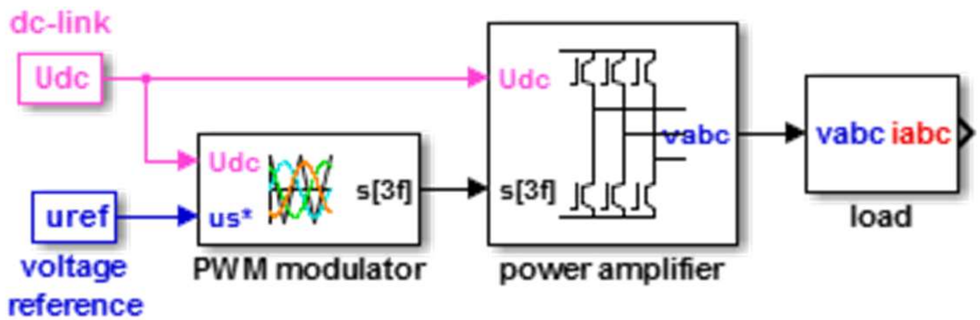
L9: AC power + 3φ modulation



Single phase power *(with sine functions)*

$$\begin{cases} u(t) = \hat{u}_{ph} \cdot cos(\omega t) \\ i(t) = \hat{\imath}_{ph} \cdot cos(\omega t - \varphi) \end{cases}$$

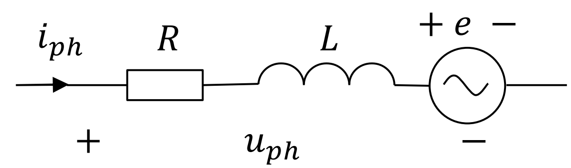
$$p(t) = \hat{u}_{ph} \cdot \hat{\imath}_{ph} \cdot (\cos(\omega t) \cdot \cos(\omega t - \varphi)) =$$

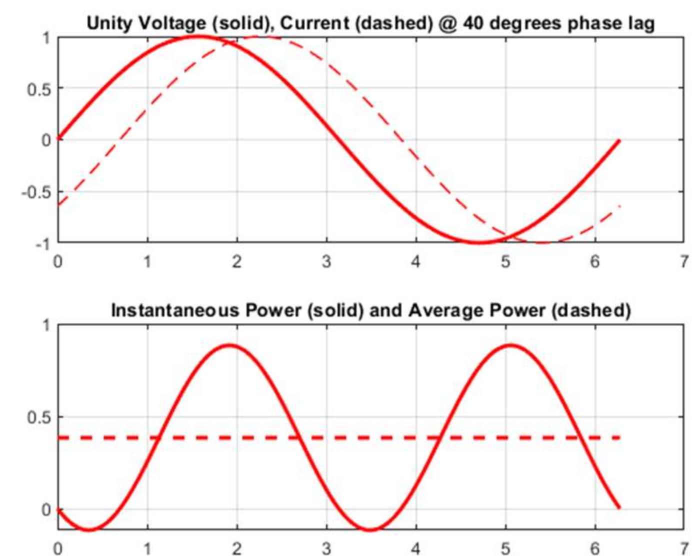
$$= \left\{ \cos(x) \cdot \cos(x - y) = \frac{\cos(y) + \cos(2 \cdot x + y)}{2} \right\} =$$

$$= \frac{\hat{u}_{ph} \cdot \hat{\imath}_{ph}}{2} \cdot (\cos(\varphi) + \cos(2\omega t - \varphi)) =$$

$$= \frac{\sqrt{2} \cdot U_{ph} \cdot \sqrt{2} \cdot I_{ph}}{2} \cdot \cos(\varphi) =$$

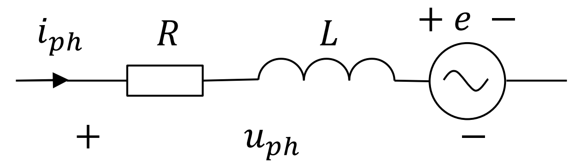
$$= U_{ph} \cdot I_{ph} \cdot \cos(\varphi) + U_{ph} \cdot I_{ph} \cdot \cos(2\omega t - \varphi)$$





Single phase power (with vectors)

$$\begin{cases} u(t) = \hat{u}_{ph} \cdot \cos(\omega t) = \hat{u}_{ph} \cdot \dfrac{e^{j\omega t} + e^{-j\omega t}}{2} \\ i(t) = \hat{\imath}_{ph} \cdot \cos(\omega t - \varphi) = \hat{\imath}_{ph} \cdot \dfrac{e^{j\omega t - j} + e^{-j\omega t + j\varphi}}{2} \end{cases}$$



$$p(t) = u(t) \cdot i(t) = \hat{u}_{ph} \cdot \hat{\imath}_{ph} \cdot \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) \cdot \left(\frac{e^{j\omega t - j\varphi} + e^{-j\omega t + j\varphi}}{2}\right)$$

$$= \frac{U_{ph} \cdot I_{ph}}{2} \cdot \left[e^{j\omega t + j\omega t - j\varphi} + e^{j\omega t - j\omega t + j\varphi} + e^{-j\omega t + j\omega t - j\varphi} + e^{-j\omega t - j\omega t + j\varphi}\right] =$$

$$= \frac{U_{ph} \cdot I_{ph}}{2} \cdot \left[\left(e^{2j\omega t - j\varphi} + e^{-(2j\omega t - j\ \ )}\right) + \left(e^{j\varphi} + e^{-j\varphi}\right)\right] =$$

$$= U_{ph} \cdot I_{ph} \cdot \cos(\varphi) + U_{ph} \cdot I_{ph} \cdot \cos(2\omega t - \varphi)$$

Single phase active and reactive power

- **Voltage and current**

$$\begin{cases} u(t) = \hat{u}_{ph} \cdot \cos(\omega t) \\ i(t) = \hat{\imath}_{ph} \cdot \cos(\omega t - \varphi) \end{cases}$$

- **Active power**

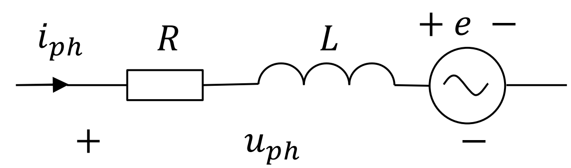
$$P_{ave} = U_m \cdot I_m \cdot \cos(\varphi)$$

