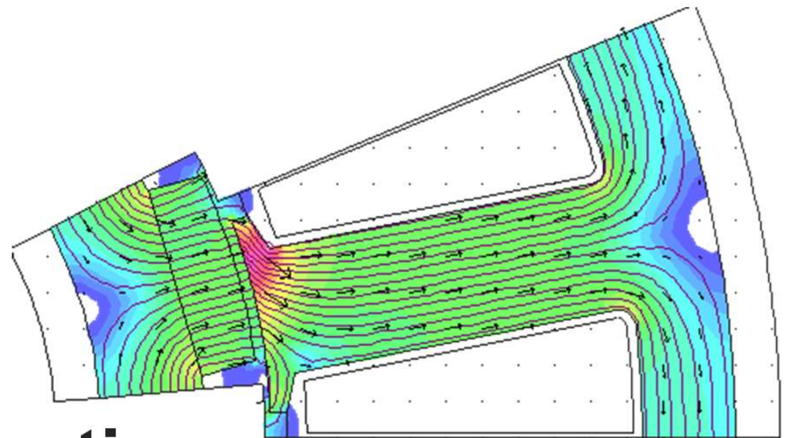


7. Torque generation

Electromagnetic and magnetic forces
Rotating magnetic field

EIEN25 Power Electronics
Devices, Converters, Control and Applications

Energy density

8.1 p 258-259

- Medium ability to maintain magnetic field or/and electric field
- The permittivity ε is for polarization whereas the permeability μ is for magnetization
- Flux density in the air-gap
  - $B_g=1T \rightarrow \sim 4 \cdot 10^5 J/m^3 > PM$
- Break down field for air
  - $E_b=3kV/mm \rightarrow \sim 4 \cdot 10^1\ J/m^3$
- Compare energy density [MJ/Lit] and specific energy [MJ/kg]

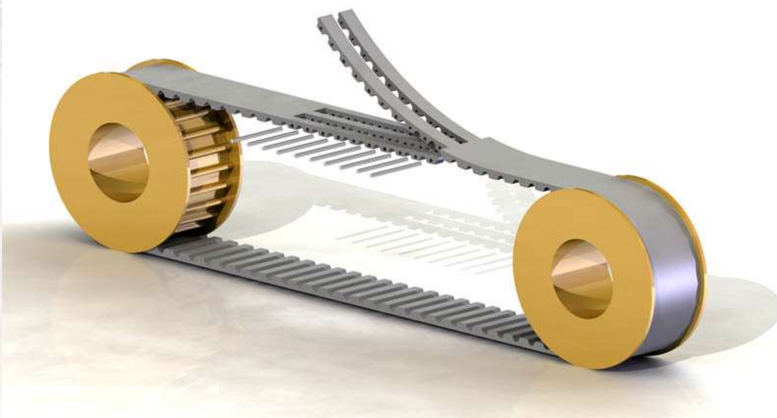
$$\eta_{HB} = \frac{B^2}{2\mu_0} \qquad \mu_0 = 4\pi 10^{-7} \left[\frac{H}{m} = \frac{Vs}{Am}\right]$$

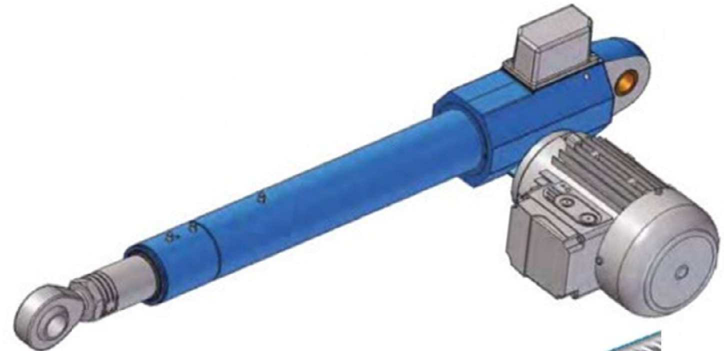
$$\eta_{ED} = \frac{\varepsilon_0 E^2}{2} \qquad \varepsilon_0 = \frac{1}{c_0^2 \mu_0} \approx \frac{1}{36\pi} 10^{-9} \left[\frac{F}{m} = \frac{As}{Vm}\right]$$

Storage material	MJ/L	MJ/kg
Liquid hydrogen	10	142
Diesel	35.8	48
Lithium metal battery	4.3	1.8
Lithium-ion battery	2.6	0.8
Carbohydrates	43	17
Magnetic Flux Density (1T) in vacuum		$4 \cdot 10^{-1}$
Electric Field (3 kW/m) in vacuum		$4 \cdot 10^{-5}$

