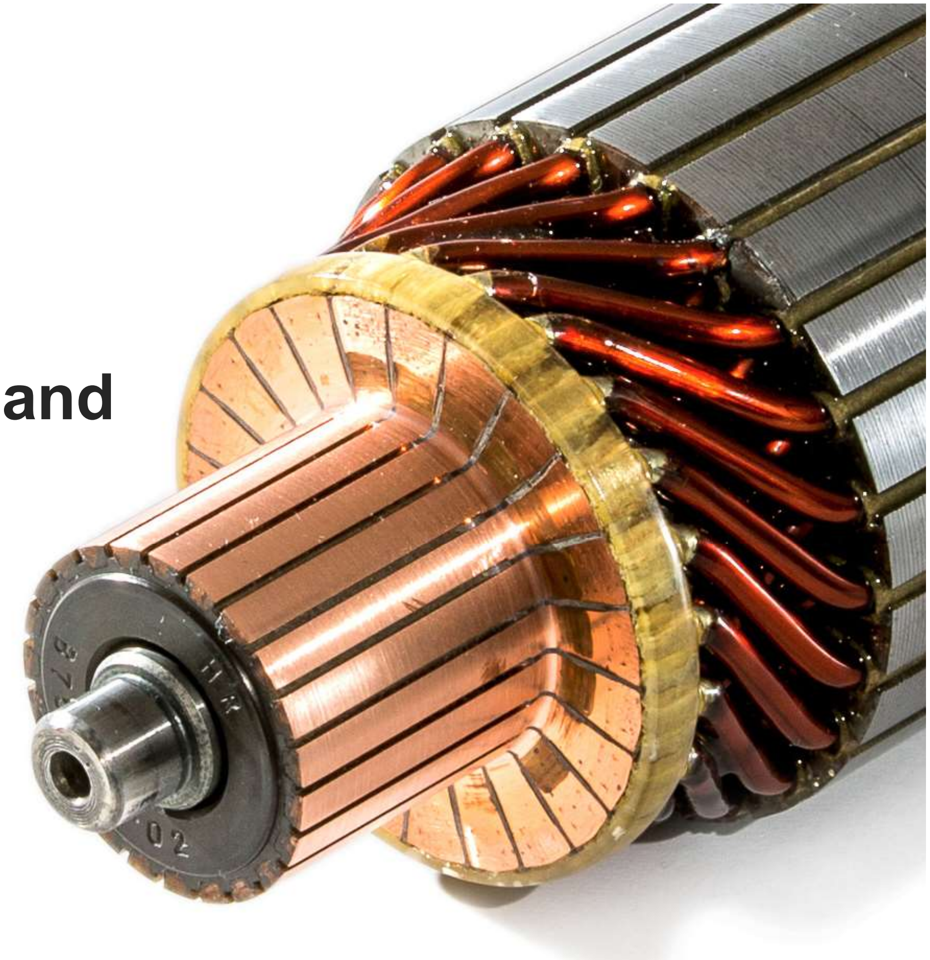
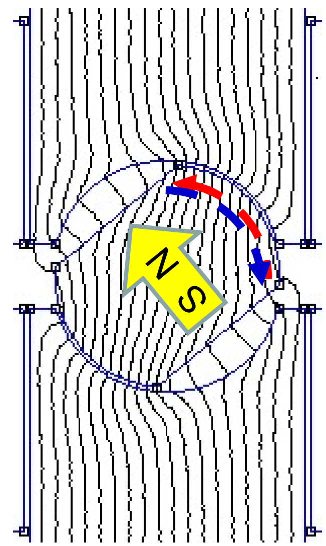
L8 The DC Machine and related Control



Our Theoretical Background

- We make Torque with
  - *Lorentz forces, and*
  - *Reluctance forces*
- We create rotation
  - *With a 3-phase winding (symmetrically distributed in space), and*
  - *A 3-phase current (symmetrically distributed in time)*
- We model the 3-phase winding with Vectors
- But …
  - *Today we will replace the 3-phase winding with another solution …*