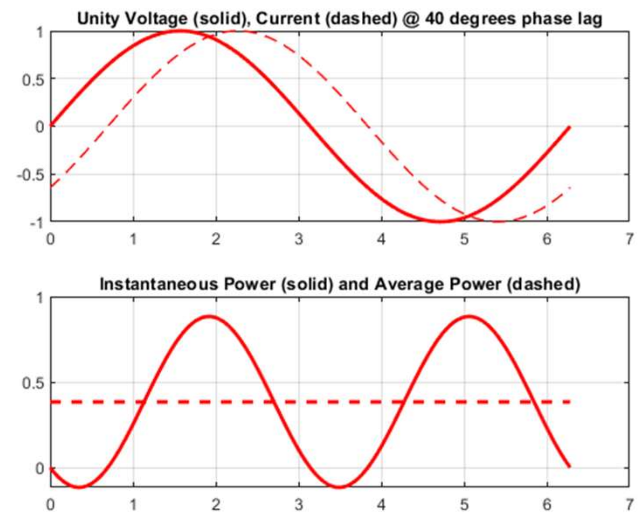
- **Reactive power**

$$Q_{av} = U_m \cdot I_m \cdot \sin(\varphi)$$

- **Apparent power**

$$S_{ave} = U_m \cdot I_m$$



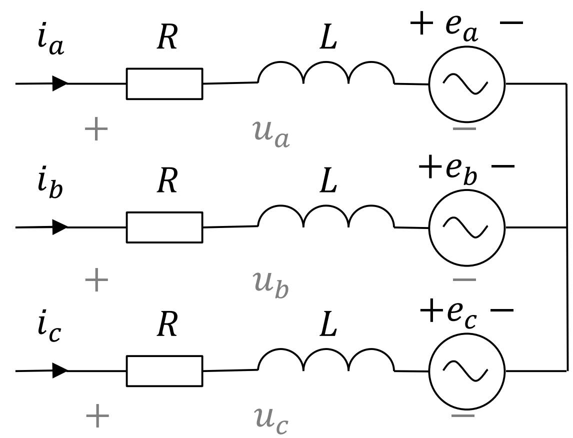


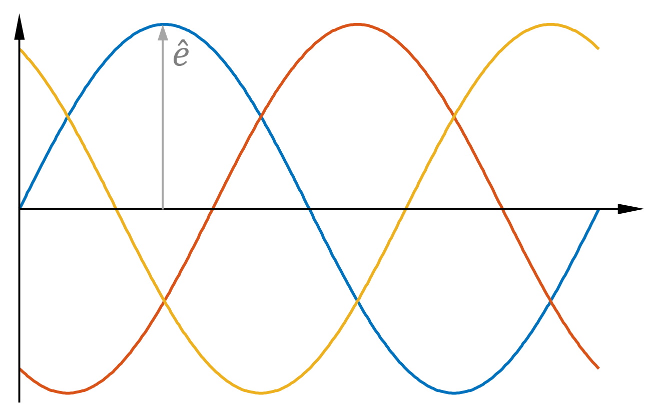
Three phase voltage and current

$$\begin{cases} u_a(t) = \hat{u}_{ph} \cdot \cos(\omega t) = \sqrt{2} \cdot U_{ph} \cdot \cos(\omega t) \\ u_b(t) = \hat{u}_{ph} \cdot \cos\left(\omega t - \frac{2\pi}{3}\right) = \sqrt{2} \cdot U_{ph} \cdot \cos\left(\omega t - \frac{2\pi}{3}\right) \\ u_c(t) = \hat{u}_{ph} \cdot \cos\left(\omega t - \frac{4\pi}{3}\right) = \sqrt{2} \cdot U_{ph} \cdot \cos\left(\omega t - \frac{4\pi}{3}\right) \end{cases}$$

$$\begin{cases} i_a(t) = \hat{\imath}_{ph} \cdot \cos(\omega t - \varphi) = \sqrt{2} \cdot I_{ph} \cdot \cos(\omega t - \varphi) \\ i_b(t) = \hat{\imath}_{ph} \cdot \cos\left(\omega t - \frac{2\pi}{3} - \varphi\right) = \sqrt{2} \cdot I_{ph} \cdot \cos\left(\omega t - \frac{2\pi}{3} - \varphi\right) \\ i_c(t) = \hat{\imath}_{ph} \cdot \cos\left(\omega t - \frac{4\pi}{3} - \varphi\right) = \sqrt{2} \cdot I_{ph} \cdot \cos\left(\omega t - \frac{4\pi}{3} - \varphi\right) \end{cases}$$

$$\begin{cases} e_a = \hat{e} \cdot \cos(\omega \cdot t) = \sqrt{2} \cdot E_{ph} \cdot \cos(\omega \cdot t) \\ e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) = \sqrt{2} \cdot E_{ph} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \\ e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) = \sqrt{2} \cdot E_{ph} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \end{cases}$$





Three phase power

$$p(t) = \hat{u}_{ph} \cdot \hat{\imath}_{ph} \cdot \left(\cos(\omega t) \cdot \cos(\omega t - \varphi) + \cos\left(\omega t - \frac{2\pi}{3}\right) \cdot \cos\left(\omega t - \frac{2\pi}{3} - \varphi\right) + \cos\left(\omega t - \frac{4\pi}{3}\right) \cdot \cos\left(\omega t - \frac{4\pi}{3} - \varphi\right) \right) =$$

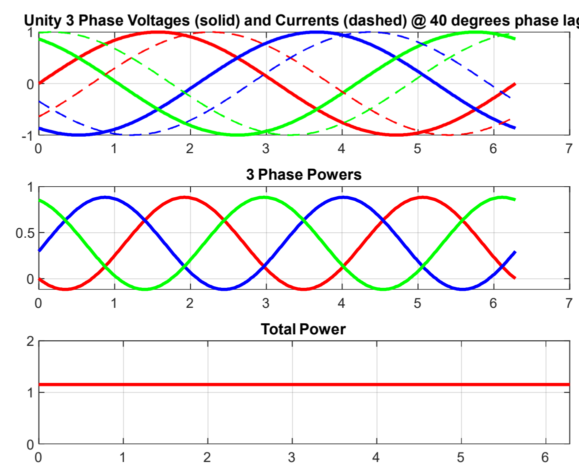
$$= \left\{ \cos(x) \cdot \cos(x - y) = \frac{\cos(y) + \cos(2 \cdot x + y)}{2} \right\} =$$

$$= \frac{\hat{u}_{ph} \cdot \hat{\imath}_{ph}}{2} \cdot \left(\cos(\varphi) + \cos(2\omega t - \varphi) + \cos(\varphi) + \cos\left(2\omega t - \frac{4\pi}{3} - \varphi\right) + \cos(\varphi) + \cos\left(2\omega t - \frac{8\pi}{3} - \varphi\right) \right) =$$