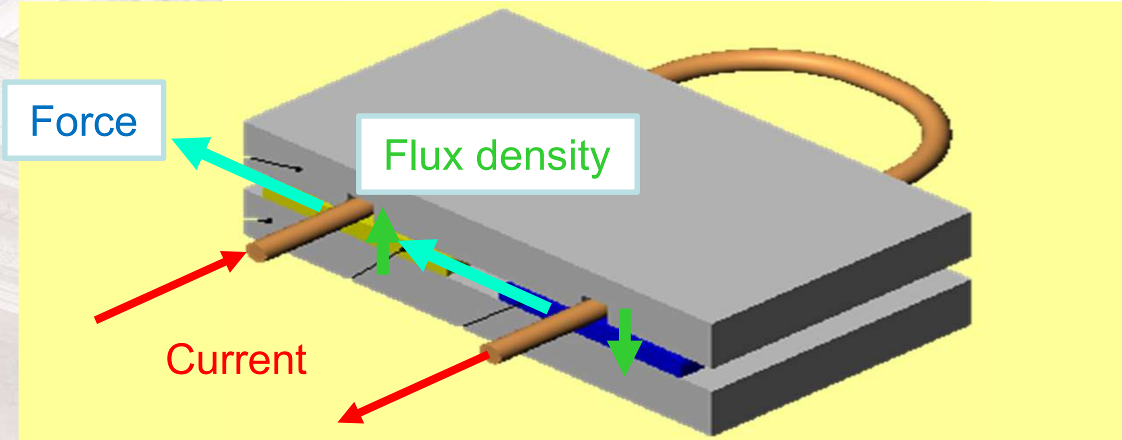
Linear Motion

- In many applications the “most wanted”
- Often translated from a rotation





Generic Force



Force = Flux density * Current * length *(twice)*

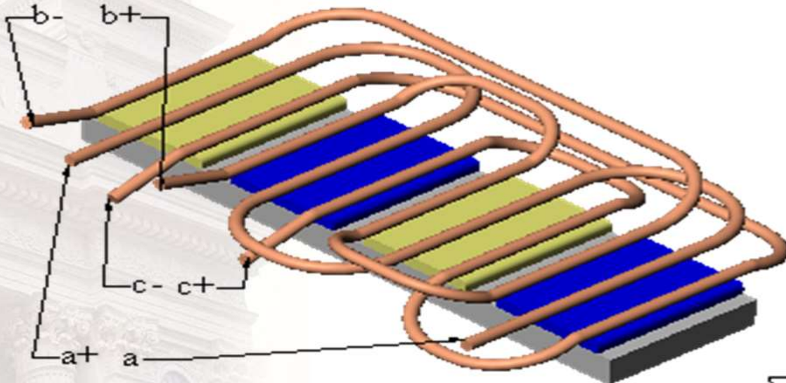
Power = Force * Speed = Voltage * current

Voltage = Force *speed / current
= Flux density * length * speed

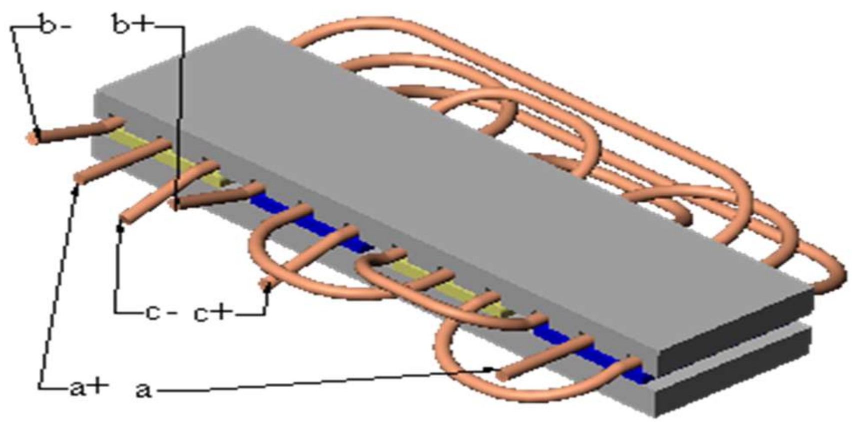
Lorentz force*)
= Force on current in magnetic field

$^{*)}\ \boldsymbol{F} = q \cdot (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