Lorentz and Reluctance forces

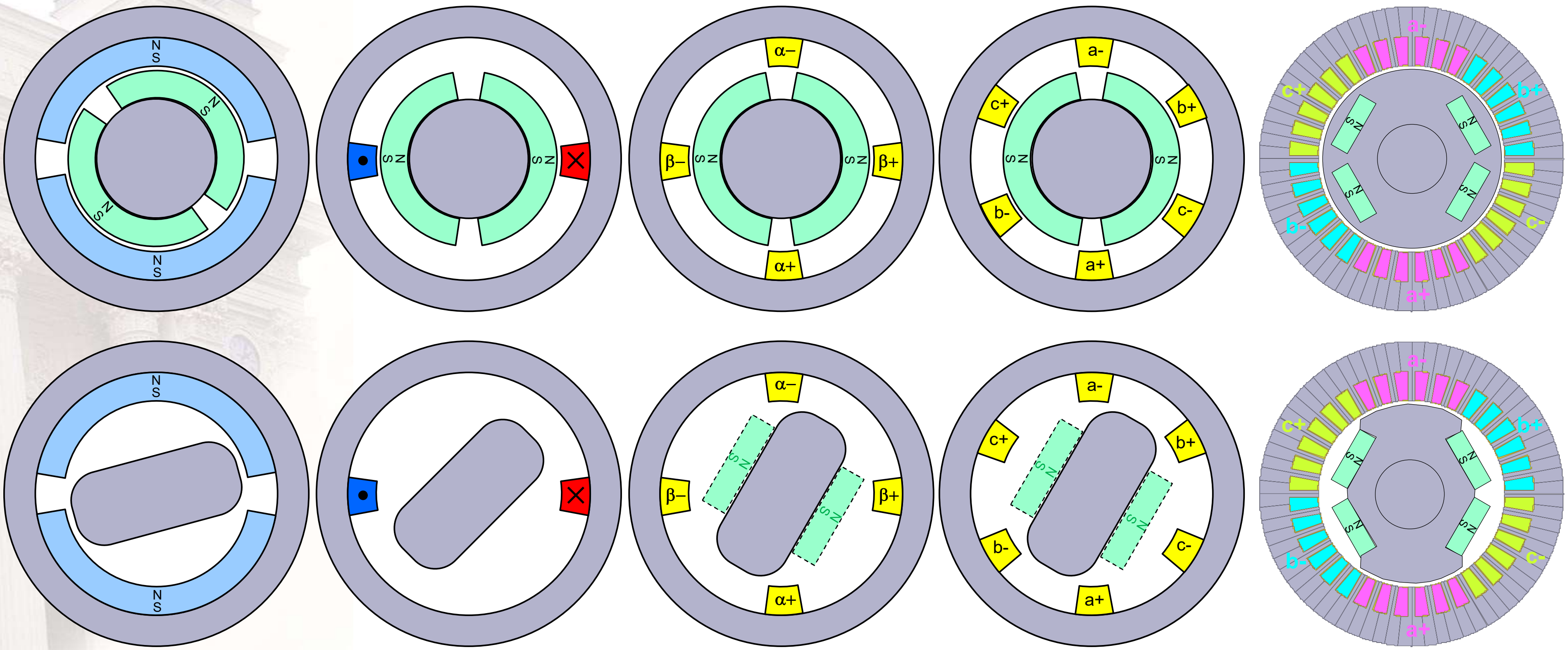
PM
PM rotor

1-phase stator
PM rotor

2-phase stator
PM rotor

3-phase stator
PM rotor

3-phase stator
Reluctance & PM rotor



PM stator
Reluctance rotor

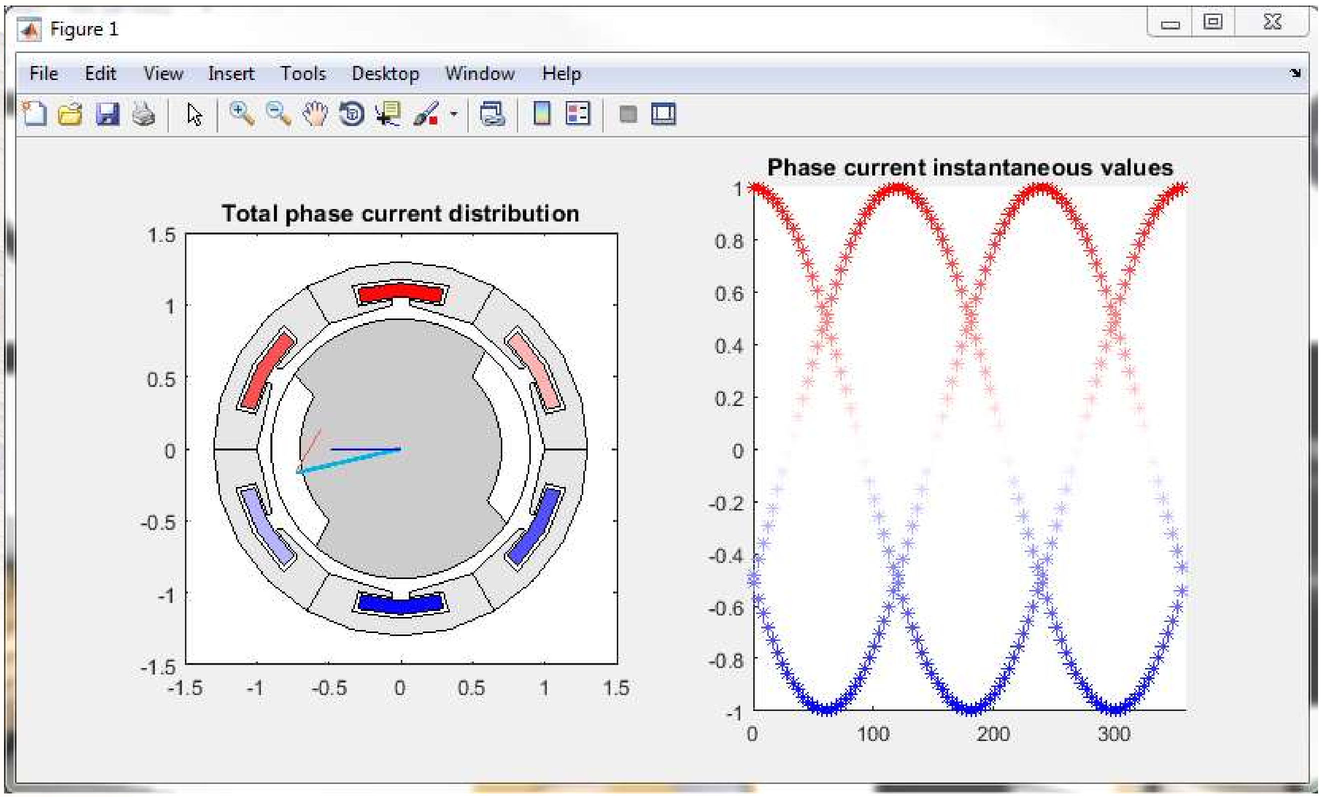
1-phase stator
Reluctance rotor

2-phase stator
Reluctance & PM rotor

3-phase stator
Reluctance & PM rotor

3-phase stator
Reluctance & PM rotor

Rotation by means of 3 phase windings and currents



3-phase winding currents as vectors

- **Assign each winding a direction**
- **Scale each equations contribution with a unity vector in each direction**
- **Add the equations, vectorially …**

$$\sqrt{\frac{2}{3}}\cdot\left(u_a = R_s\cdot i_a + \frac{d\psi_a}{dt} = R_s\cdot i_a + \frac{d}{dt}(\psi_{\delta a} + L_{s\lambda}\cdot i_a)\right)$$