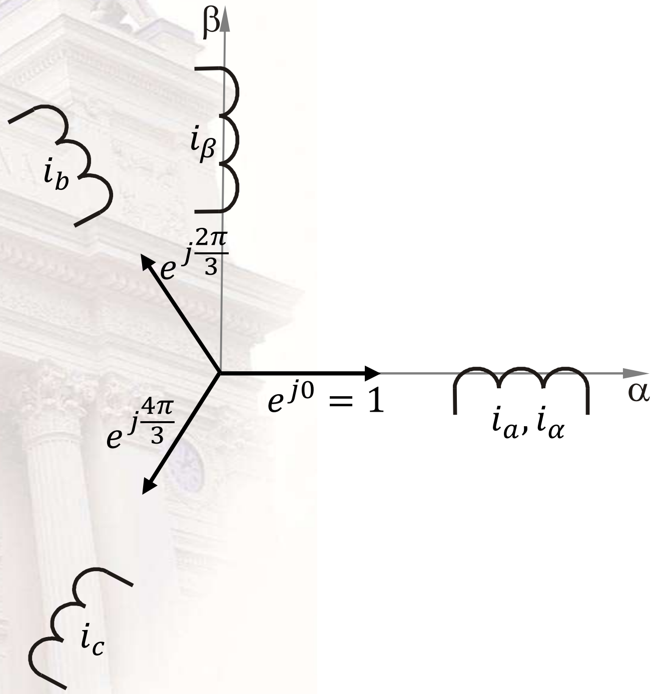
$$= \frac{\hat{u}_{ph} \cdot \hat{\imath}_{ph}}{2} \cdot \left(3 \cdot \cos(\varphi) + \underbrace{\cos(2\omega t - \varphi) + \cos\left(2\omega t - \frac{2\pi}{3} - \varphi\right) + \cos\left(2\omega t - \frac{4\pi}{3} - \varphi\right)}_{=0} \right) =$$

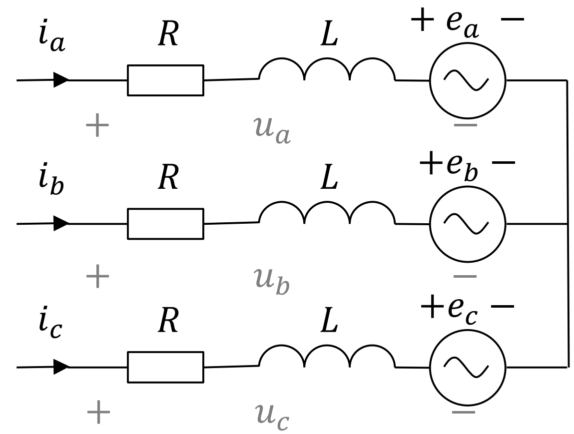
$$= 3 \cdot \frac{\sqrt{2} \cdot U_{ph} \cdot \sqrt{2} \cdot I_{ph}}{2} \cdot \cos(\varphi) =$$

$$= 3 \cdot U_{ph} \cdot I_{ph} \cdot \cos(\varphi) = \sqrt{3} \cdot U_{ph\_ph} \cdot I_{ph} \cdot \cos(\varphi)$$



The generic 3-phase load





$$\sqrt{\frac{2}{3}}\cdot e^{j\,0}\cdot\left(u_a = R\cdot i_a + L\cdot\frac{di_a}{dt} + e_a\right)$$

$$\sqrt{\frac{2}{3}}\cdot e^{j\frac{2\pi}{3}}\cdot\left(u_b = R\cdot i_b + L\cdot\frac{di_b}{dt} + e_b\right)$$

$$+\quad\sqrt{\frac{2}{3}}\cdot e^{j\frac{4\pi}{3}}\cdot\left(u_c = R\cdot i_c + L\cdot\frac{di_c}{dt} + e_c\right)$$

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{d\vec{i}}{dt} + \vec{e}$$

$$p(t) = u_a\cdot i_a + u_b\cdot i_b + u_c\cdot i_c = u_\alpha\cdot i_\alpha + u_\beta\cdot i_\beta$$

Example, grid voltage vector

$$\vec{e} = \sqrt{\frac{2}{3}} \cdot \left(e_a + e_b \cdot e^{j\frac{2\pi}{3}} + e_c \cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\cos(\omega \cdot t) + \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right) =$$

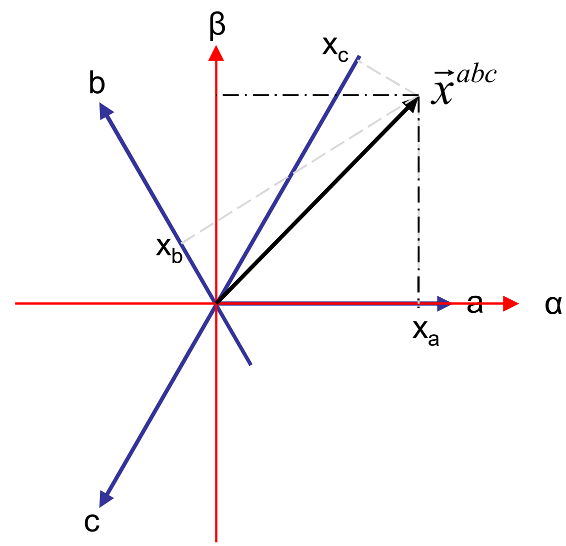
$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\begin{array}{l} \cos(\omega \cdot t) + \left(\cos(\omega \cdot t) \cdot \cos\left(\frac{2\pi}{3}\right) + \sin(\omega \cdot t) \cdot \sin\left(\frac{2\pi}{3}\right)\right) \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) \\ + \left(\cos(\omega \cdot t) \cdot \cos\left(\frac{4\pi}{3}\right) + \sin(\omega \cdot t) \cdot \sin\left(\frac{4\pi}{3}\right)\right) \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \end{array}\right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\cos(\omega \cdot t) \cdot \left(1 + \frac{1}{4} + \frac{1}{4}\right) + j \cdot \sin(\omega \cdot t) \cdot \left(\frac{3}{4} + \frac{3}{4}\right)\right) =$$

$$= \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot (\cos(\omega \cdot t) + j \cdot \sin(\omega \cdot t)) = E \cdot e^{j\omega t}$$

