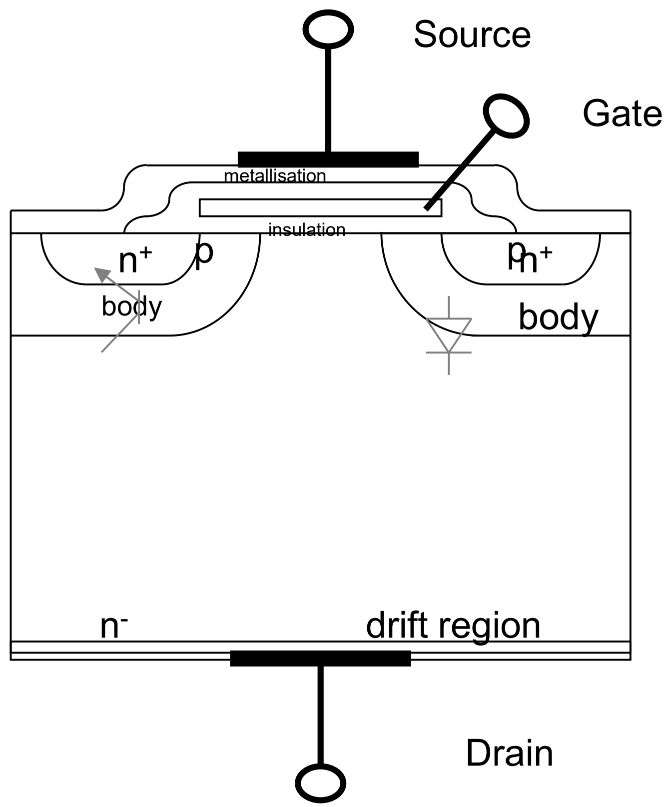
Solution Exam 2012-05-21 2c

Forward converter with snubbers
and common mode (CM) and differential mode (DM) filter



Solution Exam 2012-05-21 2d

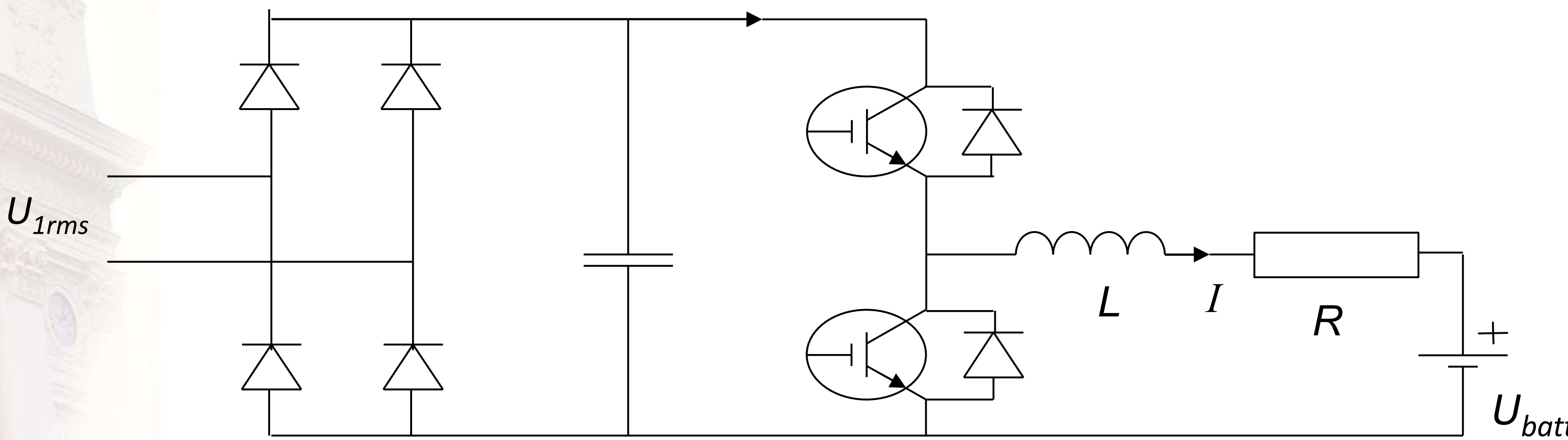
Diffusion structure of a MOSFET



The npn-transistor structure is formed of the n^+, the p (body) and the n^- (drift region), which cannot be turned off.
The gate metallisation short circuits the n+ and the p (body) to avoid turning on this unwanted transistor

Exercise Exam 20120521 3_1 - The buck converter as battery charger

A battery charger is supplied from a symmetrical single phase system.
A dc voltage is created by a two pulse diode bridge and a 2-quadrant dc-converter is used for the charge current control.



Data:

U_{1rms}= the phase-voltage rms value = 220 V, 50 Hz.
The switching frequency is f = 4 kHz.
L = 4 mH and R=0 Ohms.
U_{batt} = 100 V and is approximated to be independent of the charge current.

Exam 20120521 3_2 - The buck converter as battery charger

a) What dc link voltage *Ud* will you get I) when the charging current is zero and II) when the charging current is non-zero with a perfectly smooth rectified current ? (2p)

b) Start with the electrical equation for the load and derive a suitable current control algorithm, giving all approximations you use. (4p)

c) Draw a current step from 0 A till 10 A in the load current. The modulating wave (*um*), the voltage reference (*u**), the output voltage (*u*) and current (*ibatt*) must be shown. Indicate the sampling frequency you use in relation to the switching frequency. (4p)

Solution Exam 2012-05-21 3a

Average dc voltage with average dc current

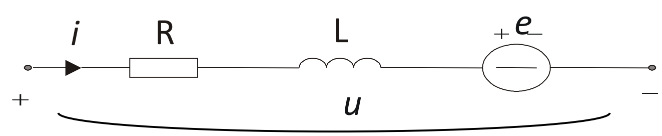
$$U_{dc\_ave} = \frac{2}{\pi} \cdot 220 \cdot \sqrt{2}\ V = 198\ V$$

Max dc voltage with zero dc current

$$U_{dc\_\max} = 220 \cdot \sqrt{2}\, V = 311 V$$

Solution Exam 20120521 3b_1

Current controller with fast computer



$$u = R \cdot i + L \cdot \frac{di}{dt} + e$$

$$\frac{\int_{k \cdot T_s}^{(k+1)T_s} u \cdot dt}{T_s} = \frac{R \cdot \int_{k \cdot T_s}^{(k+1)T_s} i \cdot dt + L \cdot \int_{k \cdot T_s}^{(k+1)T_s} \frac{di}{dt} \cdot dt + \int_{k \cdot T_s}^{(k+1)T_s} e \cdot dt}{T_s}$$

$$\overline{u}(k, k+1) = R \cdot \overline{i}(k, k+1) + L \cdot \frac{i(k+1) - i(k)}{T_s} + \overline{e}(k, k+1)$$

Solution Exam 20120521 3b_2

$$\overline{u}(k,k+1) = u^*(k) \qquad (a)$$

$$i(k+1) = i^*(k) \qquad (b)$$

$$\overline{i}(k,k+1) = \frac{i^*(k)+i(k)}{2} \qquad (c)$$

$$\overline{e}(k,k+1) = e(k) \qquad (d)$$

$$i(k) = \sum_{n=0}^{n=k-1} \left(i^*(n) - i(n)\right) \qquad (e)$$

$$u^*(k) = \{R=0\} = L \cdot \frac{i^*(k)-i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\left(i^*(k)-i(k)\right)}_{Proportional} + \underbrace{e(k)}_{\substack{Feed\\forward}}$$

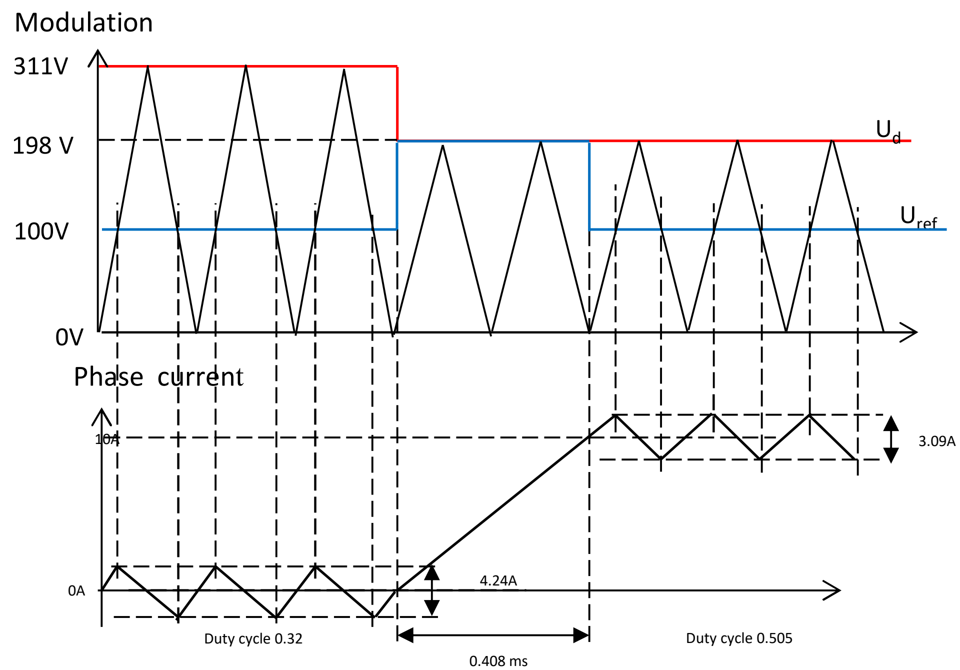
$$u^*(k) = \{R=0\} = L \cdot \frac{i^*(k)-i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\Delta i}_{Proportional} + \underbrace{e(k)}_{\substack{Feed\\forward}}$$

Solution Exam 20120521 3c

Constant 0 A
*Rectifier dc-voltage 220*1.414=311 V*
Voltage ref with const 0 A =100 V
Duty cycle 100/311=0.32
*On pulse 0.25 *0.32=0.080 ms*
*Current ripple =(311-100)/0.004*0.00008=4.24A*

Load current 0 to 10 A
*Rectifier dc-voltage 2/3.14*220*1.414=198 V*
Inductive voltage drop at current step =98 V Time to reach 10 A t=10.004/98=0.408 ms*
More than one sample time, set duty cycle =1

Constant 10 A
*Rectifier dc-voltage 2/3.14*220*1.414=198 V*
Duty cycle with 10 A = 100/198=0.505
*On pulse 0.25 *0.505=0.126 ms*
Voltage ref at const 10 A=100 V
Inductive voltage drop at current step =98 V
Current ripple=(198-100)/.004.000126=3.09 A*



Exam 20120521 4 - 4Q Converter & 3 Phase

In a 4Q DC/DC converter using PWM bipolar voltage switching, the bridge load consist of a constant voltage E (e.g. the back emf of a dc-motor) and an inductor L_a, the inductor resistance can be neglected.
The switching frequency is f_s, and the DC link voltage is V_d.

a. Calculate the maximum peak-to-peak load current ripple, expressed in V_d, L_a and f_s,. (5 p.)

b. Draw the circuit of a current control block for a generic three phase RLE load.
The drawing shall include three phase inverter, reference and load current measurement.
It must be clear in which blocks the different frame transformations occur. (5 p.)