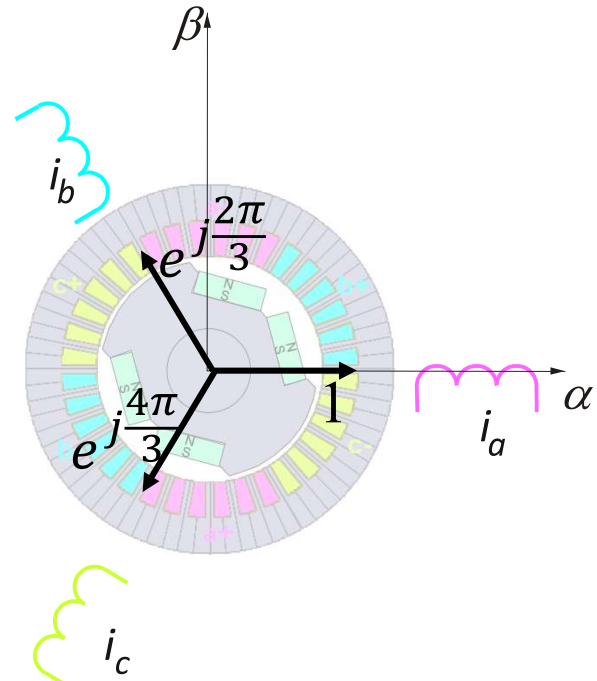
$$e^{j\cdot\frac{2\pi}{3}}\cdot\sqrt{\frac{2}{3}}\cdot\left(u_b = R_s\cdot i_b + \frac{d\psi_b}{dt} = R_s\cdot i_b + \frac{d}{dt}(\psi_{\delta b} + L_{s\lambda}\cdot i_b)\right)$$

$$+\quad e^{j\cdot\frac{4\pi}{3}}\cdot\sqrt{\frac{2}{3}}\cdot\left(u_c = R_s\cdot i_c + \frac{d\psi_c}{dt} = R_s\cdot i_c + \frac{d}{dt}(\psi_{\delta c} + L_{s\lambda}\cdot i_c)\right)$$

$$\vec{u}_s^{\alpha\beta} = R_s\cdot\vec{i}_s^{\alpha\beta} + \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = R_s\cdot\vec{i}_s^{\alpha\beta} + \frac{d}{dt}\left(\vec{\psi}_\delta^{\alpha\beta} + L_{s\lambda}\cdot\vec{i}_s^{\alpha\beta}\right)$$

PM-flux linkage + the stator currents own contribution to the air gap flux linkage