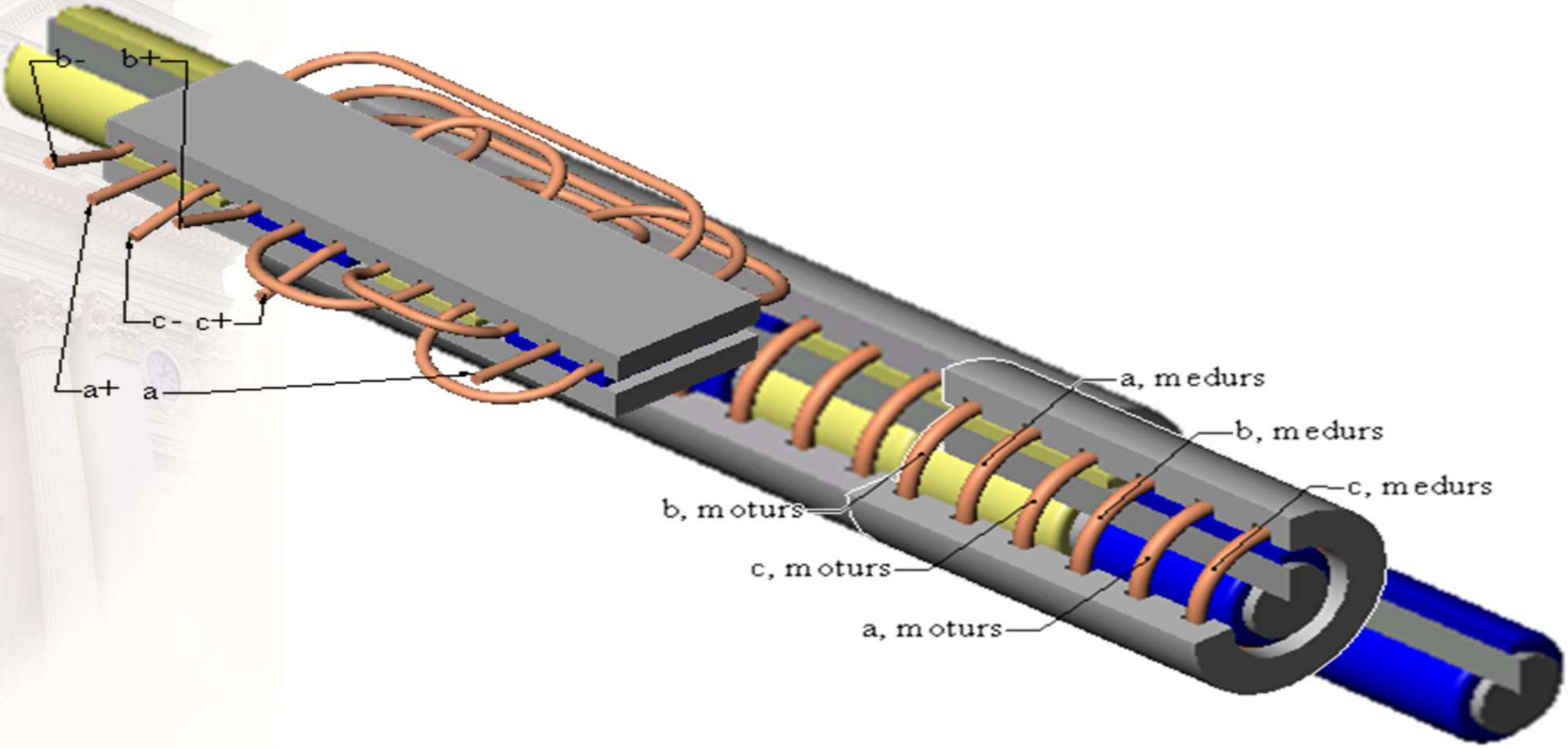
Multi Phase, otherwise it stops



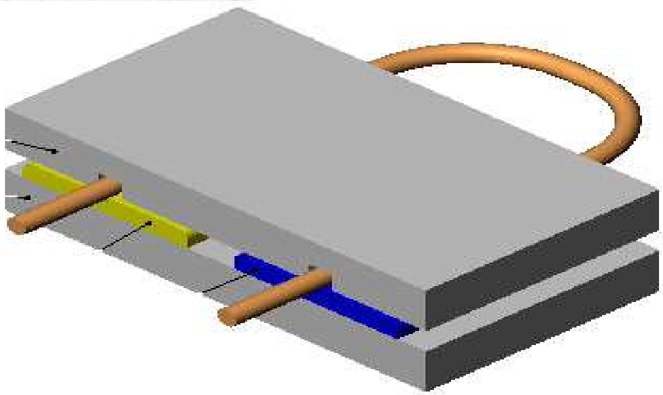
End windings don't contribute to torque, but takes a lot of space …

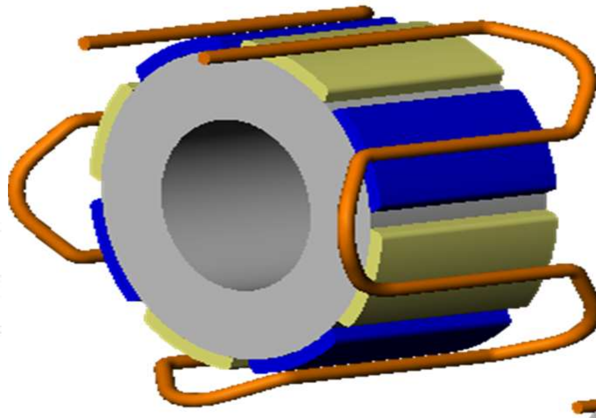


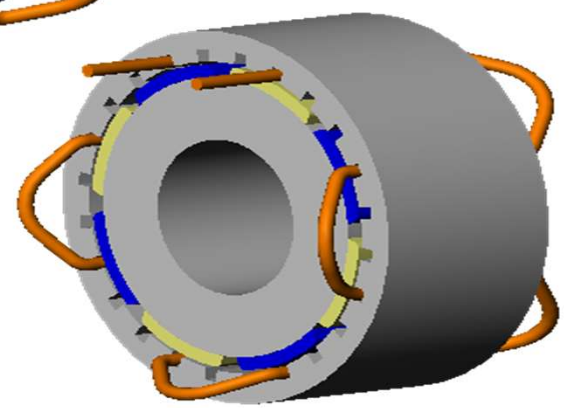
Linear movement from generic force



Rotating movement from generic force







Conclusions on force and movement

- ***The same generic circuit accomplish both linear and rotating movement.***
- ***One phase is not enough for continuous force.***
- ***Qualitative:***