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Solution Exam 2012-05-21 4a

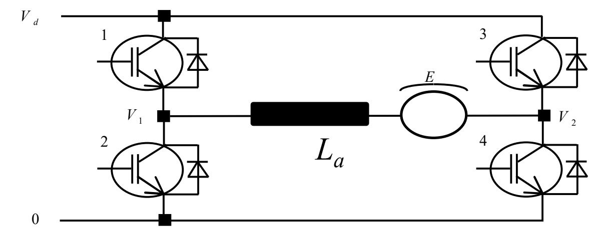
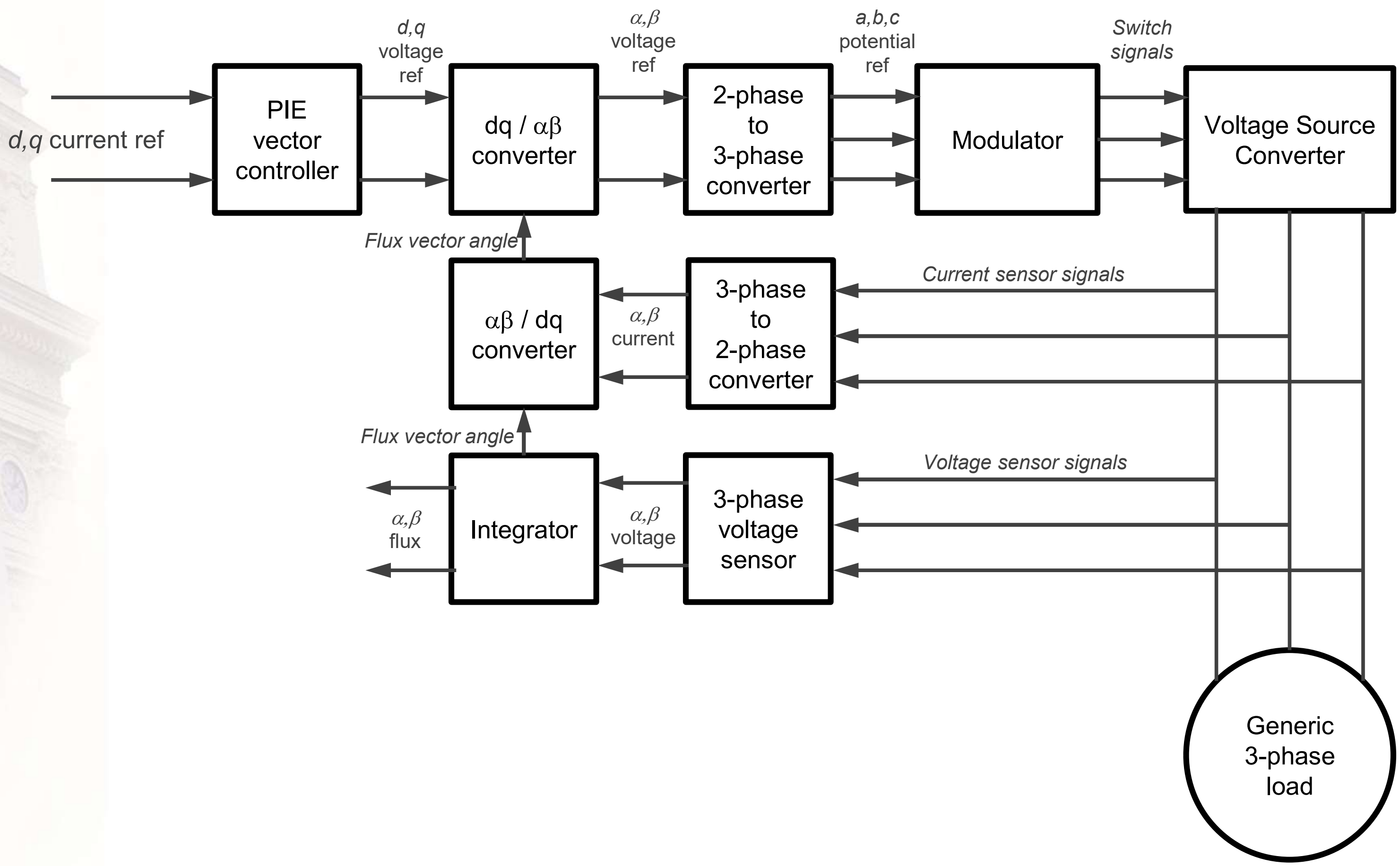


Fig 1

Control ratio	x
On – pulse duration	$\Delta t = x \cdot T_{s\_per} = \dfrac{x}{f_s}$
Phase voltages	$V_{1\_avg} = x \cdot V_d$
	$V_{2\_avg} = (1-x) \cdot V_d = V_d - x \cdot V_d$
Voltage over motor	$e = V_{1\_avg} - V_{2\_avg} = x \cdot V_d - V_d + x \cdot V_d = 2 \cdot V_d \cdot x - V_d$
At current rise, switch 1 and 4 are turned – on	$V_1 = V_d$
	$V_2 = 0$
Voltage over inductor	$V_L = V_1 - e - V_2 = V_d - e = V_d - 2 \cdot V_d \cdot x + V_d = 2 \cdot V_d \cdot (1-x)$
Current ripple via equation	$V_L = L\dfrac{di}{dt} \Rightarrow \Delta i = \dfrac{V_L \cdot \Delta t}{L} \Rightarrow \Delta i = \dfrac{2 \cdot V_d \cdot (1-x)}{L_a} \cdot \dfrac{x}{f_s} = \dfrac{2 \cdot V_d (x - x^2)}{f_s \cdot L_a}$
it's derivative	$\dfrac{\partial(\Delta i)}{\partial x} = \dfrac{2 \cdot V_d}{f_s \cdot L_a} \cdot (1 - 2x) \Rightarrow \dfrac{\partial(\Delta i)}{\partial x} = 0 \text{ when } x = 0.5$
it's second derivative	$\dfrac{\partial^2(\Delta i)}{\partial x^2} = -\dfrac{4 \cdot V_d}{f_s \cdot L_a} < 0 \Rightarrow \max \text{ at } x = 0.5$
Phase voltages at max	$V_{1\_avg} = 0.5 \cdot V_d = 0.5 \cdot V_d$
	$V_{2\_avg} = (1 - 0.5) \cdot V_d = 0.5 \cdot V_d$
	$e = V_{1\_avg} - V_{2\_avg} = 0.5 \cdot (V_d - V_d) = 0 = \dfrac{0}{V_d}$
Max current ripple	$\Delta i_{max} = \dfrac{2 \cdot V_d \cdot (1 - 0.5)}{L_a} \cdot \dfrac{0.5}{f_s} = \dfrac{V_d}{2 f_s L_a}$

Solution Exam 20120521 4b



Exam 20120521 5

A permanently magnetized synchronous machine with $L_{sy}>L_{sx}$ is used as a traction motor in an electric vehicle.

a. Draw the torque expression in rotor coordinates, and describe your interpretation of the terms in the expression, and how they relate to the rotor geometry and magnetization. (4p)

b. Explain, in a qualitative sense, what is the best locus for the stator current vector to minimize the amount of current needed for torque production. (3p)

c. Explain the restrictions to the stator current loci that are imposed when the need for stator voltage is higher than the maximum available voltage. (3p)

Solution Exam 20120521 5a_1

$$T = \vec{\psi}_s \, x \, \vec{i}_s = \psi_m x i_{sy} + \left(L_{mx} - L_{my}\right) \cdot i_{sx} \cdot i_{sy} \qquad (11.5)$$

ψ_m *is the permanent magetization along the positive x – axes*

i_{sx} *is the current along the permanent magetization*

i_{sy} *is the current perpendicular to the permanent magetization,*

$\pi/2$ *in positive direction*

L_{mx} *is the inductance in the x – direction*

L_{my} *is the inductance in the x – direction*

The more iron and the smaller the airgap in the

x – or y – direction, the higher is the inductance is that dircction

The permanent magetization material has no impact on the inductance

Solution Exam 20120521 5a_2

See the torque equation, the first part of the torque is achieved when the permanent flux ψ_m *is multiplied with the current* i_{sy} *.*
The second part of the torque is the so called reluctance torque. E.g. At high speed the drive system is in field weakening, and the permanent magnetisation must be reduced, which is done with a negative i_{sx}*.*
If $L_{mx}<L_{my}$ *the difference* $L_{mx}-L_{my}$ *is negative. When this difference is multiplied with the negative* i_{sx} *and the positive* i_{sy} *the result is a positive torque, called the reluctance torque.*

Solution Exam 20120521 5b

$$T = \vec{\psi}_s x \vec{i}_s = \psi_m x i_{sy} + \left(L_{mx} - L_{my}\right) \cdot i_{sx} \cdot i_{sy}$$

The first torque and the reluctance torque are added to the total torque, which can be achieved with different combinations of i_{sx} *and* i_{sy}.
The combination which gives the lowest

$$absolute \quad sum \quad of \quad i_{sx} \quad and \quad i_{sy} = \sqrt{i_{sx}^2 + i_{sy}^2}$$

is the optimal combination of i_{sx} *and* i_{sy} *for a certain torque.*

Solution Exam 20120521 5c

We want to increase the voltage, more than the available voltlage.

This can be achieved by weaken the field further, by increasing the negative current i_{sx}. *However, this results in an increased total current, beyond the max current loci.*

So, we have to reduce the i_{sy}, *to fullfill the the maximum current loci.*

See chapter 11.5

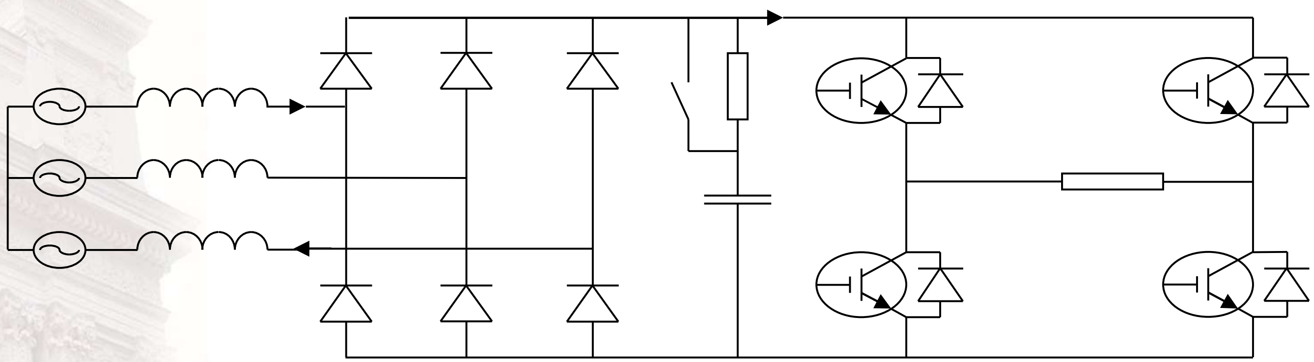
Exam 2014-05-30

Exam 20140530 1a

The four quadrant DC-DC converter

Draw a four quadrant DC DC converter with a three phase diode rectifier connected to the power grid. The Dc link capacitor and protection against too high inrush currents should be included in the drawing. The transistors are of IGBT-type. (2 p.)

Solution Exam 2014-05-30 1a



Exercise Exam 20140530 1b

The three-phase grid, to which the three phase diode rectifier is connected, has the line-to-line voltage 400 V_{rms} at 50 Hz. Calculate the dc output voltage and the maximum dc link voltage from the rectifier.

(1 p.)

Solution Exam 2014-05-30 1b

Maximum dc voltage

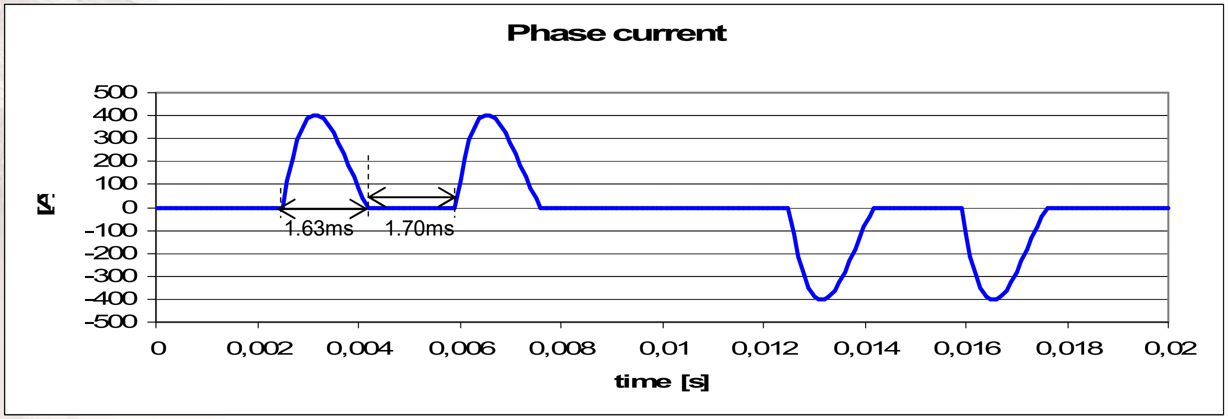
$$U_{dc\ \max} = 400 \cdot \sqrt{2}\ V = 566\ V$$

Average dc voltage

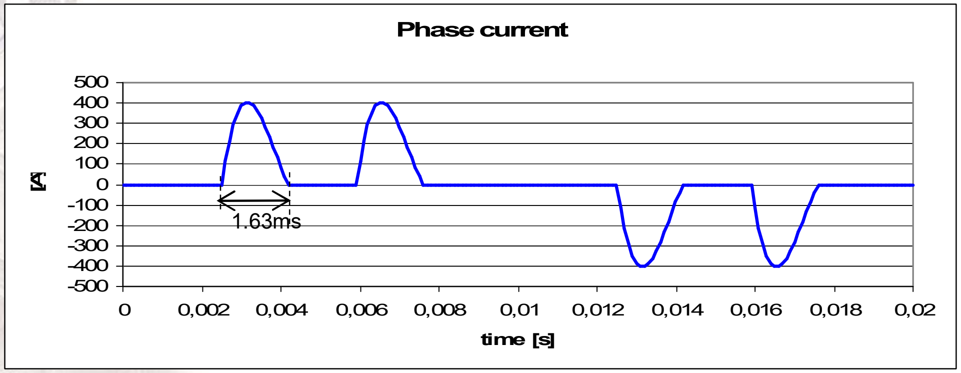
$$U_{dc\_ave} = \frac{3}{\pi} \cdot 400 \cdot \sqrt{2}\ V = 540\ V$$

Exam 20140530 1c

Calculate the rms-current and the average current through one rectifying diode (see figure 1). Calculate the rectifier diode losses. The diode threshold voltage is 1.1 V and the differential resistance is 2.0 mohm. (2 p.)