The stator currents own leakage flux linkage



Introduce the rotor reference frame (x,y) …

- **Express the stator equation in the rotor reference frame**

$$\vec{u}_s^{\alpha\beta} = R_s \cdot \vec{i}_s^{\alpha\beta} + \frac{d}{dt}\left(\vec{\psi}_\delta^{\alpha\beta} + L_{s\lambda} \cdot \vec{i}_s^{\alpha\beta}\right)$$

$$\left\{\vec{s}_s^{\alpha\beta} = \vec{s}_s^{xy} \cdot e^{j\theta_r}\right\}$$

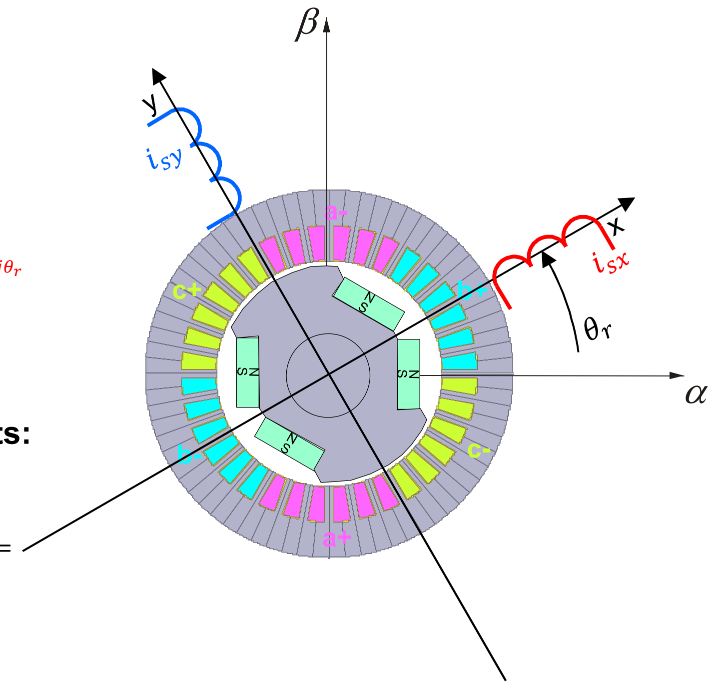
$$\begin{aligned}\vec{u}_s^{xy} \cdot e^{j\theta_r} &= R_s \cdot \vec{i}_s^{xy} \cdot e^{j\theta_r} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} \cdot e^{j\theta_r} + L_{s\lambda} \cdot \vec{i}_s^{xy} \cdot e^{j\theta_r}\right) \\ &= R_s \cdot \vec{i}_s^{xy} \cdot e^{j\theta_r} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{dq}\right) \cdot e^{j\theta_r} + j \cdot \frac{d\theta_r}{dt} \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right) \cdot e^{j\theta_r}\end{aligned}$$

$$\vec{u}_s^{xy} = R_s \cdot \vec{i}_s^{xy} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right) + j \cdot \omega_r \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)$$

- **Split up the complex equation in real- and imaginary parts:**

$$\begin{aligned}u_{sx} &= R_s \cdot i_{sx} + \frac{d}{dt}\left(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}\right) - \omega_r \cdot \left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) = \\ &= R_s \cdot i_{sx} + \frac{d}{dt}\left(\psi_m + L_{sx} \cdot i_{sx}\right) - \omega_r \cdot L_{sy} \cdot i_{sy}\end{aligned}$$

$$\begin{aligned}u_{sy} &= R_s \cdot i_{sy} + \frac{d}{dt}\left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) + \omega_r \cdot \left(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}\right) = \\ &= R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot \left(\psi_m + L_{sx} \cdot i_{sx}\right)\end{aligned}$$



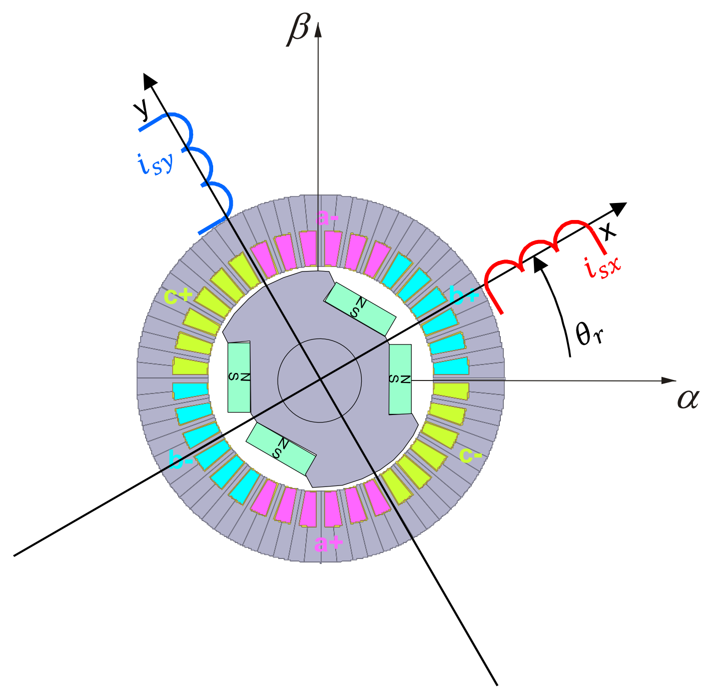
The 3-phase winding is electronically commutated