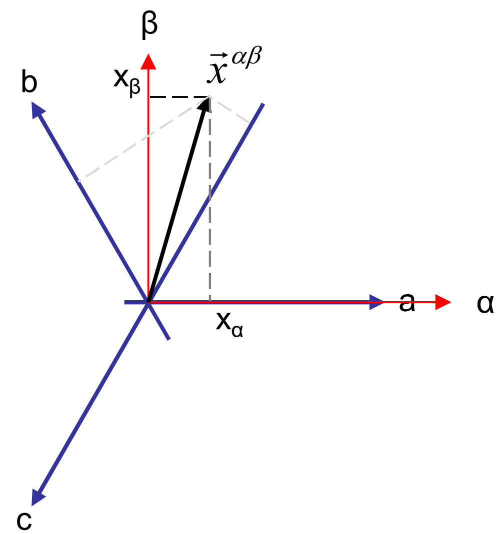
Transformation example: abc→αβ

$$
\begin{cases}
\vec{x}^{\alpha\beta} = x_\alpha + j \cdot x_b = \sqrt{\frac{2}{3}} \cdot \left(x_a \cdot e^{j0} + x_b \cdot e^{j\frac{2\pi}{3}} + x_c \cdot e^{j\frac{4\pi}{3}}\right) = \\
= \sqrt{\frac{2}{3}} \cdot \left(x_a + x_b \cdot \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) + x_c \cdot \left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right)\right) \\
x_\alpha = \sqrt{\frac{2}{3}} \cdot \left(x_a - \frac{x_b}{2} - \frac{x_c}{2}\right) = \sqrt{\frac{2}{3}} \cdot x_a - \frac{\sqrt{2}}{2 \cdot \sqrt{3}} \cdot x_b - \frac{\sqrt{2}}{2 \cdot \sqrt{3}} x_c = \\
= \sqrt{\frac{2}{3}} \cdot x_a - \frac{1}{\sqrt{6}} \cdot (x_b + x_c) \\
x_\beta = \sqrt{\frac{2}{3}} \cdot \left(\frac{\sqrt{3}}{2} \cdot x_b - \frac{\sqrt{3}}{2} \cdot x_c\right) = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} \cdot x_b - \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} \cdot x_c = \\
= \frac{1}{\sqrt{2}} \cdot (x_b - x_c)
\end{cases}
$$

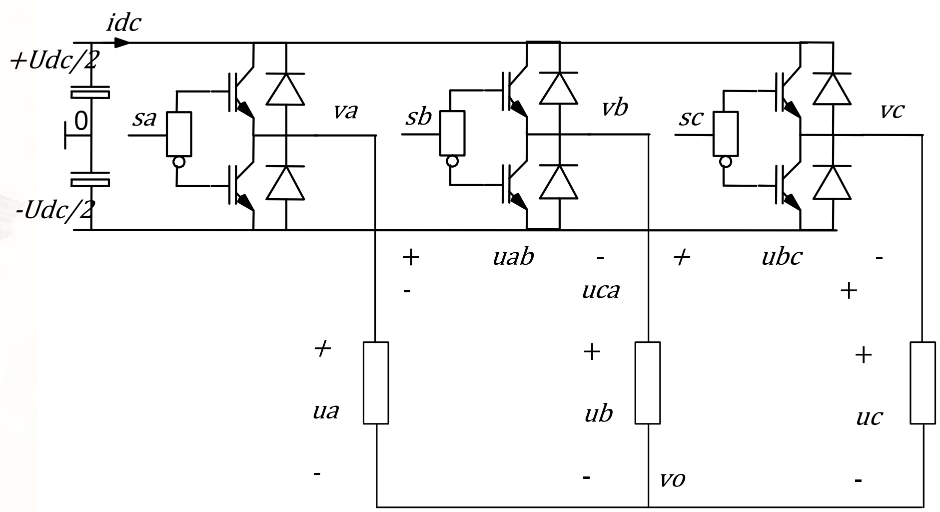


Transformation example: αβ→abc

$$\begin{cases} x_a = \sqrt{\dfrac{2}{3}} \cdot x_\alpha \\ x_b = \sqrt{\dfrac{2}{3}} \cdot \left(-\dfrac{x_\alpha}{2} + x_\beta \cdot \dfrac{\sqrt{3}}{2} \right) = -\dfrac{x_\alpha}{\sqrt{6}} + x_\beta \cdot \dfrac{1}{\sqrt{2}} \\ x_c = \sqrt{\dfrac{2}{3}} \cdot \left(-\dfrac{x_\alpha}{2} - x_\beta \cdot \dfrac{\sqrt{3}}{2} \right) = -\dfrac{x_\alpha}{\sqrt{6}} - x_\beta \cdot \dfrac{1}{\sqrt{2}} \end{cases}$$