Solution Exam 2014-05-30 1c



Rectifier diode

Threshold voltage	*1.1 V*
Differential resistance	*2.0 mohm*
I_{rms}	*114.2 A*
Average current	*41,5 A*

$$I_{diode\ rms} = \sqrt{\frac{2 \cdot 0.00163}{0.02} \cdot \left(\frac{400}{\sqrt{2}}\right)^2} = 114.2\ A$$

$$I_{diode\ ave} = \left\{ Average\ of\ \sin us = \frac{\int_0^{\pi} \sin(x)\,dx}{\pi} = \frac{(\cos(0) - \cos(\pi))}{\pi} = \frac{2}{\pi} \approx 0.637 \right\} = \frac{2 \cdot 0.00163}{0.02} \cdot 0.637 \cdot 400 = 41.5\ A$$

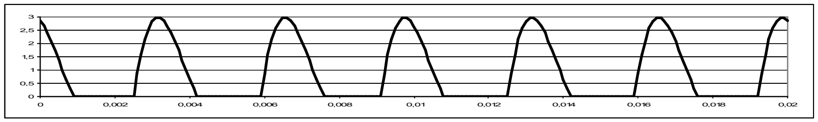
Rectifier diode power loss

$$P_{rectifier\ diode} = V_{threshold} \cdot I_{ave} + R_{diff} \cdot I_{rms}^2 = 1.1 \cdot 41.5 + 0.002 \cdot 114.2^2 = 71.7\ W$$

Exam 20140530 1d

Calculate the IGBT component losses of each IGBT in the four quadrant converter.
The duty cycle of the converter is 70%.
The switching frequency is 2.5 kHz.
The threshold voltage of the IGBT transistor equals 1.6 V
and its differential resistance equals 1.0 mohm.
The turn-on loss of the IGBT transistor equals 65 mJ and
its turn-off loss equals 82 mJ.
These turn-on and turn-off losses are nominal values
at 900 V dclink voltage and 180 A turn-on and turn-off current.
The threshold voltage of the IGBT diode equals 1.0 V and
the differential resistance of this diode equals 10 mohm.
The IGBT diode turn-on can be neglected and its
turn-off losses equals 25 mJ, at 900 V dclink voltage and 180 A. (3 p.)

Solution Exam 2014-05-30 1d



Dc current to the dc link $I_{dc} = \frac{6 \cdot 0.00163 \cdot 400}{0.02} \cdot 0.637 = 124.6\ A$

Duty cycle *70%*

IGBT and diode on state current *124.6/0.7= 178 A*

Conduction percentage of IGBT transistor (incl freewheeling) *70+30/2=85%*

Conduction percentage of IGBT diode (when freewheeling) 30/2=15%

Switching frequency *2,5 kHz*

<u>IGBT transistor</u>

Threshold voltage *1.6V*

Differential resistance *1.0 mohm*

On state voltage at 178 A *1.78 V*

Turn on energy at 900 V and 180 A *65 mJ*

Turn off energy at 900 V and 180 A *82 mJ*

<u>IGBT diode</u>

Threshold voltage *1.0 V*

Differential resistance *10. mohm*

On state voltage at 178 A *2.78 V*

Turn on energy at 900 V and 180 A *0 mJ*

Turn off energy at 900 V and 180 A *25 mJ*

<u>Power loss</u>

$$\begin{cases} P_{trans\_loss} = 1.78 \cdot 178 \cdot 0.85 + 2500 \cdot \frac{(65 + 82) \cdot 10^{-3} \cdot 540 \cdot 178}{900 \cdot 180} = 487\ W \\ P_{diode\_loss} = 2.78 \cdot 178 \cdot 0.15 + 2500 \cdot \frac{25 \cdot 10^{-3} \cdot 540 \cdot 178}{900 \cdot 180} = 111.3\ W \end{cases}$$

Exam 20140530 1e

Which is the junction temperature of the IGBT transistor and of the IGBT diode,
and which is the junction temperature of the rectifying diodes?
The thermal resistance of the heatsink equals 0.024 K/W?
The thermal resistance of the IGBT transistor equals 0.07 K/W?
The thermal resistance of the IGBT diode equals 0.16 K/W?
The thermal resistance of the rectifier diode equals 0.14 K/W?
The ambient temperature is 35 ^{o}C.
The rectifier diodes and the four quadrant converter IGBTs share the heatsink. (2 p.)

Solution Exam 2014-05-30 1e

Rectifier diode (6)	IGBT diode (2)	IGBT transistor (2)
Loss each 71.7W	Loss each 111.3 W	Loss each 487 W
Rth diode 0.14 C/W	Rth diode 0.16 C/W	Rth trans 0.07C/W
Temp diff 10.0 ^oC	Temp diff 17.8 ^oC	Temp diff 34.1 ^oC

Heatsink

Contribution fron 6 rectifier diodes and from two IGBT.

Ambient temperature	*35 ^oC*
Total loss to heatsink	*6*71.7+2*487+2*111.3=1627 W*
Rth heatsink	*0.024 C/W*
Temperature heatsink	*1627 *0.024+35=74 ^oC*

Junction temperature

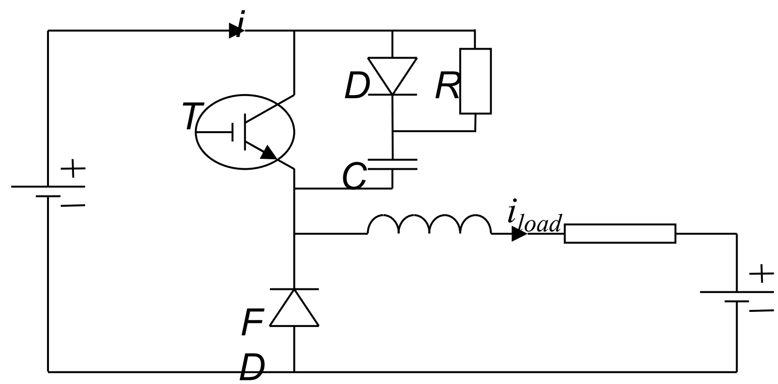
Rectifier diode	*74 +10.0 = 84 ^oC*
IGBT diode	*74 +17.8 = 92 ^oC*
IGBT transistor	*74 +34.1 = 108 ^oC*

Exam 20140530 2a

Snubbers

Draw an IGBTequipped step down chopper
(buck converter) with an RCD snubber.
The dclink voltage on the supply side is 200V and
the load voltage is 150 V.
Give a detailed description of how the RCD
charge-discharge snubber should operate.
Explain why the snubbers are needed (2 p.)

Solution Exam 2014-05-30 2a



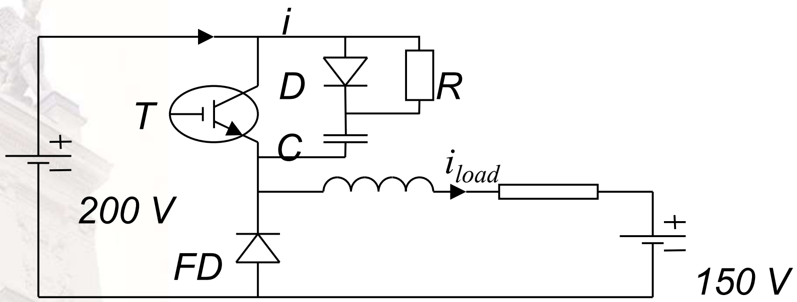
The buck converter with RCD snubber
At turn off of transistor T, the current i commtutates over to the capacitor C via diode D. The capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current i_{load} flows through diode FD and the current i=0.
A turn on of the transistor T, the capacitor C is discharged via the transistor T and resistor R. The diode FD becomes reverse biased and the current i commutates to the transistor T.

Exercise Exam 20140530 2b

Snubbers

Calculate the snubber capacitor for the commutation time 0.01 ms.
The load current is 12 A, assumed constant during the commutation.
Calculate the snubber resistor so the discharge time (3 time constants)
of the snubber capacitor is less than the IGBT on state time.
The switch frequency is 1.5 kHz (4 p.)

Solution Exam 2014-05-30 2b



Load current	*12 A*
Supply voltage	*200 V*
Load voltage	*150 V*
Commutation time	*0.01 ms*
Switching frequency	*1.5 kHz*

<u>At turn off</u> of transistor T, the capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current commutates to the freewheeling diode.

$$i = C \cdot \frac{du}{dt} \quad \Rightarrow \quad C = \frac{i \cdot dt}{du} = \frac{12 \cdot 10 \cdot 10^{-6}}{200} = 0.6\ \mu F$$

<u>A turn on</u> of the transistor the current i commutates to the transistor T, and the capacitor C is discharged via the the transistor T and resistor R. As the load voltage is 150V the duty cycle is 75%. The switching frequency is 1.5 kHz and the on state time is 0.5 ms, and thus the time constant =0.17 ms

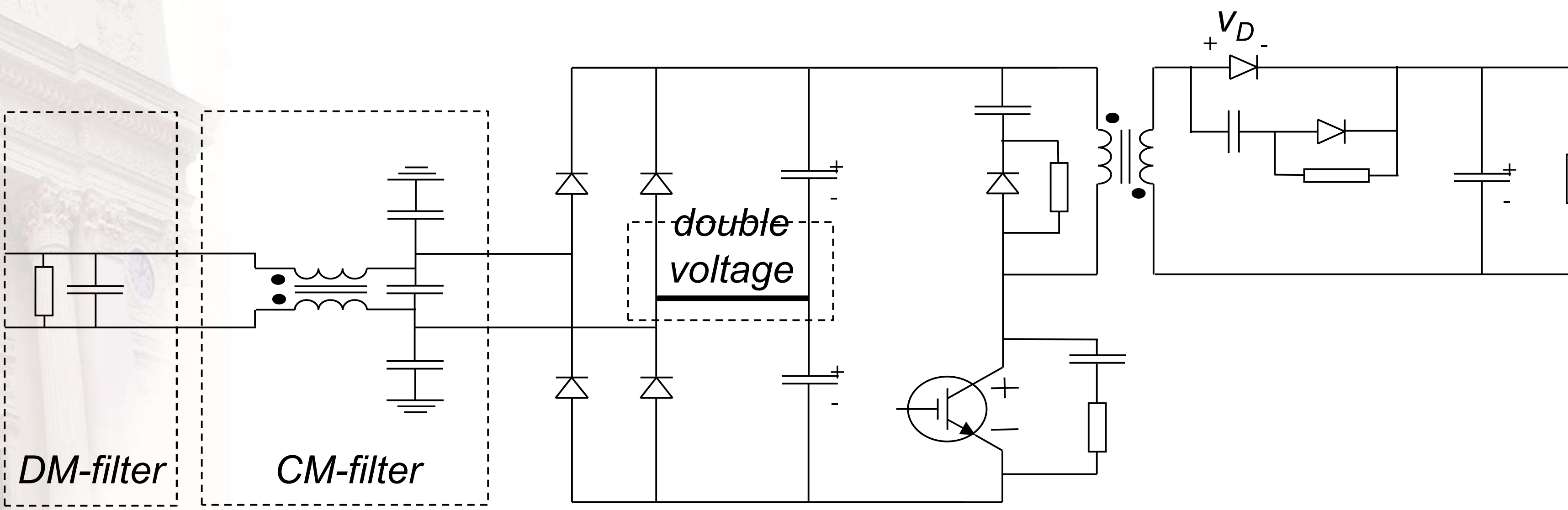
$$\tau = C \cdot R \quad \Rightarrow \quad R = \frac{\tau}{C} = \frac{170 \cdot 10^{-6}}{0.6 \cdot 10^{-6}} = 283\ \Omega$$

Exam 20140530 2c

Snubbers

Draw the main circuit of a flyback converter. The circuit should include DM-filter (differential mode) ,CM (common mode) filter, rectifier, dc link capacitors,
alternative connection for voltage doubling connection, switch transformer
(one primary and one secondary winding is enough), switch transistor, flyback
diode and a simple output filter, The circuit should also include snubbers. (2 p.)