- The i_{sx} and i_{sy} currents cannot be supplied directly!
- Instead, they have to be supplied as 3 phase currents
- The translation is made in two steps:
- First, from *xy* to $\alpha\beta$:

$$\vec{i}_s^{xy} = i_{sx} + j i_{sy} = i_s e^{j\gamma}$$
$$\vec{i}_s^{\alpha\beta} = \vec{i}_s^{xy} e^{j\theta_r} = i_s e^{j(\gamma+\theta_r)} = i_{s\alpha} + j i_{s\beta}$$
$$i_{s\alpha} + j i_{s\beta} = i_s \cos(\omega_r t + \gamma) + j i_s \sin(\omega_r t + \gamma)$$

- Then, from $\alpha\beta$ to *abc :*

$$i_a = \sqrt{\frac{2}{3}} i_{s\alpha}$$
$$i_b = \sqrt{\frac{2}{3}}\left(-\frac{1}{2} i_{s\alpha} + \frac{\sqrt{3}}{2} i_{s\beta}\right)$$
$$i_c = \sqrt{\frac{2}{3}}\left(-\frac{1}{2} i_{s\alpha} - \frac{\sqrt{3}}{2} i_{s\beta}\right)$$



What if the stator winding could be mechanically commutated … ???

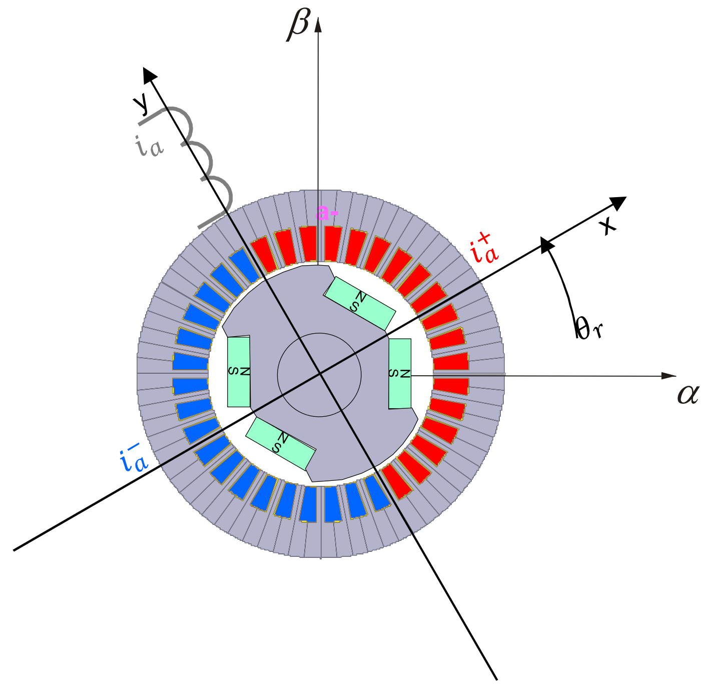
- Take the equation from the y-axis (i.e NO i_{sx} current!) …

$$u_{sy} = R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot (\psi_m + L_{sx} \cdot i_{sx})$$