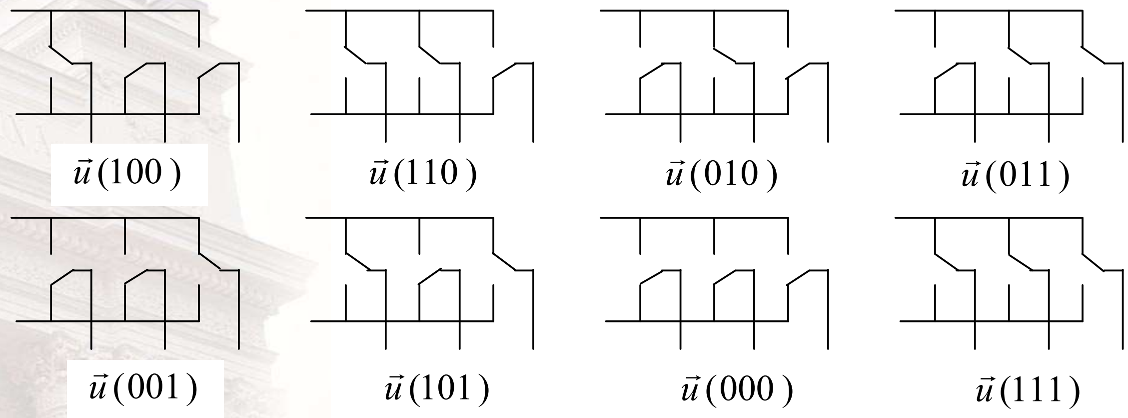


3-phase converter



$$v_o = \frac{v_a + v_b + v_c}{3}$$

3-phase output voltage → as vector : I

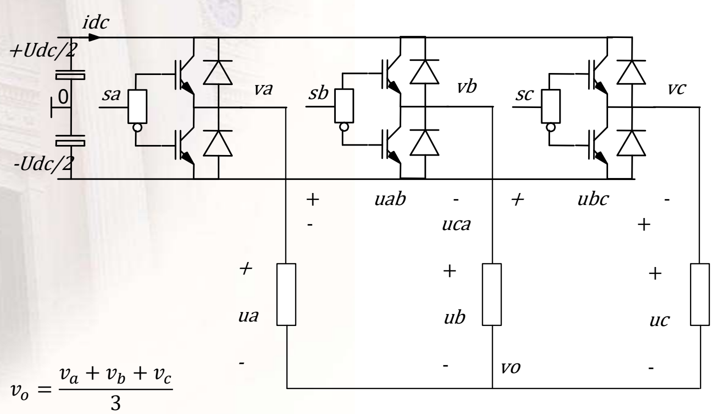


va	vb	vc	vo	ua	ub	uc	uα	uβ
-1	-1	-1						
-1	-1	1						
-1	1	-1						
-1	1	1						
1								
1	-1	1						
1	1							
1	1							

x Udc/2

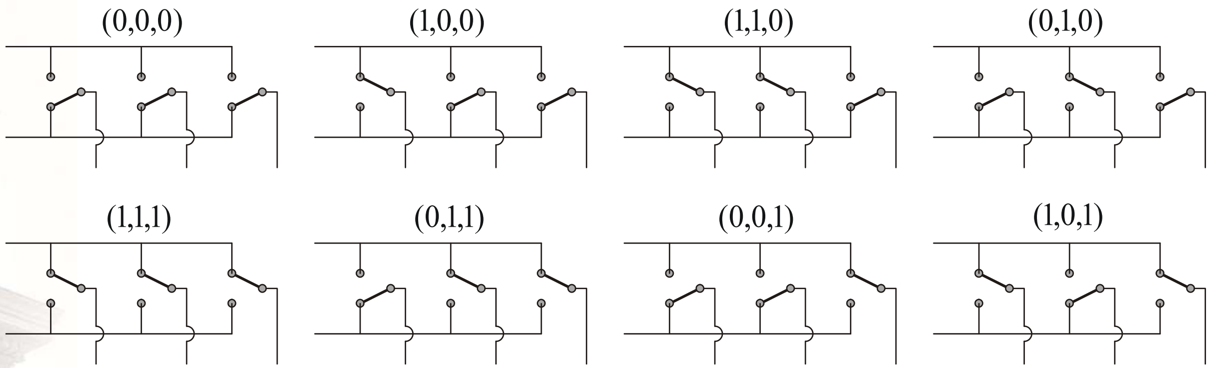
ua = va-vo

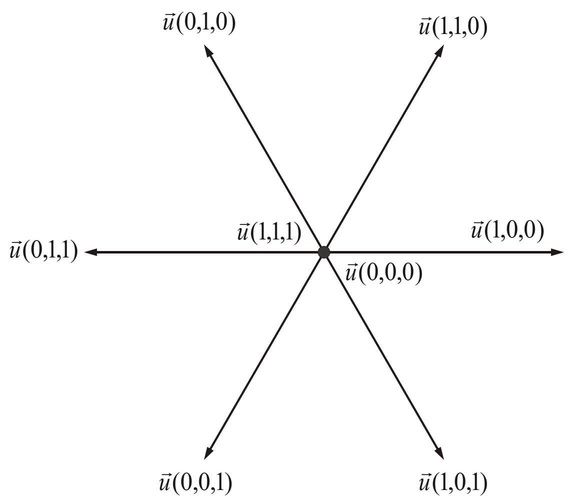
vo =(va+vb+vc)/3



$$\vec{u}^{\alpha\beta} = \sqrt{\frac{3}{2}} \cdot u_a + j\frac{1}{\sqrt{2}} \cdot (u_b - u_c) = u_\alpha + j \cdot u_\beta$$

3-phase converters – 8 switch states





$$\vec{u}(1,0,0) = \sqrt{\frac{2}{3}}U_{dc} = -\vec{u}(0,1,1)$$

$$\vec{u}(0,1,0) = \sqrt{\frac{2}{3}}U_{dc} \cdot e^{j\frac{2\cdot\pi}{3}} = -\vec{u}(1,0,1)$$

$$\vec{u}(0,0,1) = \sqrt{\frac{2}{3}}U_{dc} \cdot e^{j\frac{4\cdot\pi}{3}} = -\vec{u}(1,1,0)$$

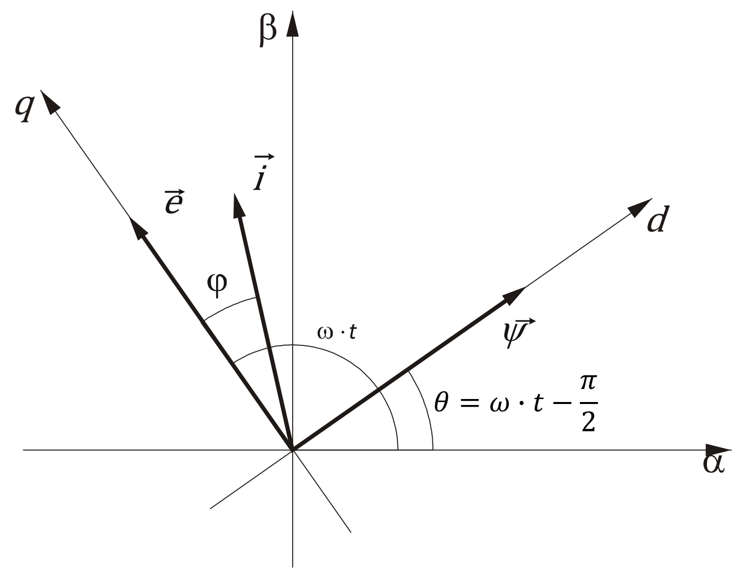
$$\vec{u}(0,0,0) = 0 = \vec{u}(1,1,1)$$

Rotating reference frame

- Use the integral of the grid back emf vector:

$$\vec{\psi} = \int_0^t \vec{e} \cdot dt = \int_0^t E \cdot e^{j\omega \cdot t} dt = \frac{\vec{e}}{j \cdot \omega}$$

$$= \frac{E}{\omega} e^{j\left(\omega \cdot t - \frac{\pi}{2}\right)}$$



Introduce the grid flux reference frame (d,q) …

- **Express the stator equation in the rotor reference frame**

$$\vec{u}^{\alpha\beta} = R\cdot\vec{\imath}^{\alpha\beta} + L\cdot\frac{d\vec{\imath}^{\alpha\beta}}{dt} + \vec{e}^{\alpha\beta}$$

$$\left\{\vec{s}^{\alpha\beta} = \vec{s}^{dq}\cdot e^{j\theta}\ \right\}$$

$$\vec{u}_s^{dq}\cdot e^{j\theta} = R\cdot\vec{\imath}^{dq}\cdot e^{j\theta} + L\cdot\frac{d}{dt}\left(\vec{\imath}^{dq}\cdot e^{j\theta}\right) + \vec{e}^{dq}\cdot e^{j\theta} =$$

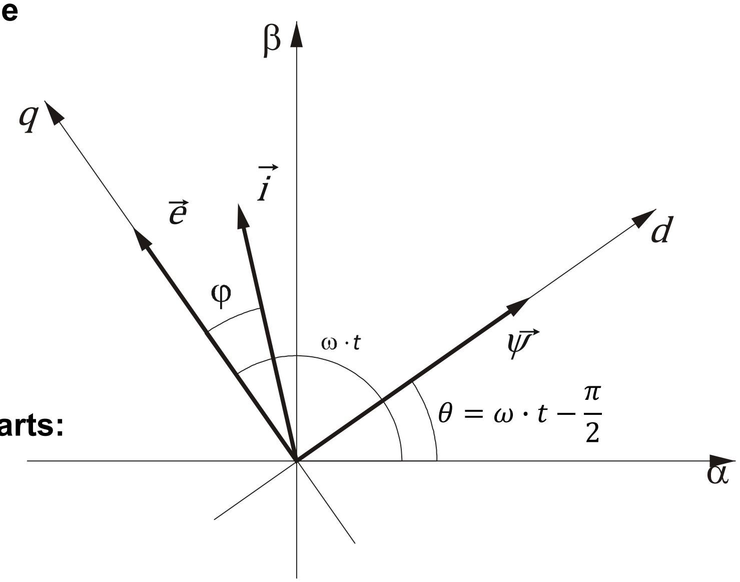
$$= R\cdot\vec{\imath}^{dq}\cdot e^{j\theta} + L\cdot\frac{d\vec{\imath}^{dq}}{dt}\cdot e^{j\theta} + j\cdot\frac{d\theta}{dt}\cdot L\cdot\vec{\imath}^{dq}\cdot e^{j\theta} + \vec{e}^{dq}\cdot e^{j\theta} =$$

$$\vec{u}_s^{dq} = R\cdot\vec{\imath}^{dq} + L\cdot\frac{d\vec{\imath}^{dq}}{dt} + j\cdot\omega\cdot L\cdot\vec{\imath}^{dq} + \vec{e}^{dq}$$

- **Split up the complex equation in real- and imaginary parts:**

$$u_d = R\cdot i_d + L\cdot\frac{di_d}{dt} - \omega\cdot L\cdot i_q$$

$$u_q = R\cdot i_q + L\cdot\frac{di_q}{dt} + \omega\cdot L\cdot i_d + e_q$$



Active power ...

- **Express the terminal power of the load**

$$\vec{u} = R \cdot \vec{\imath} + L \cdot \frac{d\vec{\imath}}{dt} + j \cdot \omega \cdot L \cdot \vec{\imath} + \vec{e}$$

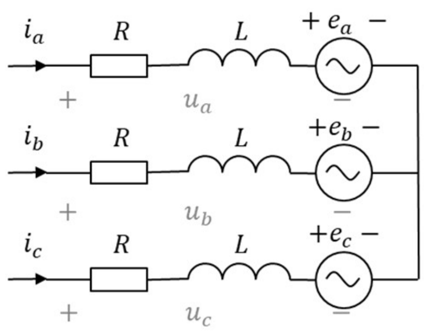
$$p(t) = \mathrm{Re}\{\vec{u} \cdot \vec{\imath}^{*}\} = \mathrm{Re}\left\{R \cdot \vec{\imath} \cdot \vec{\imath}^{*} + L \cdot \frac{d\vec{\imath}}{dt} \cdot \vec{\imath}^{*} + j \cdot \omega \cdot L \cdot \vec{\imath} \cdot \vec{\imath}^{*} + \vec{e} \cdot \vec{\imath}^{*}\right\} =$$

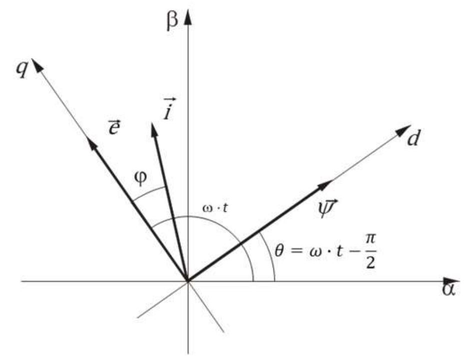
$$= \underbrace{Ri_d^2 + Ri_q^2}_{1} + \underbrace{L\frac{di_d}{dt}i_d + L\frac{di_q}{dt}i_q}_{2} + \underbrace{e_q i_q}_{3}$$

Resistive losses

Energizing inductances

Power absorbed by the grid back emf





Stationarity:

$$p(t) = E \cdot |\vec{\imath}| \cdot \cos(\varphi) = E \cdot \sqrt{\frac{3}{2}} \cdot \left|\hat{\imath}_{phase}\right| \cdot \cos(\varphi) = \sqrt{3} \cdot E \cdot I_{rms,phase} \cdot \cos(\varphi)$$