Solution Exam 2014-05-30 2c



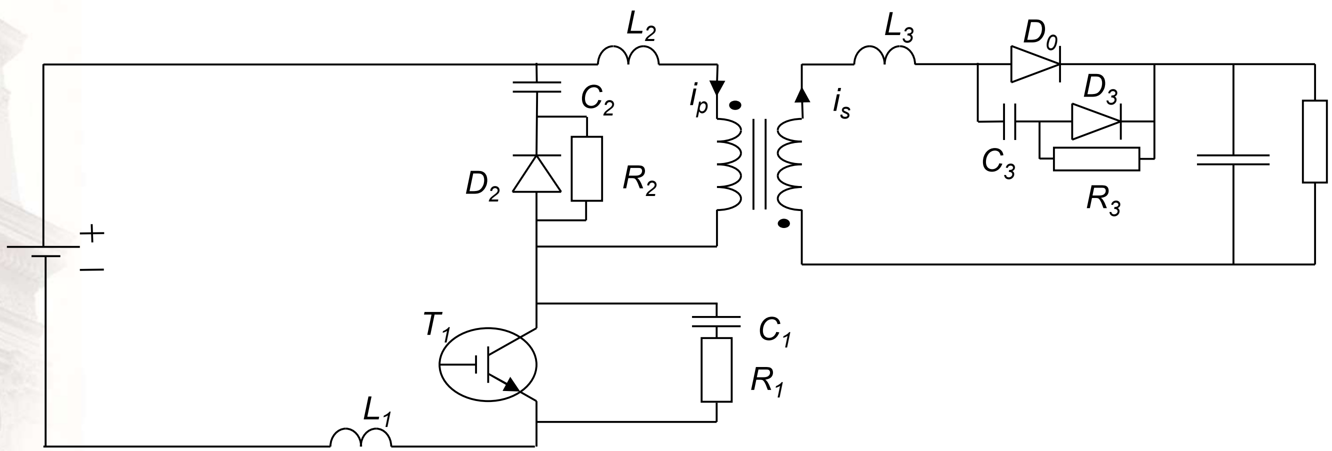
Exam 20140530 2d

Snubbers

Describe, in detail, the operation of the flyback converter snubbers you have used. Describe in detail how the current is flowing in the snubber and the voltages n the snubber (2 p.)

Solution Exam 2014-05-30 2d

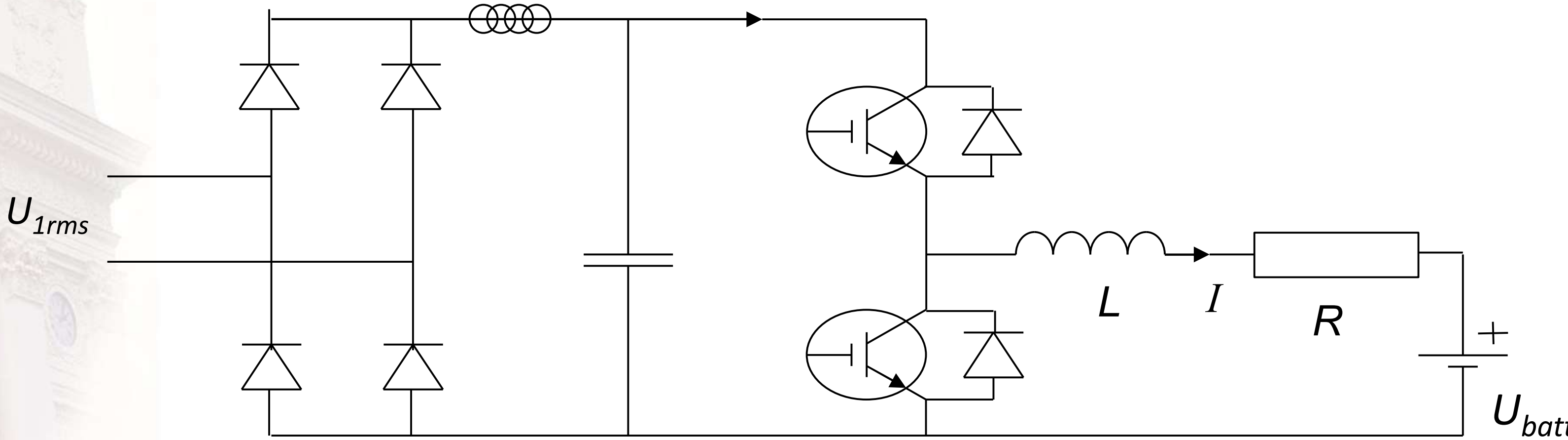
Fly-back converter withSnubber operation



For the description of the snubber operation the stray inductance L_1 between the switch transistor and the supply/dclink, and the transformer leakage inductance, L_2 on primary side and L_3 on secondary side are added as discrete component in the circuit drawing above.

Exam 2014-05-30 3_1

The buck converter as battery charger
A battery charger is supplied from a symmetrical single phase system.
A dc voltage is created by a two pulse diode bridge and a 2-quadrant dc-converter is used for the charge current control.



Data: U_{1rms}= the phase-voltage rms value = 220 V, 50 Hz.
The switching frequency is f = 4 kHz.
L = 4 mH and R=0 Ohms.
U_{batt} = 100 V and is approximated to be independent of the charge current.

Exam 2014-05-30 3_2

The buck converter as battery charger

a) What dc link voltage *Ud* will you get I) when the charging current is zero and II) when the charging current is non-zero with a perfectly smooth rectified current ? (2p)

b) Start with the electrical equation for the load and derive a suitable current control algorithm, giving all approximations you use. (4p)

c) Draw a current step from 0 A till 10 A in the load current. The modulating wave (*um*), the voltage reference (*u**), the output voltage (*u*) and current (*ibatt*) must be shown. Indicate the sampling frequency you use in relation to the switching frequency. (4p)

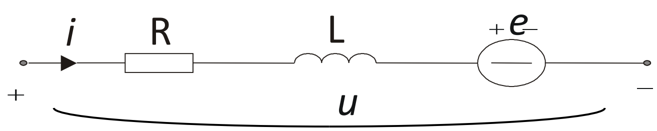
Solution Exam 2014-05-30 3a

Average dc voltage with average dc current $$U_{dc\_ave} = \frac{2}{\pi} \cdot 220 \cdot \sqrt{2}\ V = 198\ V$$

Max dc voltage with zero dc current $$U_{dc\_\max} = 220 \cdot \sqrt{2}\ V = 311\ V$$

Solution Exam 2014-05-30 3b_1

Current controller with fast computer



$$u = R \cdot i + L \cdot \frac{di}{dt} + e$$

$$\frac{\int_{k \cdot T_s}^{(k+1)T_s} u \cdot dt}{T_s} = \frac{R \cdot \int_{k \cdot T_s}^{(k+1)T_s} i \cdot dt + L \cdot \int_{k \cdot T_s}^{(k+1)T_s} \frac{di}{dt} \cdot dt + \int_{k \cdot T_s}^{(k+1)T_s} e \cdot dt}{T_s}$$

$$\overline{u}(k, k+1) = R \cdot \overline{i}(k, k+1) + L \cdot \frac{i(k+1) - i(k)}{T_s} + \overline{e}(k, k+1)$$

Solution Exam 2014-05-30 3b_2

$$\overline{u}(k,k+1) = u^*(k) \qquad (a)$$

$$i(k+1) = i^*(k) \qquad (b)$$

$$\overline{i}(k,k+1) = \frac{i^*(k) + i(k)}{2} \qquad (c)$$

$$\overline{e}(k,k+1) = e(k) \qquad (d)$$

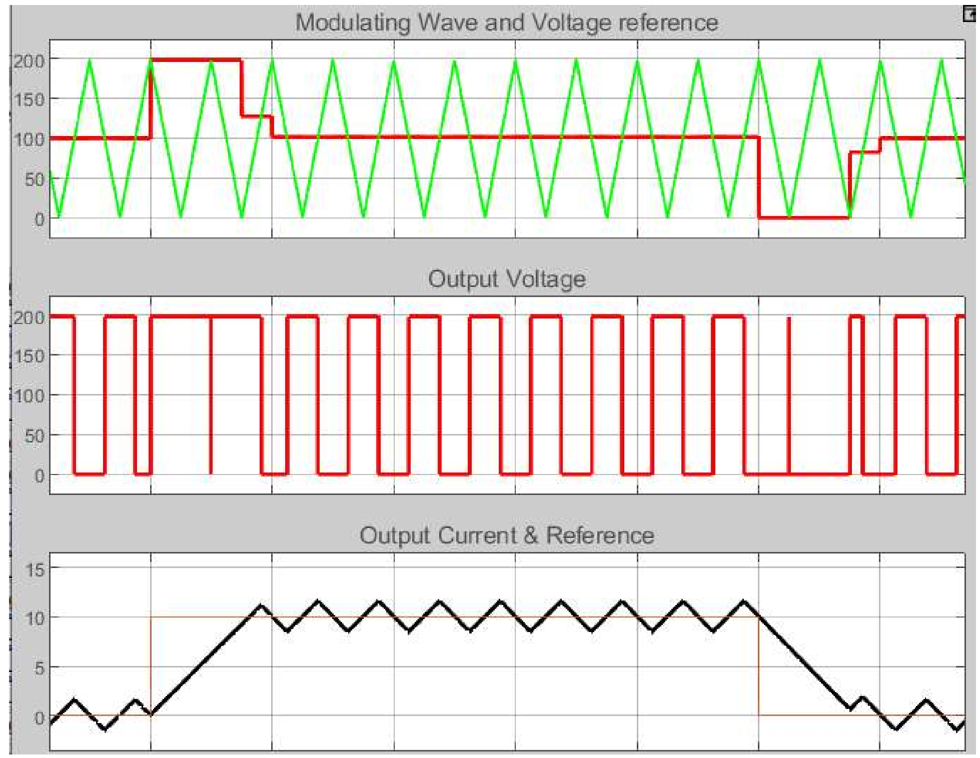
$$i(k) = \sum_{n=0}^{n=k-1} \left(i^*(n) - i(n)\right) \qquad (e)$$

$$u^*(k)\{R=0\} = L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\left(i^*(k) - i(k)\right)}_{\text{Proportional}} + \underbrace{e(k)}_{\substack{Feed \\ forward}}$$

$$u^*(k) = \{R=0\} = L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\Delta i}_{\text{Proportional}} + \underbrace{e(k)}_{\substack{Feed \\ forward}}$$

Solution Exam 2014-05-30 3c

- Assuming that the DC link voltage is 198 V, the Inductance L=4 mH and the switching frequency is 4 kHz,
- Ts = 125 microseconds
- The positive step voltage reference should be u* = 4e-3/125e-6*(10-0) + 100 = 320 +100 V. The voltage margin of 98 V has to be repeated 3 times, i.e. the voltage reference 198 V three times and then 100+26 V the last time. Even here, the example not correctly illustrated. It should look like this:



Exercise Exam 2014-05-30 4a

Three phase system and 4QC

A symmetric three phase voltage:

$$\begin{cases} e_a = \hat{e} \cdot \cos(\omega \cdot t) \\ e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \\ e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \end{cases}$$

Show that these voltages form a rotating vector with constant length and constant speed in the complex (α,β) frame. (5 p.)

Solution Exam 2014-05-30 4a

$$\begin{cases} e_a = \hat{e}\cdot COS\ (\omega\cdot t) \\ e_b = \hat{e}\cdot COS\ (\omega\cdot t - \frac{2\pi}{3}) \\ e_c = \hat{e}\cdot COS\ (\omega\cdot t - \frac{4\pi}{3}) \end{cases}$$

$$\vec{e} = \sqrt{\frac{2}{3}}\cdot\left(e_a + e_b\cdot e^{j\frac{2\pi}{3}} + e_c\cdot e^{j\frac{4\pi}{3}}\right) = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \cos\left(\omega t - \frac{4\pi}{3}\right)\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \left\{\cos(\omega t)\cdot\cos\left(\frac{2\pi}{3}\right) + \sin(\omega t)\cdot\sin\left(\frac{2\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \left\{\cos(\omega t)\cdot\cos\left(\frac{4\pi}{3}\right) + \sin(\omega t)\cdot\sin\left(\frac{4\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right) + \sin(\omega t)\cdot\left(\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right) + \sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(1 + \frac{1}{4} - j\cdot\frac{\sqrt{3}}{4} + \frac{1}{4} + j\cdot\frac{\sqrt{3}}{4}\right) + \sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{4} + j\cdot\frac{3}{4} + \frac{\sqrt{3}}{4} + j\cdot\frac{3}{4}\right)\right] = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(\frac{3}{2}\right) + \sin(\omega t)\cdot\left(j\cdot\frac{3}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot[\cos(\omega t) + j\cdot\sin(\omega t)] = \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

Alternative solution

$$\vec{e} = \sqrt{\frac{2}{3}}\cdot\left(e_a + e_b\cdot e^{j\frac{2\pi}{3}} + e_c\cdot e^{j\frac{4\pi}{3}}\right) = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cdot e^{j\frac{2\pi}{3}} + \cos\left(\omega t - \frac{4\pi}{3}\right)\cdot e^{j\frac{4\pi}{3}}\right] = \left\{\cos(x) = \frac{e^{jx} + e^{-jx}}{2}\right\} =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} + \frac{e^{j\left(\omega t - \frac{2\pi}{3}\right)} + e^{-j\left(\omega t - \frac{2\pi}{3}\right)}}{2}\cdot e^{j\frac{2\pi}{3}} + \frac{e^{j\left(\omega t - \frac{4\pi}{3}\right)} + e^{-j\left(\omega t - \frac{4\pi}{3}\right)}}{2}\cdot e^{j\frac{4\pi}{3}}\right] = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} + \frac{e^{j\omega t - j\frac{2\pi}{3} + j\frac{2\pi}{3}} + e^{-j\omega t + j\frac{2\pi}{3} + j\frac{2\pi}{3}}}{2} + \frac{e^{j\omega t - j\frac{4\pi}{3} + j\frac{4\pi}{3}} + e^{-j\omega t + j\frac{4\pi}{3} + j\frac{4\pi}{3}}}{2}\right] =$$

$$= \hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[e^{j\omega t} + e^{-j\omega t} + e^{j\omega t} + e^{-j\omega t}\cdot e^{j\frac{4\pi}{3}} + e^{j\omega t} + e^{-j\omega t}\cdot e^{j\frac{8\pi}{3}}\right] = \hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[3\cdot e^{j\omega t} + e^{-j\omega t}\cdot\underbrace{\left(1 - \frac{1}{2} + \frac{\sqrt{3}}{2} - -\frac{1}{2} - \frac{\sqrt{3}}{2}\right)}_{=0}\right] = \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

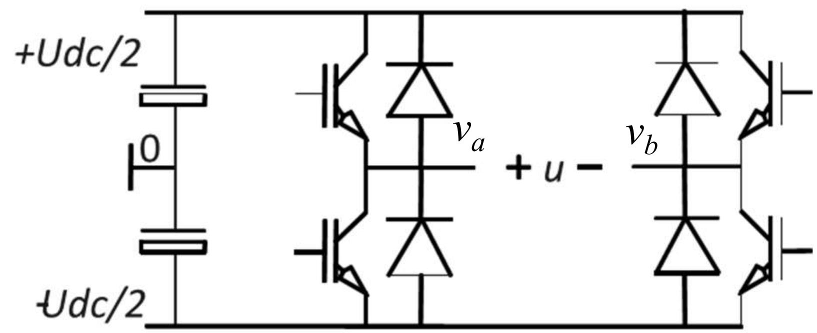
Exam 2014-05-30 4b

In a 4QC dc-dc converter using PWM bipolar voltage switching, the the bridge load consist of a constant voltage E (e.g. the back emf of a dc-motor) and an inductor L_a, the inductor resistance can be neglected. The switching frequency is f_s, and the dc-link voltage is V_d

Calculate the maximum peak-to-peak load current ripple, .expressed in V_d, L_a and f_s,. (5 p.)