- And replace the "*sy*"-index with "*a*", as in "armature" …

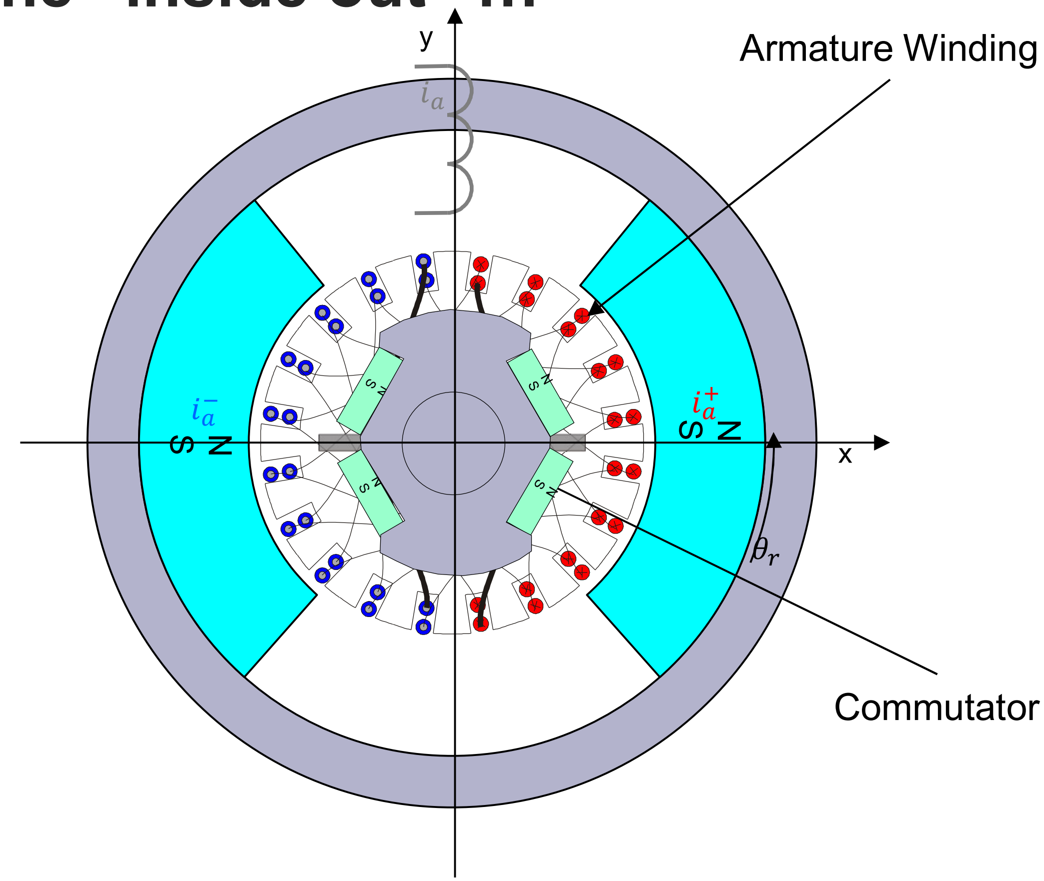
$$u_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + \omega_r \cdot \psi_m$$

- … can this be implemented … ?



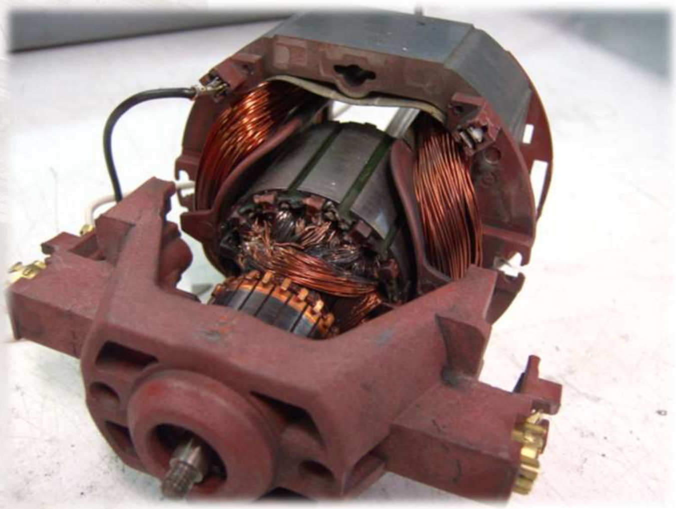
First: - Turn the machine "inside out" …

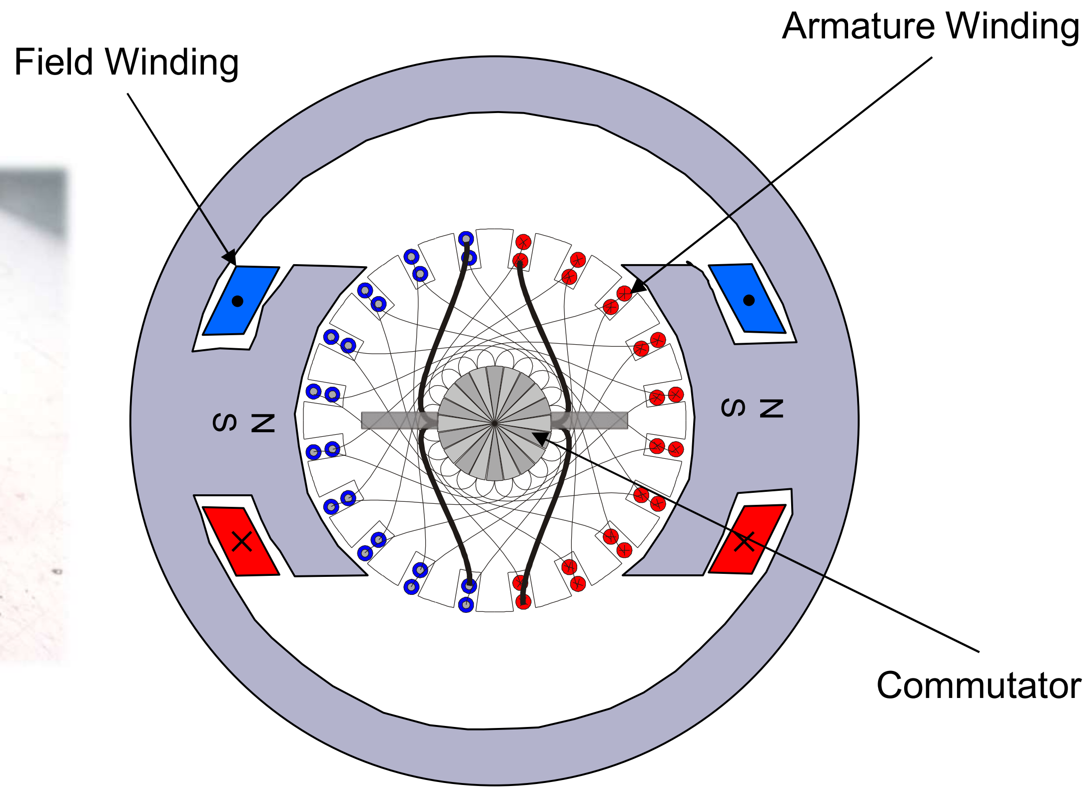
- **PM magnets in the stator**
- **Commutator winding in the rotor**
- **The commutator FORCES the current to keep the intended distribution**