Voltage ~ Speed
Current ~ Force

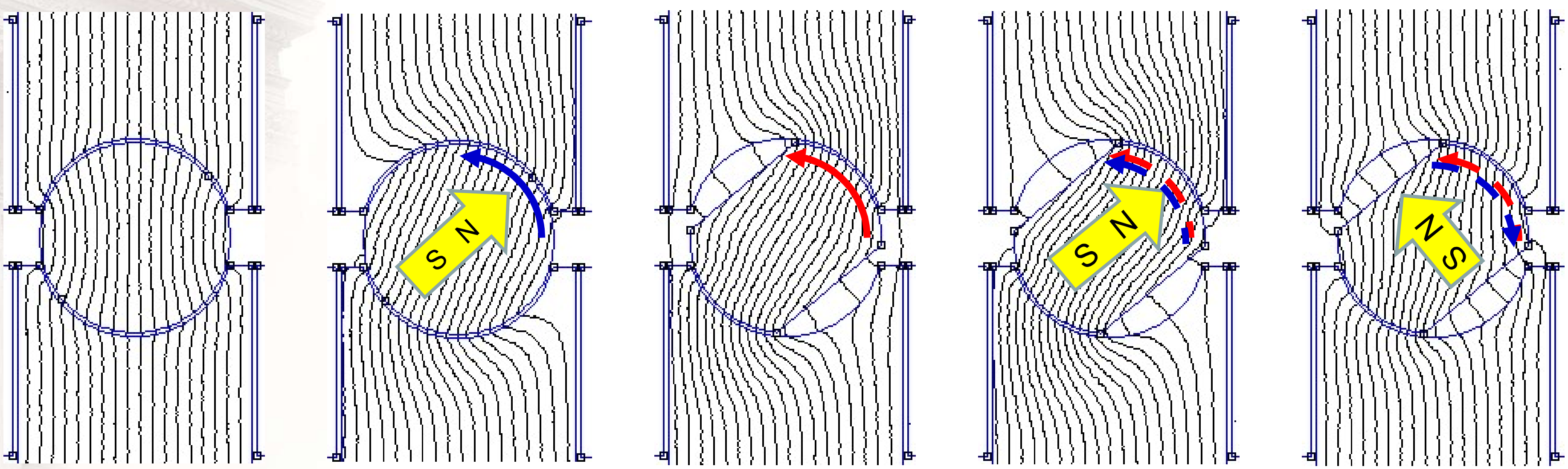
Torque Components

No Torque

Electrically or permanently magnetized = **Lorenz forces**

Only **Reluctance force**

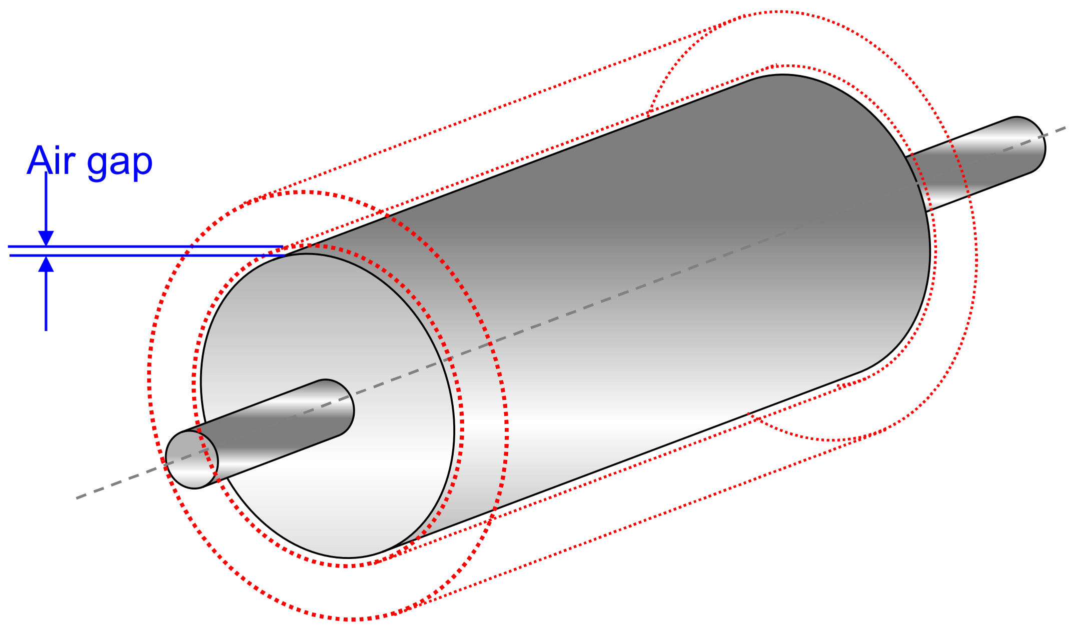
Lorentz AND **Reluctance** forces



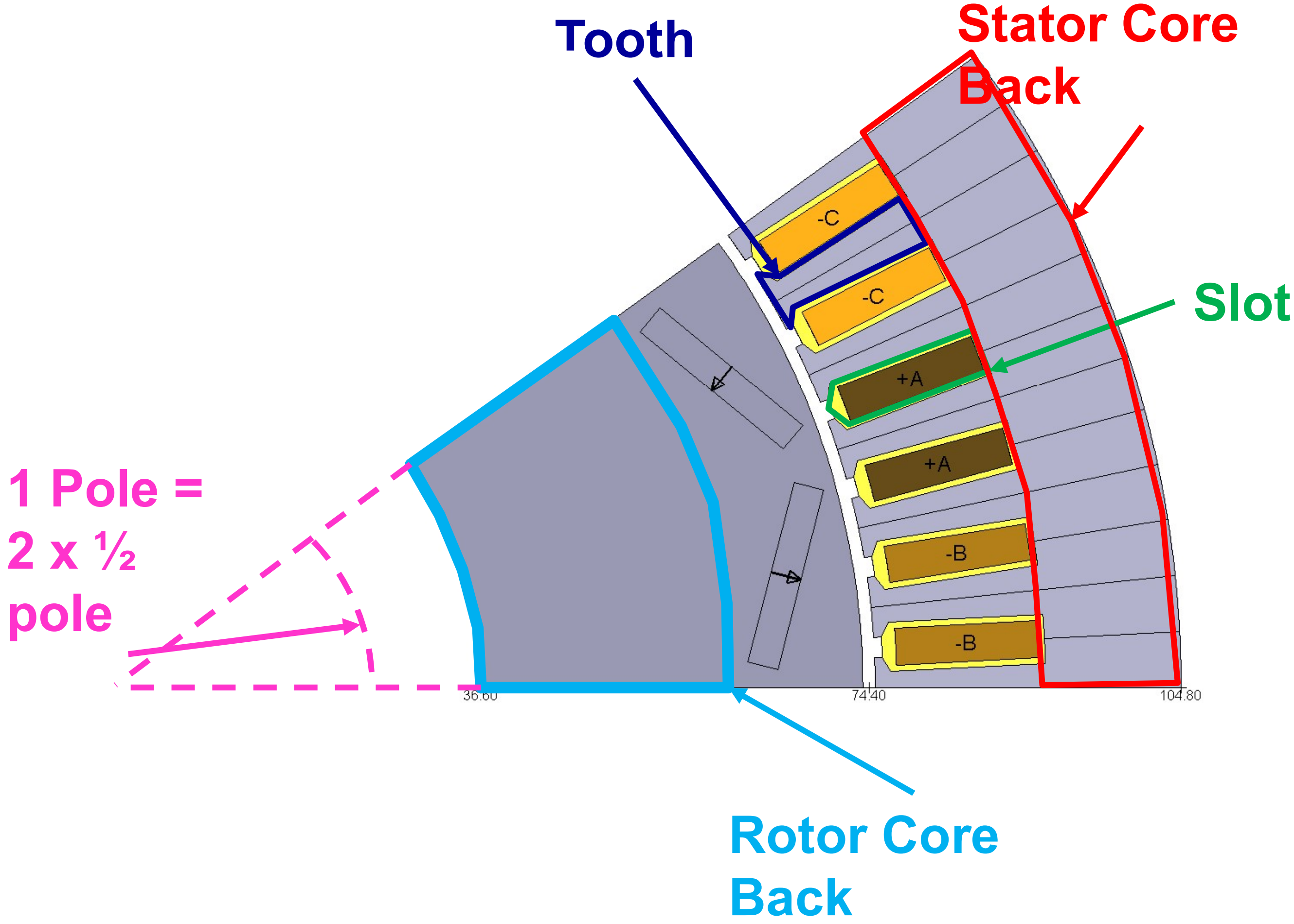
External Magnetic Field

Stator, Rotor and Airgap

- The stator is static (not moving)
- The rotor rotates
- The air gap seperates them
  - *Usually < 1 mm*



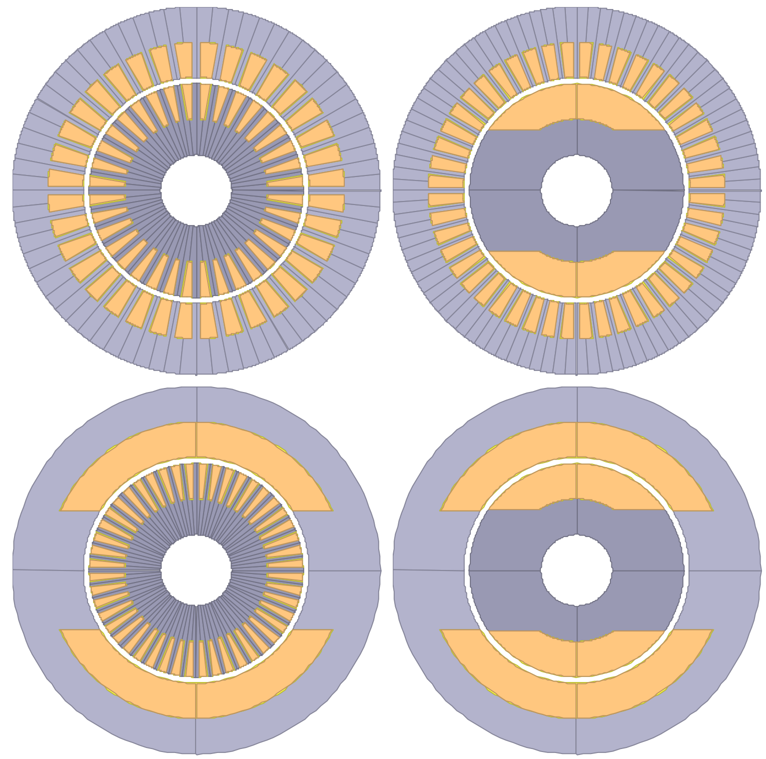
Core back, teeth etc ...