Solution Exam 2014-05-30 4b

1. Assume a load with inductance L, no resistance and a back emf *e*
2. Assume phase potential references v_a* and v_b* where $v_b^* = - v_a^* = u^*/2$ (symmetric modulation)
3. Assume stationarity
4. Calculate the current ripple as a function of the back emf



Di/dt = (Udc-e)/L
Delta_i = (Udc-e)/L*Delta_t = current ripple
Delta_t = e/Udc*1/fsw/2 (time duration of a positive pulse)
Delta_i = (Udc-e)/L*(e/Udc/fsw/2) = (Udc*e-e^2)/Udc/fsw/2;

d(Delta_i)/d(e) = (Udc-2*e)/L/Udc/fsw/2 = 0 - > e = Udc/2;

Solution Exam 2014-05-30 4b

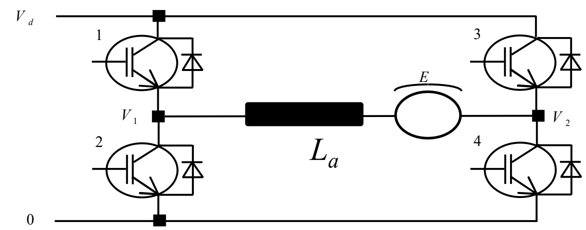


Fig 1

Control ratio x

On – pulse duration $\Delta t = x \cdot T_{s\_per} = \dfrac{x}{f_s}$

Phase voltages $V_{1\_avg} = x \cdot V_d$

$V_{2\_avg} = (1-x)\cdot V_d = V_d - x \cdot V_d$

Voltage over motor $e = V_{1\_avg} - V_{2\_avg} = x \cdot V_d - V_d + x \cdot V_d = 2 \cdot V_d \cdot x - V_d$

At current rise, switch 1 and 4 are turned – on $V_1 = V_d$

$V_2 = 0$

Voltage over inductor $V_L = V_1 - e - V_2 = V_d - e = V_d - 2 \cdot V_d \cdot x + V_d = 2 \cdot V_d \cdot (1-x)$

Current ripple via equation $V_L = L\dfrac{di}{dt} \Rightarrow \Delta i = \dfrac{V_L \cdot \Delta t}{L} \Rightarrow \Delta i = \dfrac{2 \cdot V_d \cdot (1-x)}{L_a} \cdot \dfrac{x}{f_s} = \dfrac{2 \cdot V_d (x - x^2)}{f_s \cdot L_a}$

it's derivative $\dfrac{\partial(\Delta i)}{\partial x} = \dfrac{2 \cdot V_d}{f_s \cdot L_a} \cdot (1 - 2x) \Rightarrow \dfrac{\partial(\Delta i)}{\partial x} = 0 \text{ when } x = 0.5$

it's second derivative $\dfrac{\partial^2(\Delta i)}{\partial x^2} = -\dfrac{4 \cdot V_d}{f_s \cdot L_a} < 0 \Rightarrow \max \text{ at } x = 0.5$

Phase voltages at max $V_{1\_avg} = 0.5 \cdot V_d = 0.5 \cdot V_d$

$V_{2\_avg} = (1-0.5)\cdot V_d = 0.5 \cdot V_d$

$e = V_{1\_avg} - V_{2\_avg} = 0.5 \cdot (V_d - V_d) = 0 = \dfrac{0}{V_d}$

Max current ripple $\Delta i_{\max} = \dfrac{2 \cdot V_d \cdot (1-0.5)}{L_a} \cdot \dfrac{0.5}{f_s} = \dfrac{V_d}{2 f_s L_a}$

Exam 2014-05-30 5

Permanent magnetized motor

A 50 kW motor drive is to be designed. The motor will run from a Power Electronic Converter with 500 V DC link. The bases speed must be 4000 rpm and the maximum speed 12000 rpm. The motor has 18 poles.

a) What is the highest output voltage ($U_{phase\text{-}to\text{-}phase\_rms}$) that you would use? (2p)
b) What will be the phase current in this case? (2p)
c) What is the lowest sampling frequency that the controller must use to run this motor? (2p)
d) What is the lowest switching frequency that the modulation can use? (2p)
e) What will be the motor torque at base speed and maximum speed? (2p)

All your answers must be accompanied with your calculations and motivations!

Solution Exam 20140530 5

$$5a)\ Phase-to-phase voltage e rms U_{rms\_line\_to\_line}=\frac{500}{\sqrt{2}}=354V$$

$$5b)\ 50000=\{assume\cos\phi=0.9\}=\sqrt{3}\cdot 354\cdot I\cdot 0.9\Rightarrow I=\frac{50000}{\sqrt{3}\cdot 354\cdot 0.9}=90.7A$$

$$5c)\ Mech freq=\frac{12000}{60}=200Hz$$

$Elec.freq=\{18 poles, 9 polepairs\}=9\cdot 200Hz=1800Hz$

$Sampling freq, see solution 5d.$

$5d)\ At least switching freq=6\cdot Elec.freq=10800Hz$

$Two sample per switch. freq period\Rightarrow sampling freq=21600Hz$

$$5e)\ Torque_{4000rpm}=\{P=T\cdot\omega\}=\frac{P}{\omega}=\frac{50000}{2\pi\cdot\frac{4000}{60}}=119Nm$$

$$Torque_{12000rpm}=\frac{P}{\omega}=\frac{50000}{2\pi\cdot\frac{12000}{60}}=39.8Nm$$

Exam 2017-05-30

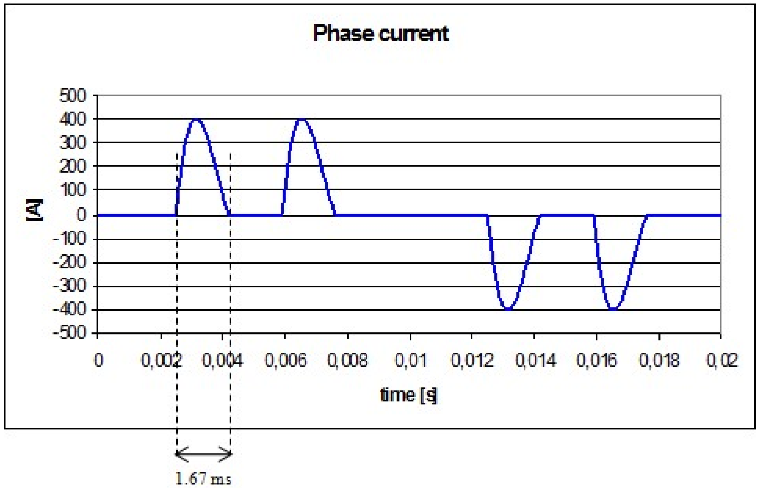
Exam 2017-05-30, 1a-c

The DC-DC buck converter

a) Draw a 1QC buck converter connected to the dc side of a three phase diode rectifier, which is connected to the power grid. The dclink capacitor and protection against too high inrush currents should be included in the drawing. The transistor is of IGBT-type. (1 p.)

b) The three-phase grid, to which the three phase diode rectifier is connected, has the line-to-line voltage 400 V_{rms} and the frequency 50 Hz. Calculate the dc output voltage and the maximum dc link voltage from the rectifier. (1 p.)

c) Calculate the rms-current and the average current through one rectifying diode (see figure 1). Calculate the rectifier diode losses. The diode threshold voltage is 0,9 V and the differential resistance is 2.0 mohm. (2 p.)



Exam 2017-05-30, 1d

The DC-DC buck converter

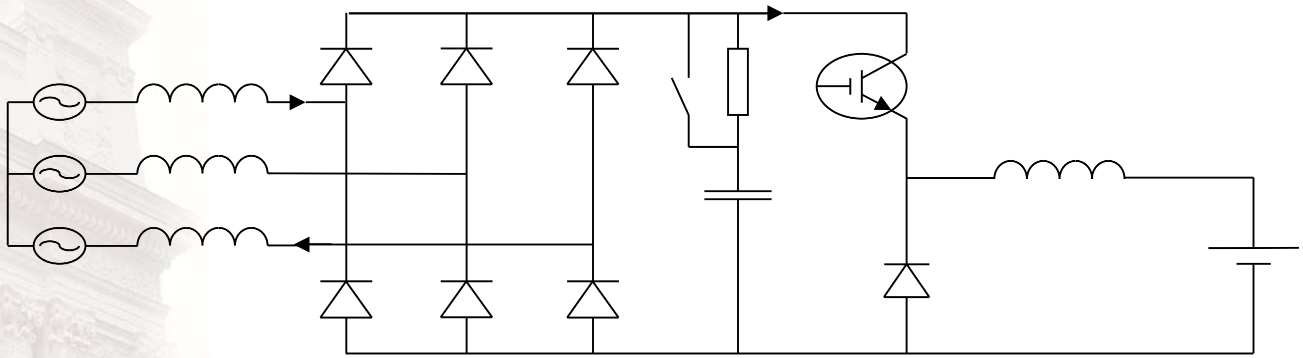
d) Calculate the losses of the IGBT transistor and of the free wheeling diode in the buck converter. The buck converter phase inductor is 1 mH, and its resistance can be neglected.
Draw a time diagram with the buck converter phase current versus time during one period of the switching frequency.
The load on the low voltage side of the buck converter is a battery with the voltage 400 V_{dc}.
The switching frequency is 2 kHz.
The threshold voltage of the IGBT transistor equals 1.1 V and its differential resistance equals 1.0 mohm. The turn-on loss of the IGBT transistor equals 60 mJ and its turn-off loss equals 80 mJ. These turn-on and turn-off losses are nominal values at 900 V dclink voltage and 180 A turn-on and turn-off current.
The threshold voltage of the free wheeling diode equals 1.3 V and the differential resistance of this diode equals 2 mohm. The free wheeling diode turn-on losses can be neglected and its turn-off losses equals 25 mJ, at 900 V dclink voltage and 180 A turn off current. (4 p.)

Exam 2017-05-30, 1e

The DC-DC buck converter

e) Which is the junction temperature of the IGBT transistor and of the free wheeling diode, and which is the junction temperature of the rectifying diodes?
The thermal resistance of the heatsink equals 0.065 K/W?
The thermal resistance of the IGBT transistor equals 0.078 K/W?
The thermal resistance of the free wheeling diode equals 0.19 K/W?
The thermal resistance of the rectifier diode equals 0.21 K/W?
The ambient temperature is 35 ^oC.
The rectifier diodes and the buck converter transistor and diode share the heatsink. (2 p.)