Then: - Replace the PM with an Electro Magnet (EM)

- **Now we are close ….**





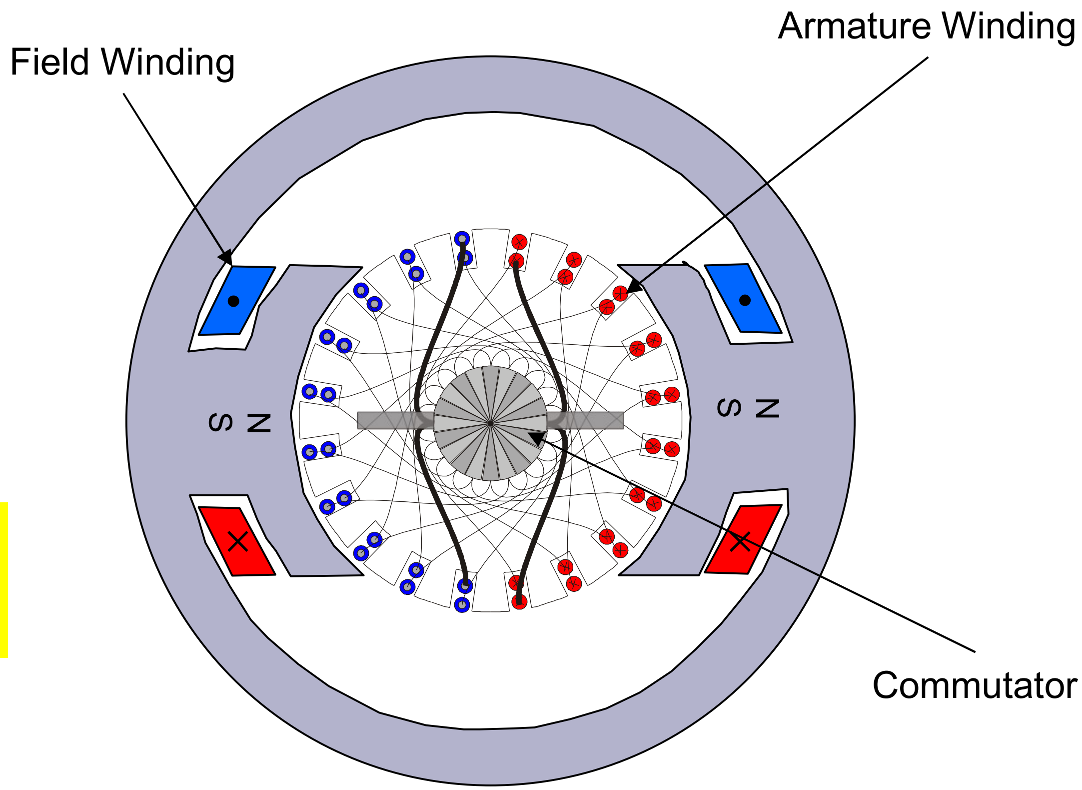
Electric Equations

$$u_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + \omega_r \cdot \psi_m$$