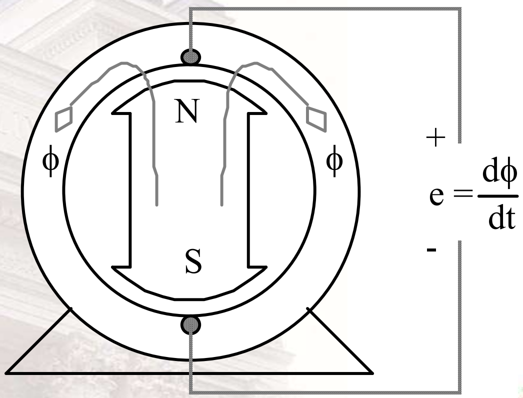
Machine layouts

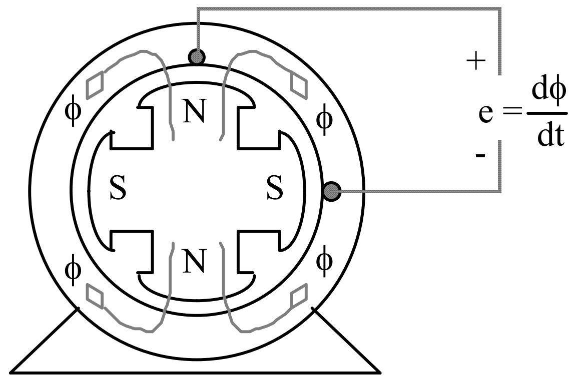
8.2 p 259-260

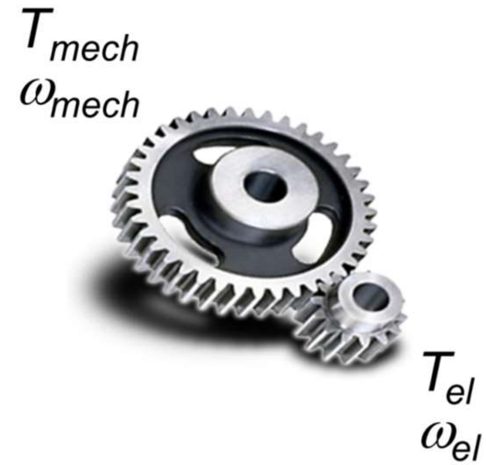
- **Machine classification**
  - *Saliency: none, single, double*
  - *Supply: single or double fed*
  - *Excitation: EM & PM*
  - *Magnetization: IM & RM*



of poles = p







$$\omega_{el} = \frac{p}{2} \cdot \omega_{mech}$$

$$T_{mech} = \frac{p}{2} \cdot T_{el}$$

$$\text{Mechanical Power} = \omega_{mech} \cdot T_{mech} = \frac{p}{2} \cdot \omega_{el} \cdot \frac{2}{p} \cdot T_{el} = \omega_{el} \cdot T_{el}$$