Solution Exam 2017-05-30 1a



Solution Exam 2017-05-30 1b

Maximum dc voltage $U_{dc\,\max} = 400 \cdot \sqrt{2}\ V = 566\ V$

Average dc voltage $U_{dc\_ave} = \frac{3}{\pi} \cdot 400 \cdot \sqrt{2}\ V = 540\ V$

Solution Exam 2017-05-30 1c

Data

Rectifier diode Threshold voltage	0.9 V
Differential resistance	2.0 mohm

$$I_{diode\ rms\_one\ half\ \sin us} = \left(\frac{400}{\sqrt{2}}\right) A = 282.8\ A$$

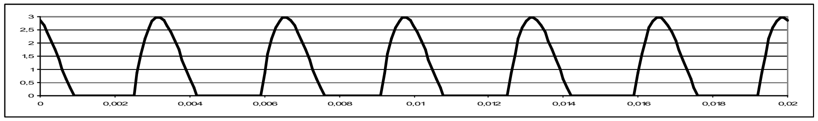
$$I_{diode\ rms} = \sqrt{\frac{2 \cdot 0.00163}{0.02} \cdot 282.8^2} = 114.2\ A$$

$$I_{diode\ ave} = \left\{ Average\ of\ \sin us = \frac{\int_0^{\pi} \sin(x)dx}{\pi} = \frac{(\cos(0) - \cos(\pi))}{\pi} = \frac{2}{\pi} \approx 0.637 \right\} = \frac{2 \cdot 0.00163}{0.02} \cdot 0.637 \cdot 400 = 41.5A$$

Rectifier diode power loss

$$P_{rectifier\ diode} = V_{threshold} \cdot I_{ave} + R_{diff} \cdot I_{rms}^2 = 0.9 \cdot 41.5 + 0.002 \cdot 114.2^2 = 63.4\ W$$

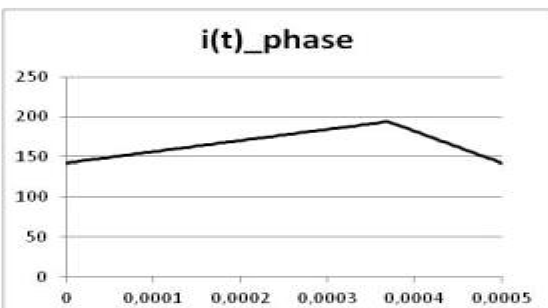
Solution Exam 2017-05-30 1d_1

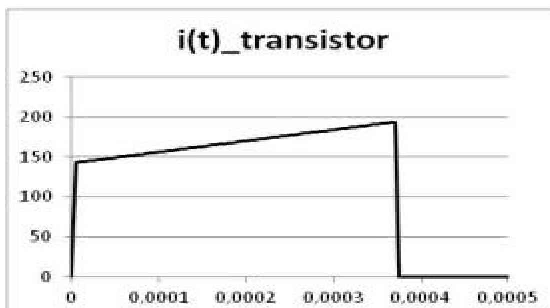


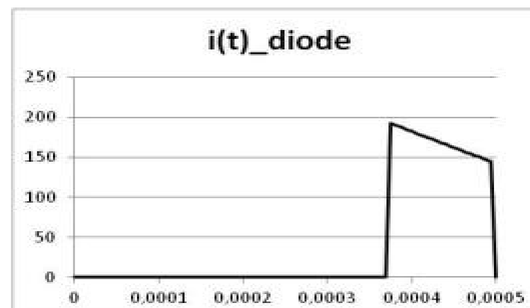
$$I_{dc} = \frac{6 \cdot 0.00163 \cdot 400}{0.02} \cdot 0.637 = 124.6\ A$$

Thc current to the dc link

Duty cycle	400/540=74%
Average transistor current	124.6/0.74=168.2 A
Switching frequency period time	1/2000=0.0005 s
Duration of transistor on	0.74/0.0005=0.00037 s
Current ripple, equ U=L*di/dt , Δi=U*Δt/L=	Δi=(540-400)*0.00037/0.001=51.8 A
Low voltage side max phase current	Imax=168.2+51.8/2=194.1 A
Low voltage side max phase current	Imin=168.2-51.8/2=142.3A

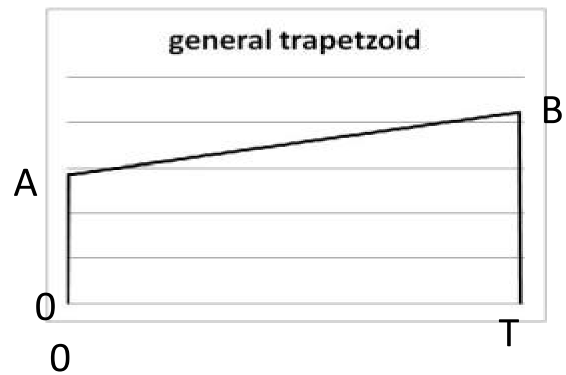






Solution Exam 2017-05-30 1d_2

Find a general expression for RMS from a time domain trapezoid shaped current



$$Equation \quad for \ the \ straight \ line \quad i(t) = \frac{(B-A)}{T}\cdot t + A$$

$$I_{rms} = \sqrt{\frac{\int_0^T \left(\frac{(B-A)}{T}\cdot t + A\right)^2 dt}{T}} = \sqrt{\left(\frac{(B^2 + A^2 - 2AB)\cdot T^3}{3T^2 \cdot T} + \frac{A^2 T}{T} + 2\cdot A\cdot\frac{(B-A)\cdot t^2}{2T\cdot T}\right)} =$$

$$= \sqrt{\left(\frac{B^2 + A^2 - 2AB + 3A^2 + 3AB - 3A^2}{3}\right)} = \sqrt{\left(\frac{A^2 + B^2 + AB}{3}\right)}$$

$$\begin{cases} I_{min} = 142.3\ A \\ I_{max} = 194.1\ A \end{cases}$$

$$I_{rms\_transistor} = \sqrt{\left(\frac{142.3^2 + 194.1^2 + 142.3\cdot 194.1}{3}\right)\cdot 0.74} = 145.3\ A \qquad I_{avg\_transistor} = \left(\frac{142.3 + 194.1}{2}\right)\cdot 0.74 = 124.5\ A$$

$$I_{rms\_diode} = \sqrt{\left(\frac{142.3^2 + 194.1^2 + 142.3\cdot 194.1}{3}\right)\cdot 0.26} = 86.1\ A \qquad I_{avg\_diode} = \left(\frac{142.3 + 194.1}{2}\right)\cdot 0.26 = 43.7\ A$$

	threshold volrage[V]	Rdiff[mohm]	Turn-on[mJ]	Turn off[mJ]	Switch losses at voltage[V]	and at current[A]
Transistor	1.1	1.0	60	80	900	180
Diode	1.3	2.0	0	25	900	180

$$\begin{cases} P_{trans\_loss} = 1.5\cdot 124.5 + 0{,}001\cdot 145.3^2 + 2000\cdot\frac{(0.060\cdot 142.3 + 0.080\cdot 194.1)\cdot 540}{900\cdot 180} = 368\ W \\ P_{diode\_loss} = 1.0\cdot 43.7 + 0{,}002\cdot 86.1^2 + 2000\cdot\frac{0.025\cdot 142.3\cdot 540}{900\cdot 180} = 82.2\ W \end{cases}$$

Solution Exam 2017-05-30 1e

Rectifier diode (6)	IGBT diode	IGBT transistor
Loss each 63.4W	Loss each 82.2 W	Loss each 368 W
Rth diode 0.25 C/W	Rth diode 0.4 C/W	Rth trans 0.2C/W
Temp diff 15.8 ^{o}C	Temp diff 32.9 ^{o}C	Temp diff 73.6 ^{o}C

Heatsink

Contribution fron 6 rectifier diodes and from one IGBT and one diode.

Ambient temperature	35 ^{o}C
Total loss to heatsink	6*63.4+368+82.2=831 W
Rth heatsink	0.07 C/W
Temperature heatsink	831 *0.07+35=93 ^{o}C

Junction temperature

Rectifier diode	93 +15.8 = 109 ^{o}C
IGBT diode	93 +32.9 = 126 ^{o}C
IGBT transistor	93 +73.6 = 167 ^{o}C

Exam 2017-05-30, 2

Snubbers and semiconductor

a) Draw an IGBTequipped step down chopper (buck converter) with an RCD snubber. Give a detailed description of how the RCD charge-discharge snubber operates at turn on and at turn-off. Explain why the snubbers are needed (2 p.)

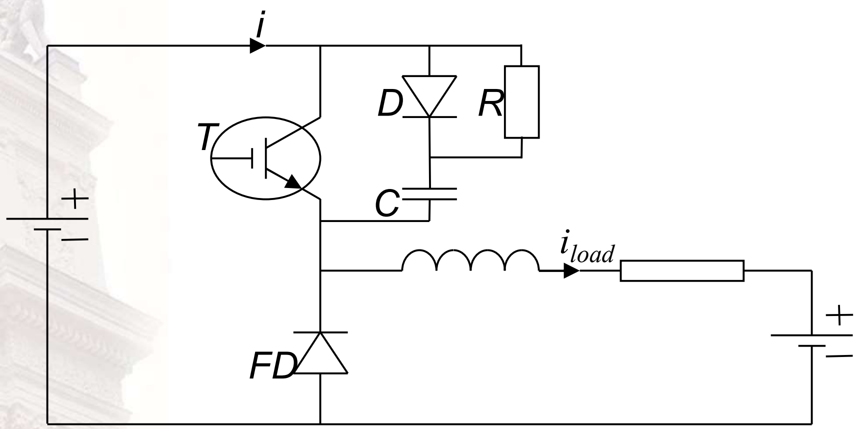
b) The DC link voltage on the supply side is 250V and the load voltage is 200 V. Calculate the snubber capacitor for the commutation time 0.015ms. The load current is 17 A, assumed constant during the commutation. Calculate the snubber resistor so the discharge time (3 time constants) of the snubber capacitor is less than the IGBT on state time. The switch frequency is 2 kHz (3 p.)

c) Draw a figure with the diffusion layers in a (n-channel) MOSFET (2 p.)

d) Where in the (n-channel) MOSFET diffusion layers structure can an <u>unwanted</u> NPN-transistor be found, and where can the anti-parallel diode be found? (2 p.) What in the MOSFET layout reduces the risk that this unwanted transistor is turned on?

e) Which layer is always present in a power semiconductor? How is it doped? (1 p.)

Solution Exam 2017-05-30, 2a



The buck converter with RCD snubber
At turn off of transistor T, the current i commtutates over to the capacitor C via diode D. The capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current i_{load} flows through diode FD and the current i=0.
A turn on of the transistor T, the capacitor C is discharged via the transistor T and resistor R. The diode FD becomes reverse biased and the current i commutates to the transistor T.

Exam 2017-05-30 2b, 1

The DC link voltage on the supply side is 250V and the load voltage is 200 V.

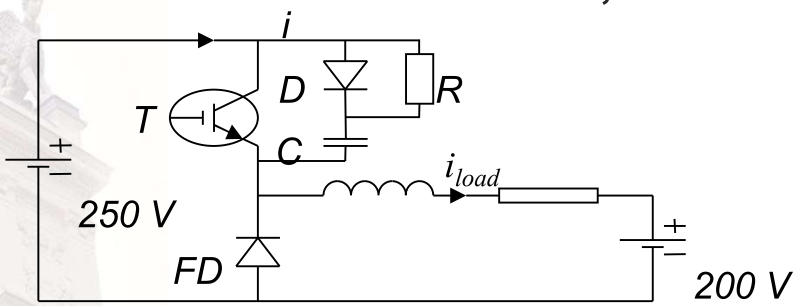
Calculate the snubber capacitor for the commutation time 0.015ms.
The load current is 17 A, assumed constant during the commutation.

Calculate the snubber resistor so the discharge time (3 time constants) of the snubber capacitor is less than the IGBT on state time.

The switching frequency is 2 kHz

(3 p.)