$$T = \psi_m \cdot i_a$$

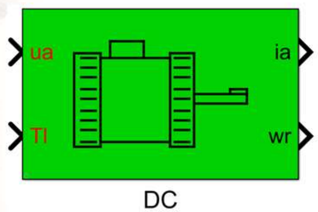
$$\psi_m = L_m \cdot i_f$$

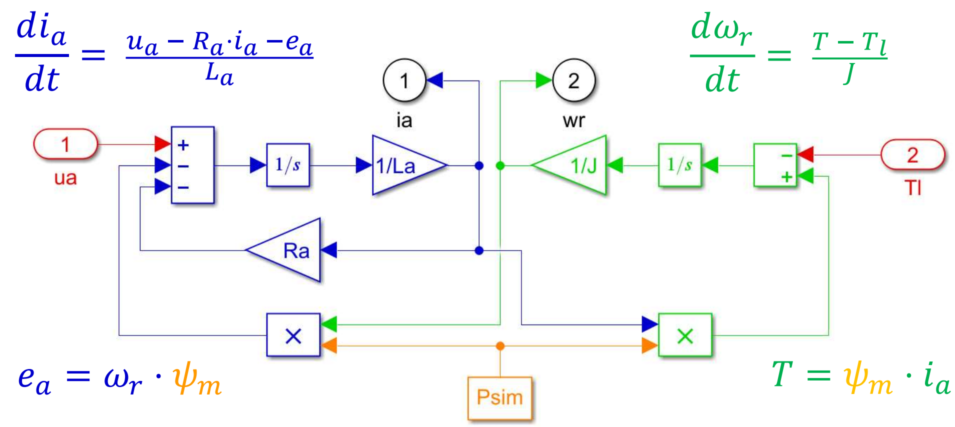
Note:
- Torque is prop. to current
- Stationary voltage is prop. to speed



Electric & Mechanic equations

- **Enjoy the symmetry of Nature!**

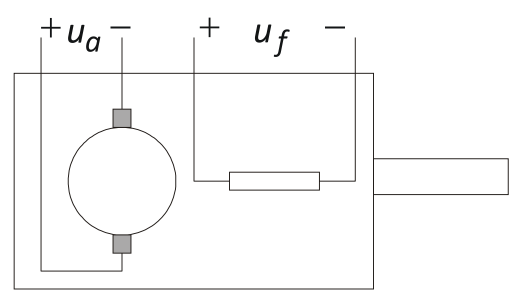




Separate field supply

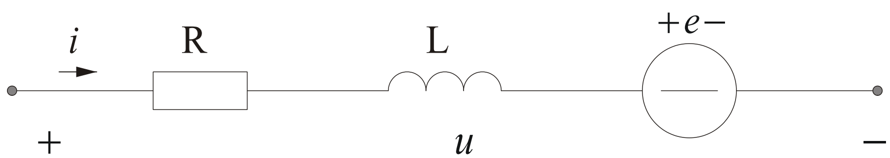
- **IF electrically magnetized, the field winding is also a dynamic system**
  - *… but with no back-emf*

$$u_f = R_f \cdot i_f + L_f \cdot \frac{di_f}{dt}$$
$$\psi_m = L_m \cdot i_f$$
$$L_f = L_m + L_{f\lambda}$$



Torque Control

- **Use the control law from the generic circuit**



$$i_a^* = \frac{T^*}{\psi_m}$$

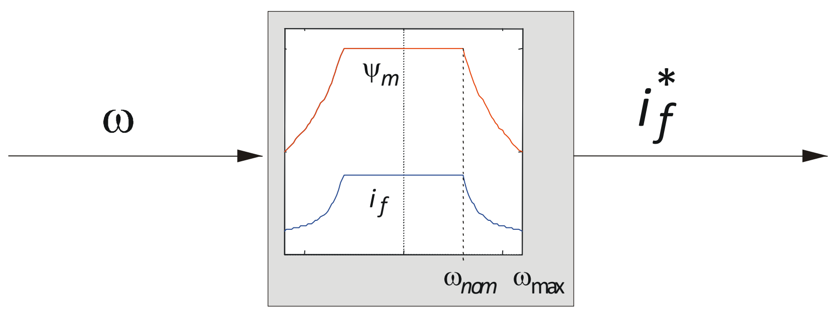
$$u_a^*(k) = \left(\frac{L_a}{T_s} + \frac{R_a}{2}\right) \cdot \left((i_a^*(k) - i_a(k)) + \frac{T_s}{\left(\frac{L_a}{R_a} + \frac{T_s}{2}\right)} \cdot \sum_{n=0}^{n=k-1} (i_a^*(n) - i_a(n)) \right) + e_a(k)$$

$$e_a(k) = \omega_r(k) \cdot \psi_m$$

Field weakening

$$e_a = \omega_r \cdot \psi_m$$

$$\psi_m^* = \begin{cases} \psi_{m,nom} if\, \omega_r < \omega_{r,nom} \\ \psi_{m,nom} \cdot \dfrac{\omega_{r,nom}}{\omega_r} if\, \omega_r > \omega_{r,nom} \end{cases}$$



Example with 2Q DC/DC supply and current control