Flux variation & electromotive force

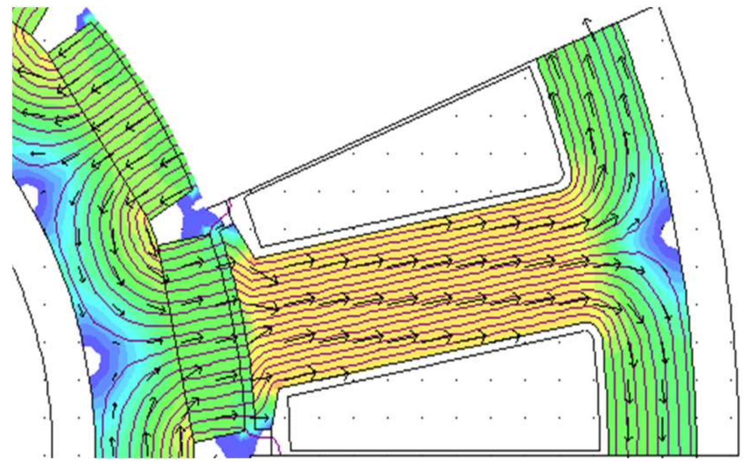
- **Faradays Law**

$$\oint_l \vec{E}\,dl = -\int \frac{\partial B}{\partial t}\,dA$$

$$E = -\frac{d\psi}{dt}$$

$$= -N_t\frac{d\varphi}{dt}$$

- **Lenz's law – any current produced by the emf tends to oppose the flux change**
- **Flux variation due to magnet movement and current change**



$$\psi = Li$$

$$e = +\frac{d\psi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt}$$

$$L_s = L_\mu + L_\sigma = KL_s + (1-K)L_s$$

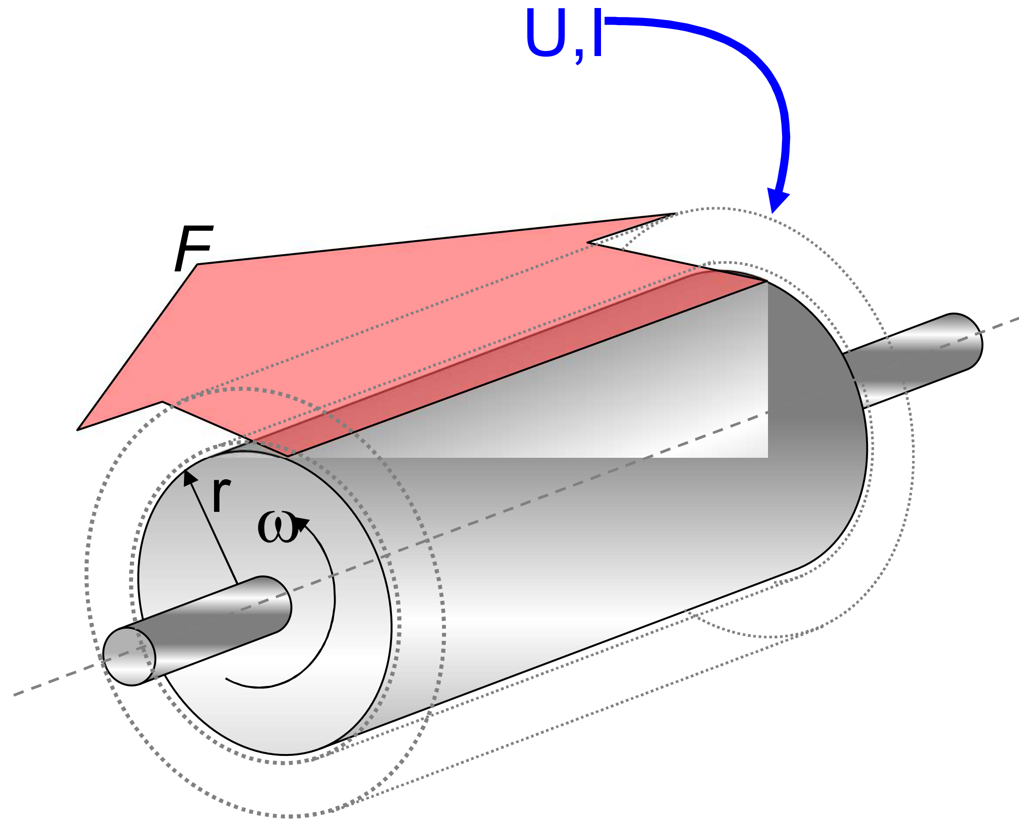
Electromechanical energy converters

8.1 p 257-258

- **Conversion of electric energy into mechanical energy or vice versa**
  - *Reversible except for the energy losses – motoring and generating mode*
  - *In the presence of magnetic field (energy density)*
  - *Mechanical motion – translational, rotational, …*
  - *Electrical EMF waveform – pulsating DC, alternating AC*