Solution Exam 2017-05-30 2b, 2



Load current	17 A
Supply voltage	250 V
Load voltage	200 V
Commutation time	0.015 ms
Switching frequency	2 kHz

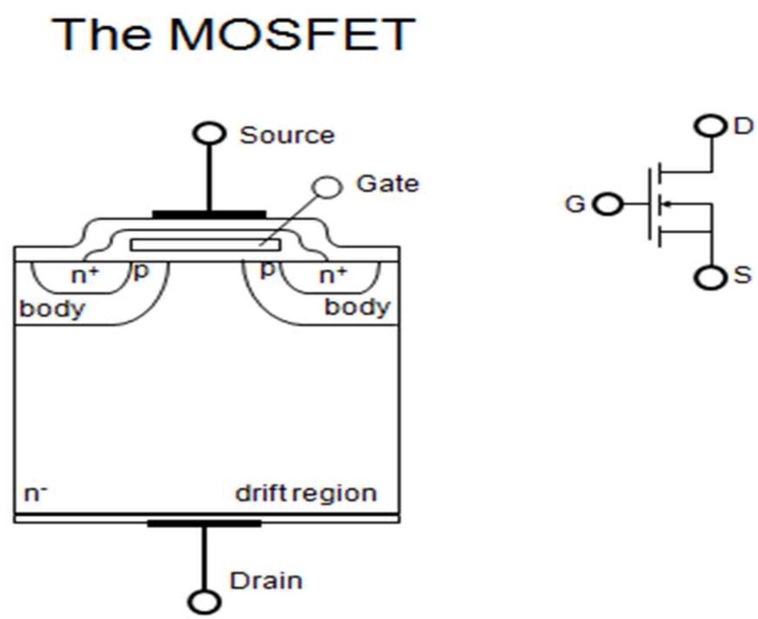
At turn off of transistor T, the capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current commutates to the freewheeling diode.

$$i = C \cdot \frac{du}{dt} \Rightarrow C = \frac{i \cdot dt}{du} = \frac{17 \cdot 15 \cdot 10^{-6}}{250} = 1.0\ \mu F$$

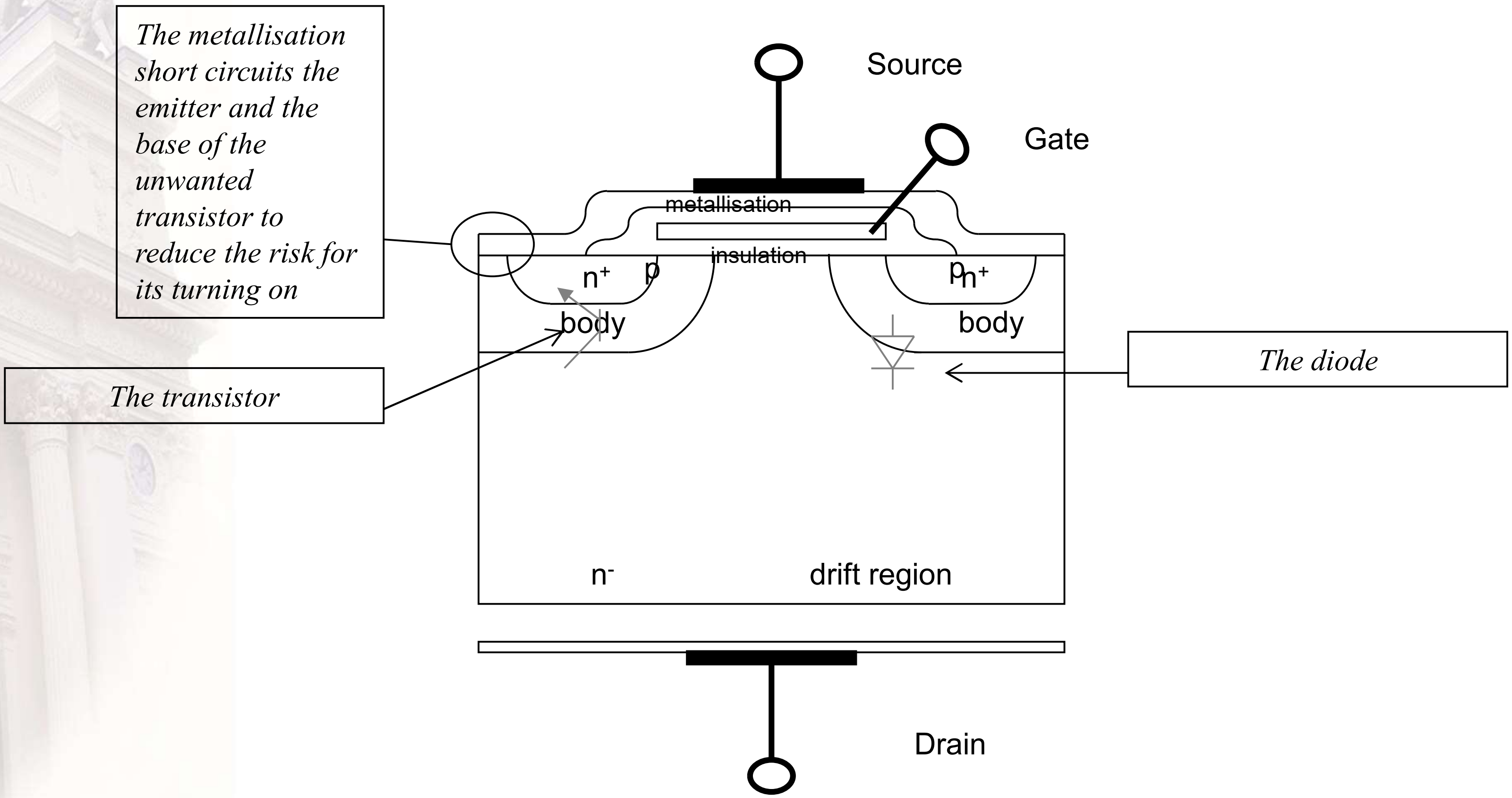
A turn on of the transistor the current i commutates to the transistor T, and the capacitor C is discharged via the the transistor T and resistor R. As the load voltage is 200V the duty cycle is 80%. The switching frequency is 2 kHz and the on state time is 0.5*0.8=0.4 ms, and thus the time constant =0.133 ms

$$\tau = C \cdot R \Rightarrow R = \frac{\tau}{C} = \frac{120 \cdot 10^{-6}}{1 \cdot 10^{-6}} = 120\ \Omega$$

Solution Exam 20170530 2c



Solution Exam 20170530 2d



Solution Exam 20170530 2e

Depletion region n^-

- *The depletion region, is an insulating region within a conductive, doped semiconductor material where the mobile charge carriers have been diffused away, or have been forced away by an electric field.*
- *The only elements left in the depletion region are ionized donor or acceptor impurities.*
- *The depletion region is so named because it is formed from a conducting region by removal of all free charge carriers, leaving none to carry a current.*

Exam 2017-05-30 3a

Three phase system

a) A symmetric three phase voltage:

$$\begin{cases} e_a = \hat{e} \cdot \cos(\omega \cdot t) \\ e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \\ e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \end{cases}$$

b) Show that these voltages form a rotating vector with constant length and constant speed in the complex (α,β) frame. (5 p.)

c) Draw the circuit of a current control block for a generic three phase RLE load. The drawing shall include three phase converter, reference and load current measurement. It must be clear in which blocks the different frame transformations occur. (5 p.)

Solution Exam 2017-05-30 3a

$$\begin{cases} e_a = \hat{e}\cdot COS\ (\omega\cdot t) \\ e_b = \hat{e}\cdot COS\ (\omega\cdot t - \frac{2\pi}{3}) \\ e_c = \hat{e}\cdot COS\ (\omega\cdot t - \frac{4\pi}{3}) \end{cases}$$

$$\vec{e} = \sqrt{\frac{2}{3}}\cdot\left(e_a + e_b\cdot e^{j\frac{2\pi}{3}} + e_c\cdot e^{j\frac{4\pi}{3}}\right) = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \cos\left(\omega t - \frac{4\pi}{3}\right)\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \left\{\cos(\omega t)\cdot\cos\left(\frac{2\pi}{3}\right) + \sin(\omega t)\cdot\sin\left(\frac{2\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \left\{\cos(\omega t)\cdot\cos\left(\frac{4\pi}{3}\right) + \sin(\omega t)\cdot\sin\left(\frac{4\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right) + \sin(\omega t)\cdot\left(\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right) + \sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(1 + \frac{1}{4} - j\cdot\frac{\sqrt{3}}{4} + \frac{1}{4} + j\cdot\frac{\sqrt{3}}{4}\right) + \sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{4} + j\cdot\frac{3}{4} + \frac{\sqrt{3}}{4} + j\cdot\frac{3}{4}\right)\right] = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(\frac{3}{2}\right) + \sin(\omega t)\cdot\left(j\cdot\frac{3}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot\left[\cos(\omega t) + j\cdot\sin(\omega t)\right] = \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

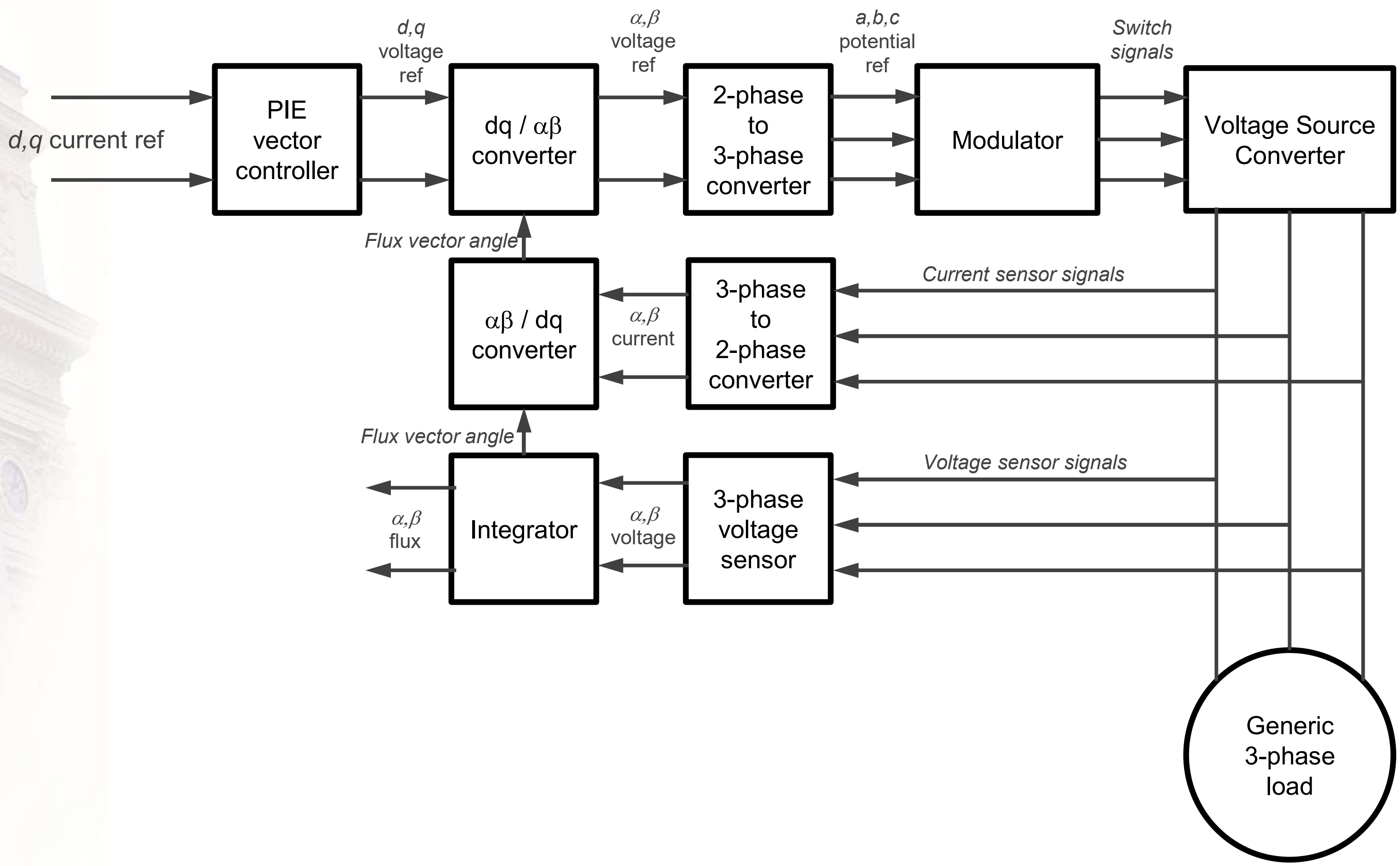
Alternativ e solution

$$\vec{e} = \sqrt{\frac{2}{3}}\cdot\left(e_a + e_b\cdot e^{j\frac{2\pi}{3}} + e_c\cdot e^{j\frac{4\pi}{3}}\right) = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cdot e^{j\frac{2\pi}{3}} + \cos\left(\omega t - \frac{4\pi}{3}\right)\cdot e^{j\frac{4\pi}{3}}\right] = \left\{\cos(x) = \frac{e^{jx} + e^{-jx}}{2}\right\} =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} + \frac{e^{j\left(\omega t - \frac{2\pi}{3}\right)} + e^{-j\left(\omega t - \frac{2\pi}{3}\right)}}{2}\cdot e^{j\frac{2\pi}{3}} + \frac{e^{j\left(\omega t - \frac{4\pi}{3}\right)} + e^{-j\left(\omega t - \frac{4\pi}{3}\right)}}{2}\cdot e^{j\frac{4\pi}{3}}\right] = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} + \frac{e^{j\omega t - j\frac{2\pi}{3} + j\frac{2\pi}{3}} + e^{-j\omega t + j\frac{2\pi}{3} + j\frac{2\pi}{3}}}{2} + \frac{e^{j\omega t - j\frac{4\pi}{3} + j\frac{4\pi}{3}} + e^{-j\omega t + j\frac{4\pi}{3} + j\frac{4\pi}{3}}}{2}\right] =$$

$$= \hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[e^{j\omega t} + e^{-j\omega t} + e^{j\omega t} + e^{-j\omega t}\cdot e^{j\frac{4\pi}{3}} + e^{j\omega t} + e^{-j\omega t}\cdot e^{j\frac{8\pi}{3}}\right] = \hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[3\cdot e^{j\omega t} + e^{-j\omega t}\cdot\underbrace{\left(1 - \frac{1}{2} + \frac{\sqrt{3}}{2} - -\frac{1}{2} - \frac{\sqrt{3}}{2}\right)}_{=0}\right] = \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