3-phase converters - sinusoidal references

- **Assume a rotating voltage reference vector**

$$\vec{u}^* = u^* \cdot e^{j\omega t} = u^* \cdot (\cos(\omega t) + j \cdot \sin(\omega t)) = u_\alpha^* + j \cdot u_\beta^*$$

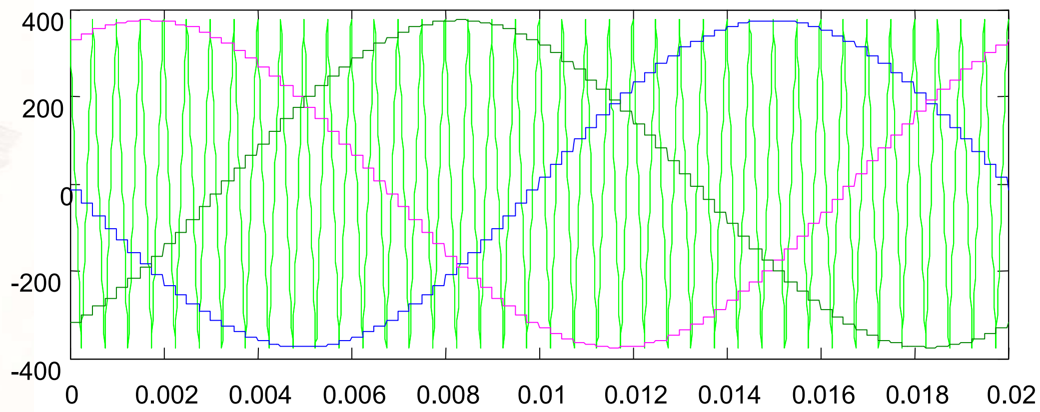
$$u_a^* = \sqrt{\frac{2}{3}} u_\alpha^*$$
$$u_b^* = \frac{1}{\sqrt{2}} u_\beta^* - \frac{1}{\sqrt{6}} u_\alpha^*$$
$$u_c^* = -\frac{1}{\sqrt{2}} u_\beta^* - \frac{1}{\sqrt{6}} u_\alpha^*$$

$\Rightarrow$

$$u_a^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t)$$
$$u_b^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t - \frac{2\pi}{3})$$
$$u_c^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t - \frac{4\pi}{3})$$

3-phase converters modulation

Simplest with sinusoidal references...

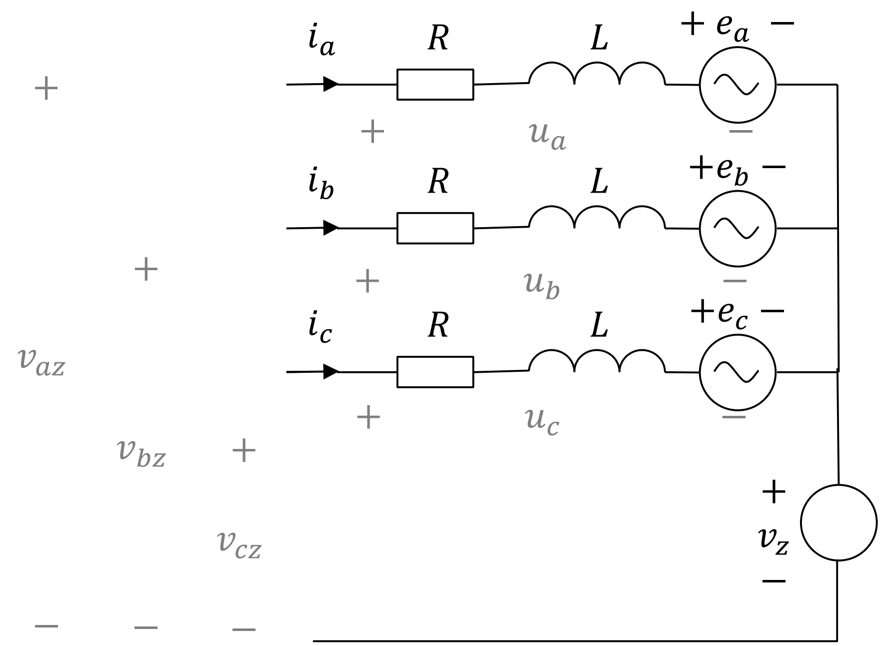


... but the DC link voltage is badly utilized.

3-phase converters – symmetrization

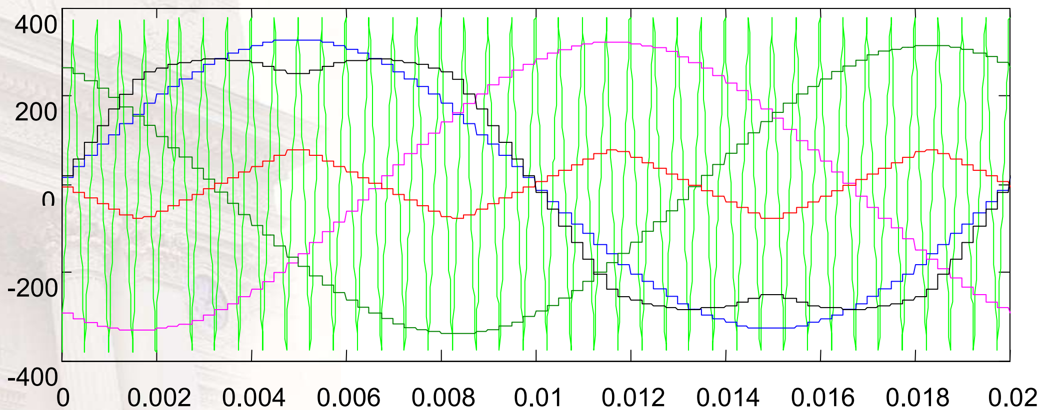
- 3 phase potentials, only 2 vector components. One degree of freedom to be used for other purposes.