Torque and Power

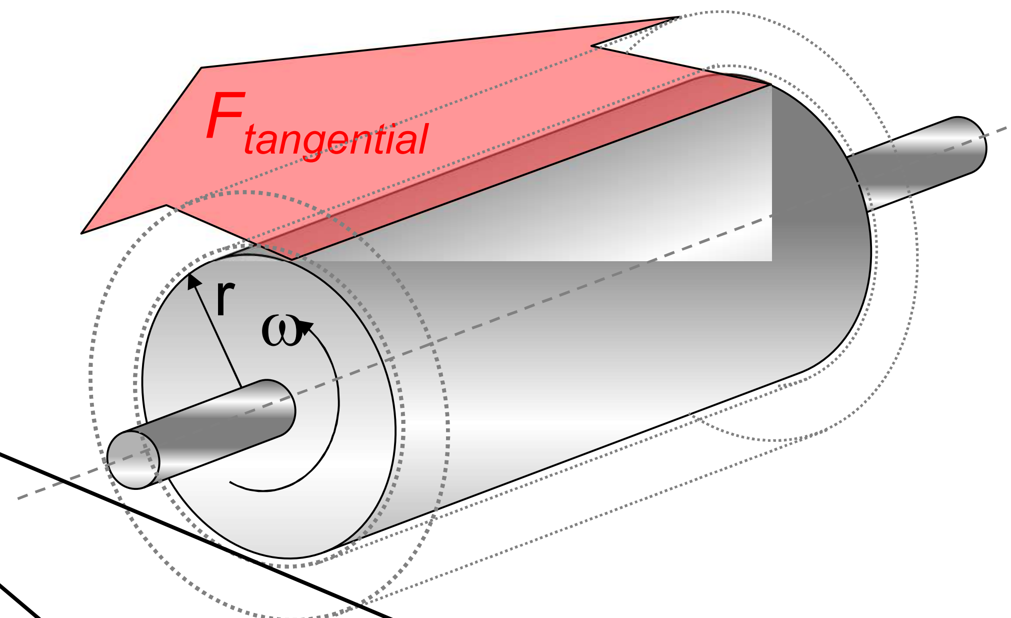
- **Torque = Force * radius**
  on the shaft
  T = F * r
- **Power = Torque*Speed**
  on the shaft
  P = T * ω
  but, also …
- **Power = Voltage * Current**
  on the electrical terminals
  P = U * I