Solution Exam 20170530 3b



Exam 2017-05-30 4

The buck converter as battery charger

a) A DC/DC Converter has a DC link voltage of 100 V and can be either a 2Q or a 4Q converter supplying a load consisting of a 625 mH inductance in series with a 20 V back emf. The converter is carrier wave modulated with a 4 kHz modulation frequency and equipped with a current controller. A current step from 0 to 12 A is made and then back to 0 A again after 4 modulation periods.

b) Calculate the voltage reference for a few modulation periods before the positive step, for the positive step, for the time in between the steps, for the negative step and for a few modulation periods after the negative step in the 2Q case. (3p)

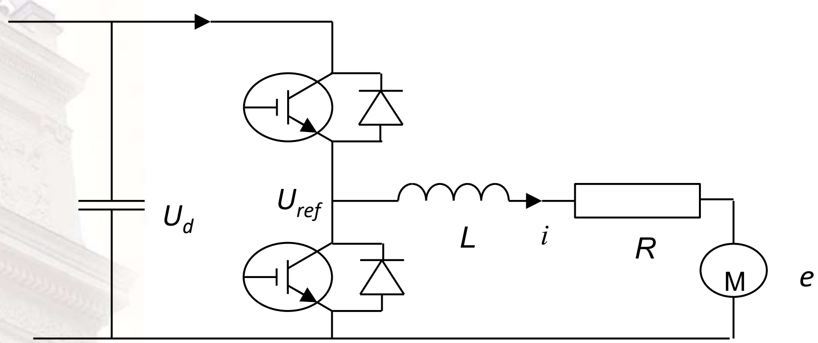
c) Draw the current to the load in the 2Q case, from two modulation periods before the positive current step to two modulation periods after the negative step. (2p)

d) Calculate the voltage reference for a few modulation periods before the positive step, for the positive step, for the time in between the steps, for the negative step and for a few modulation periods after the negative step in the 4Q case. (3p)

e) Draw the current to the load in the 4Q case, from two modulation periods before the positive current step to two modulation periods after the negative step. (2p)

f) In both b) and d) the current ripple must be correctly calculated.

Solution Exam 2017-05-30 4a



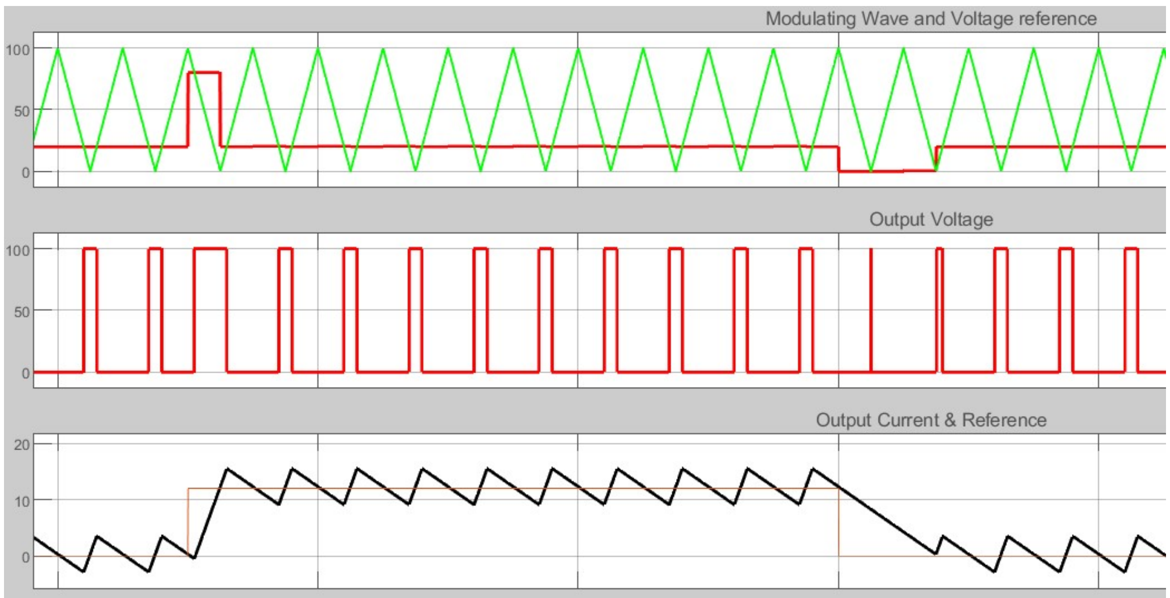
Data: f_{sw} = 4 kHz.
L = 0.625 Mh
R=0
U_d = 100 V
e = 20 V

Equation $U = L \cdot \frac{di}{dt} + e$

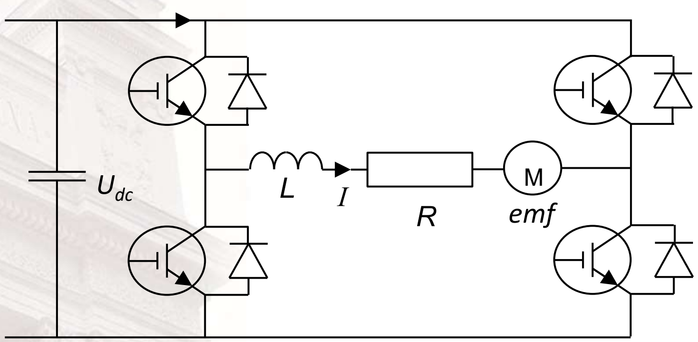
Before the pos. current step i=0 A, constant, di/dt=0	U_{ref}=e=20 V
At the positive current step, use max voltage	U_{ref}=U_d=100 V
At the constant current 12 A, di/dt=0, and R=0	U_{ref}=e=20 V
At the negative current step, use zero voltage	U_{ref}=0 V
After the neg. Current step i=0 A, constant, di/dt=0	U_{ref}=e=20 V

Solution Exam 2017-05-30 4b

The sampling period is 1/4000/2 = 125 microseconds
The positive step voltage reference should be u* = 625e-6/125e-6*(12-0) + 20 = 80 V. Since there is a 100 V DC link one sampling period should be enough. I see that the solution is not correctly drawn. The voltage reference step should not exceed the carrier but reach 80 V only.



Solution Exam 2017-05-30 4c



Data: f_{sw} = 4 kHz.
L = 0.625 Mh
R=0
U_{dc} = 100 V
e = 20 V

Equation $$U = L \cdot \frac{di}{dt} + e$$

Before the pos. current step i=0 A, constant, di/dt=0	U_{ref}=e=20 V
At the positive current step, use max voltage	U_{ref}=U_{dc}=100 V
At the constant current 12 A, di/dt=0, and R=0	U_{ref}=e=20 V
At the negative current step, use minimum voltage	U_{ref}=-100 V
After the neg. current step i=0 A, constant, di/dt=0	U_{ref}=e=20 V

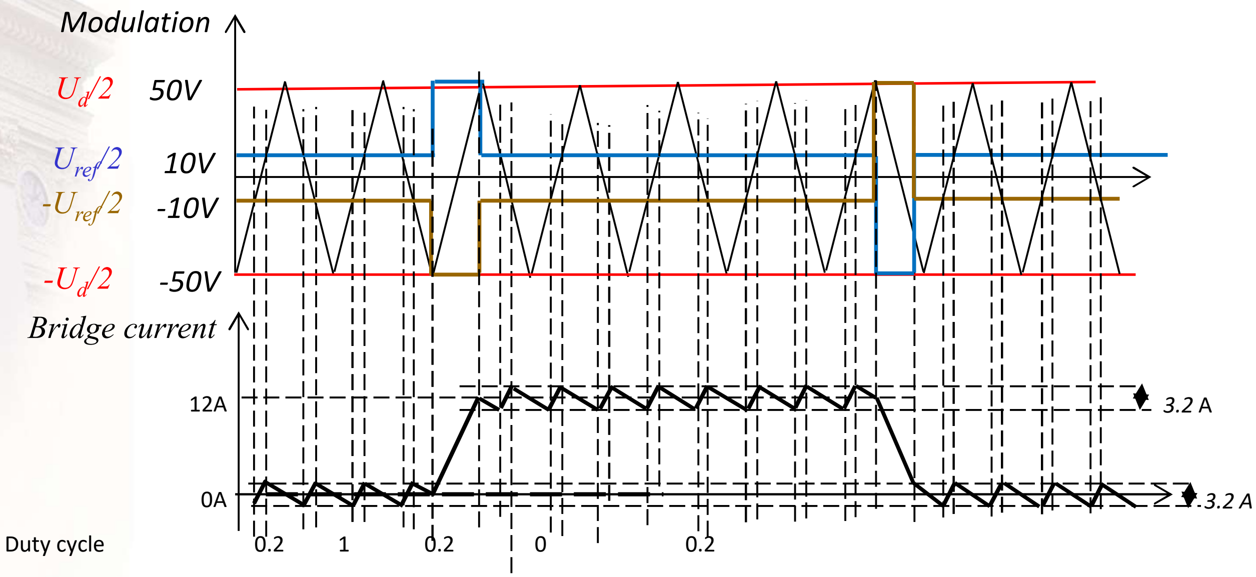
Solution Exam 2017-05-30 4d

$$Duty\ cycle\quad D = \frac{20}{100} = 0.2$$

$$U = L \cdot \frac{di}{dt} + e \approx L \cdot \frac{\Delta i}{\Delta t} + e \Rightarrow \Delta i = \frac{(U - e)}{L} \cdot \Delta t$$

*See figure. Time for current rise = Duty cycle * half the switching frequency period*

$$Current\ ripple\quad \Delta i = \frac{(U - e)}{L} \cdot \Delta t = \left\{ \Delta t = \frac{0.5 \cdot D}{f_{sw}} \right\} = \frac{(100 - 20)}{0.625 \cdot 10^{-3}} \cdot \frac{0.1}{4 \cdot 10^{3}} = 3.2\ A$$



Exam 2017-05-30 5_1

Permanently magnetized synchronous machine

A 120 kW motor drive is to be designed. The motor will run from a Power Electronic Converter with 800 V DC link. The bases speed must be 5000 rpm and the maximum speed 12000 rpm.

a) What is the rated torque of the machine (2p)
b) What is the RMS Phase-to-phase voltage at rated power? (2p)
c) What is the rated phase current of the machine? (2p)

All your answers must be accompanied with your calculations and motivations!

Exam 2017-05-30 5_2

Permanent magnetized motor

d) The electric frequency of the machine at base speed is 250 Hz. What is the lowest sampling frequency that you would choose to control the machine? (2p)

e) What is a suitable switching frequency for the converter? (2p)

All your answers must be accompanied with your calculations and motivations!

Solution 20170530 5

$5a)$ *Torque of the machine* $T = \frac{P}{\omega} = \frac{120000}{\frac{5000}{60} \cdot 2\pi} = 229\ Nm$

$5b)$ *Phase − to − phase voltage rms* $U_{LL\_rms} = \frac{800}{\sqrt{2}} = 566\ V$

$5c)$ $120000 = \{assume\ \ \cos\varphi = 0.95\} = \sqrt{3} \cdot 566 \cdot I \cdot 0.95 \Rightarrow I = \frac{120000}{\sqrt{3} \cdot 566 \cdot 0.95} = 129\ A$

$5d)$ *The base speed is where the top power is achieved. At this speed the electric frequency of the motor equals* $250\ Hz$*, and itsthe mechanical frequency* $= \frac{5000}{60} = 83.33\ Hz$

The relation between the electric and the mechanical frequency $= \frac{250}{83.33}\ a = 3$

gives the result that the motor has $6-$ *poles.*

At the top motor speed 12000 *rpm the electric frequency* $= 3 \cdot \frac{12000}{60} = 600\ Hz$.

Switching freq $= \{$*at least one switching period per hexagon side*$\} = 6 \cdot 600 = 3600\ Hz$

Sampling freq $= \{2$ *samples per switching frequency period*$\} = 2 \cdot 3600\ Hz = 7200\ Hz$

$5e)$ *See* $5d$*. Switching freq* $= 3600\ Hz$

Exam 2017-05-30

Formulas:

$$\vec{s} = K \cdot \left[s_a + s_b \cdot e^{j \cdot \frac{2 \cdot \pi}{3}} + s_c \cdot e^{j \cdot \frac{4 \cdot \pi}{3}} \right] = K \cdot \left[\frac{3}{2} \cdot s_a + j \cdot \frac{\sqrt{3}}{2} (s_b - s_c) \right] = s_\alpha + j \cdot s_\beta$$

Power invariant

Three phase –> two phase conversion

$$s_\alpha = \sqrt{\frac{3}{2}} \cdot s_a$$

$$s_\beta = \frac{1}{\sqrt{2}} \cdot (s_b - s_c)$$

Power invariant

Two phase –> three phase conversion

$$s_a = \sqrt{\frac{2}{3}} \cdot s_\alpha$$

$$s_b = -\frac{1}{\sqrt{6}} s_\alpha + \frac{1}{\sqrt{2}} \cdot s_\beta$$

$$s_c = -\frac{1}{\sqrt{6}} s_\alpha - \frac{1}{\sqrt{2}} \cdot s_\beta$$