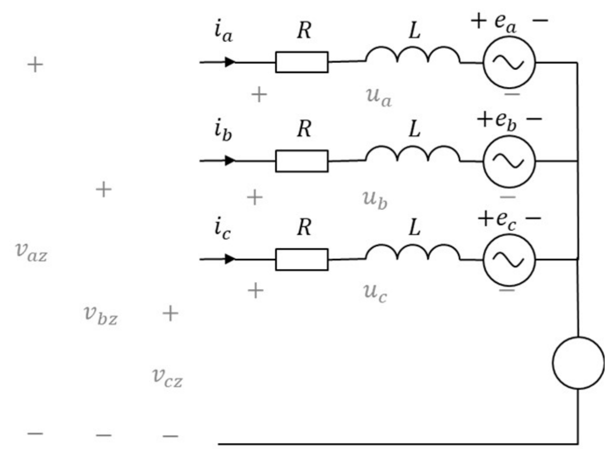
$$v_{az}^* = u_a^* - v_z^*$$
$$v_{bz}^* = u_b^* - v_z^*$$
$$v_{cz}^* = u_c^* - v_z^*$$



3-phase symmetrized modulation

$$v_z^* = \frac{\max(u_a^*, u_b^*, u_c^*) + \min(u_a^*, u_b^*, u_c^*)}{2}$$

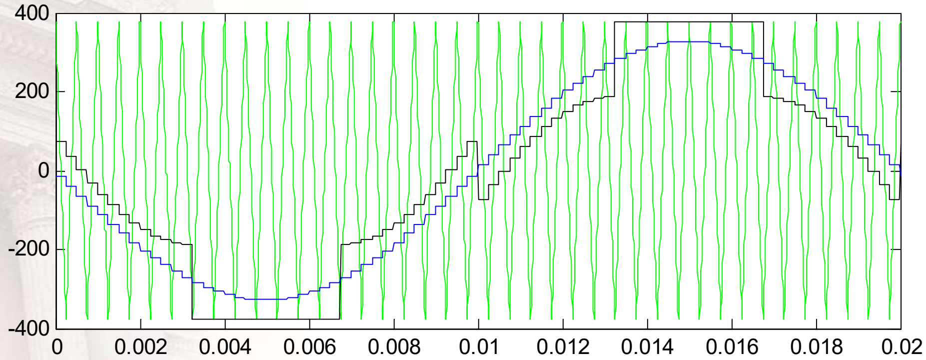




Maximum phase voltage with sinusoidal modulation : Udc/2
Maximum phase-to phase voltage with symmetrized modulation : Udc -> Phase voltage Udc/sqrt(3), i.e. 2/sqrt(3)=1.15 times larger than with sinusoidal modulation.

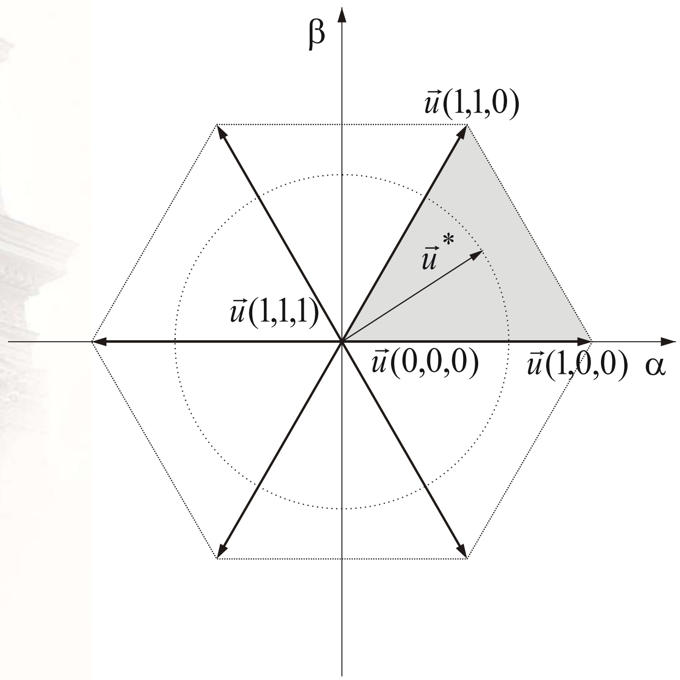
3-phase minimum switching modulation

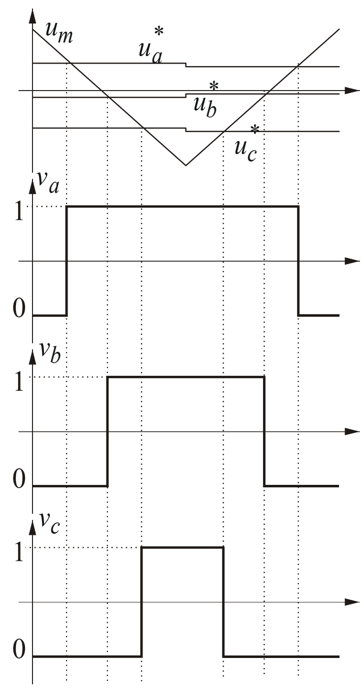
$$v_z * = -\min\left(\frac{U_{dc}}{2} - \max(u_a^*, u_b^*, u_c^*), -\frac{U_{dc}}{2} - \min(u_a^*, u_b^*, u_c^*)\right)$$



One phase is not switching for 2 60 degree intervals ...

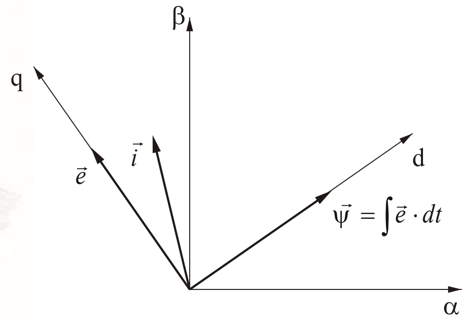
Modulation sequence vs. ripple

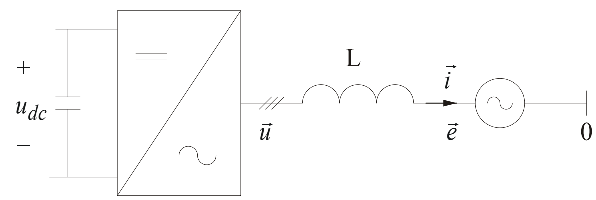




$$\vec{u}^{\alpha\beta} = R \cdot \vec{\imath}^{\alpha\beta} + L \cdot \frac{d\vec{\imath}^{\alpha\beta}}{dt} + \vec{e}^{\alpha\beta}$$

Modulation sequence vs. ripple





$$\frac{d\vec{i}}{dt} = \frac{\vec{u} - \vec{e}}{L}$$

Current ripple in the (d,q)-frame