Tangential force

- **Interaction of flux and current**
- ***"F= B i l"***

$$F_{tangential} = \frac{\pi}{2} D_{is} l_e B_{gm1} K_{s1}$$



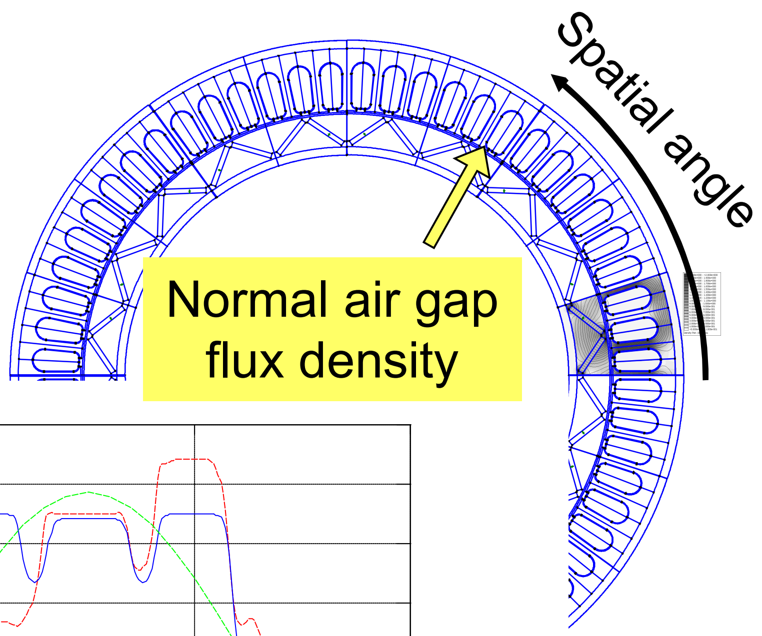
Inner stator diamete

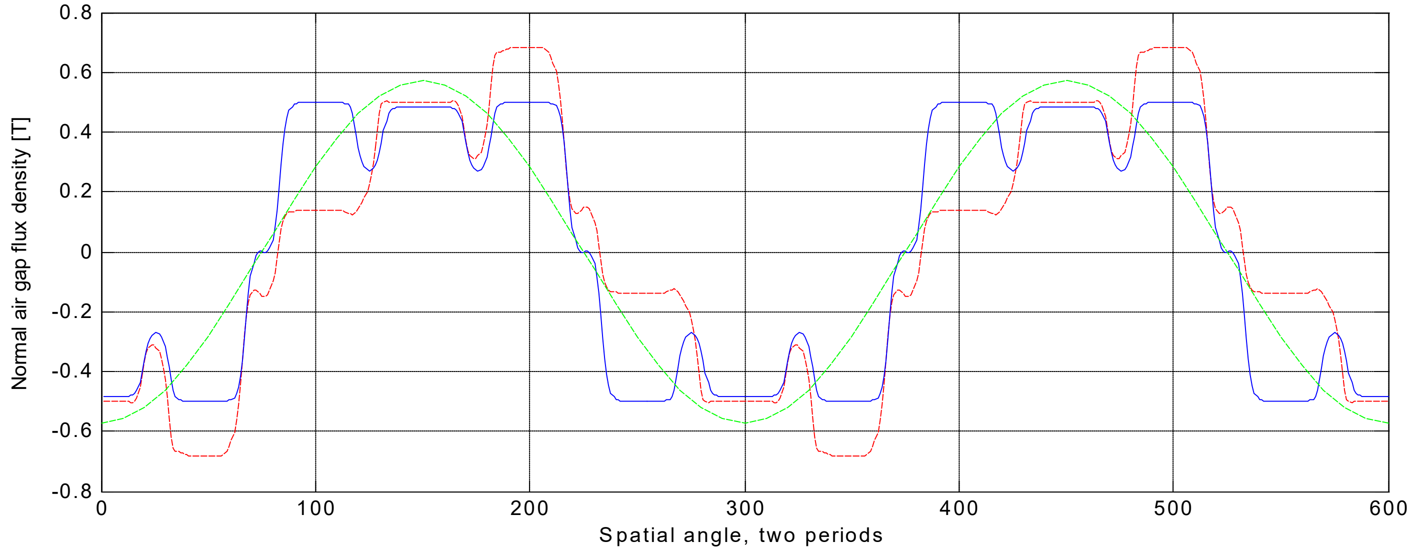
Equivalent active magnetic length

Air gap flux density

Stator Angular Current Density

Flux density in the air gap





Stator current / meter air gap periphery

Angular current density

Number of winding turns

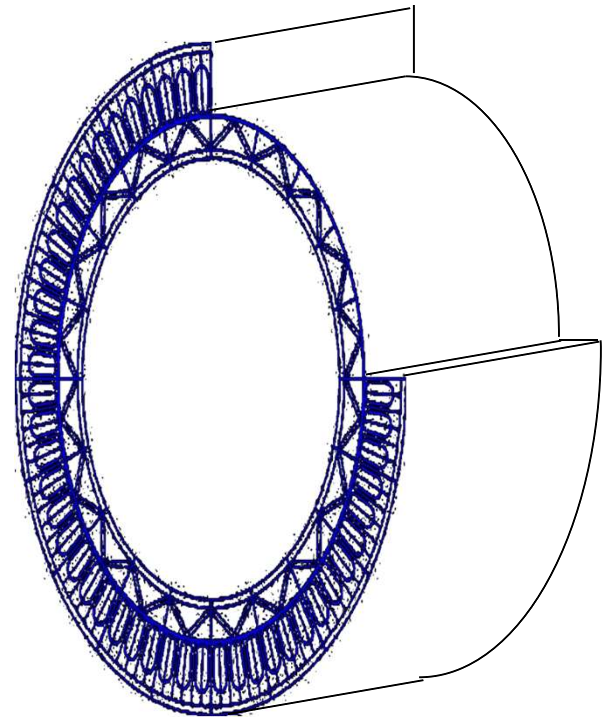
$$K_s(\theta) = \frac{6}{\pi} \cdot \frac{k_1 N_s}{D_{is}} \cdot \hat{i}_s \cdot \cos\left(\frac{p\theta}{2} - \omega_e t\right) \quad [A/m]$$

Winding factor

Number of poles

Phase current amplitude

Stator inner diameter

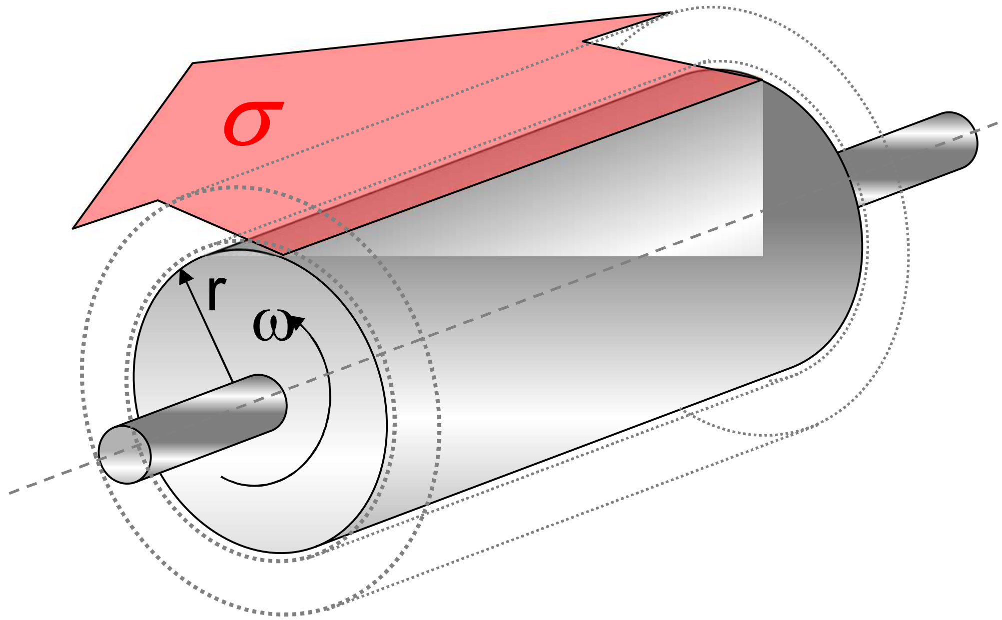


Shear Force & Torque

- Current and Flux interact for tangential force
  σ = Force/Unit area is a key figure
- A good design accomplish about
  σ = 10 000 … 30 000 [N/m^2]
- … in continuous operation and
  2…4 times that in transient operation

$$\sigma = \left(\frac{F}{A}\right)_{avg} = \frac{\frac{\pi}{2} D_{is} l_e B_{gm1} K_{s1}}{\pi D_{is} l_e} = \frac{B_{gm1} K_{s1}}{2} \quad \left[N/m^2\right]$$

$$T = \frac{\pi}{4} D_{is}^2 l_e \cdot B_{gm1} K_{s1}$$



Conclusion on torque

The torque is proportional to the:

- *Magnetic flux density – Limited by material propertied to about 1.0 ... 1.5 Tesla*
- *Spatial current ”density” – Limited by cooling capability*
- *Axial length of the machine*
- *Diameter SQUARED !*

(Axial length and diameter squared) = Rotor Volume

$$T = \frac{\pi}{4} D_{is}^{2} l_{e} \cdot B_{gm1} K_{s1}$$

Sizing Exercise ...

- **Size a machine capable of 250 Nm.**
- **Assume**
  - ***Sigma = 25000 N/m2***
  - ***Rotor outer radius = 75% of Stator outer radius***
  - ***Rotor active lengt? = Stator outer diameter***
- **How big will it be?**

```
>> Sigma = 25000;
>> T=250;
% T=2*pi*rr*lr*sigma*rr
% rr=0.75*rs
% T=2*pi*0.75*rs*2*rs*Sigma*0.75*rs =
% 4*pi*0.75^2*Sigma*rs^3

rs = (T/(4*pi*0.75^2*Sigma))^(1/3)

rs = 0.1123

i.e. 224 mm diameter, 224 mm length
```

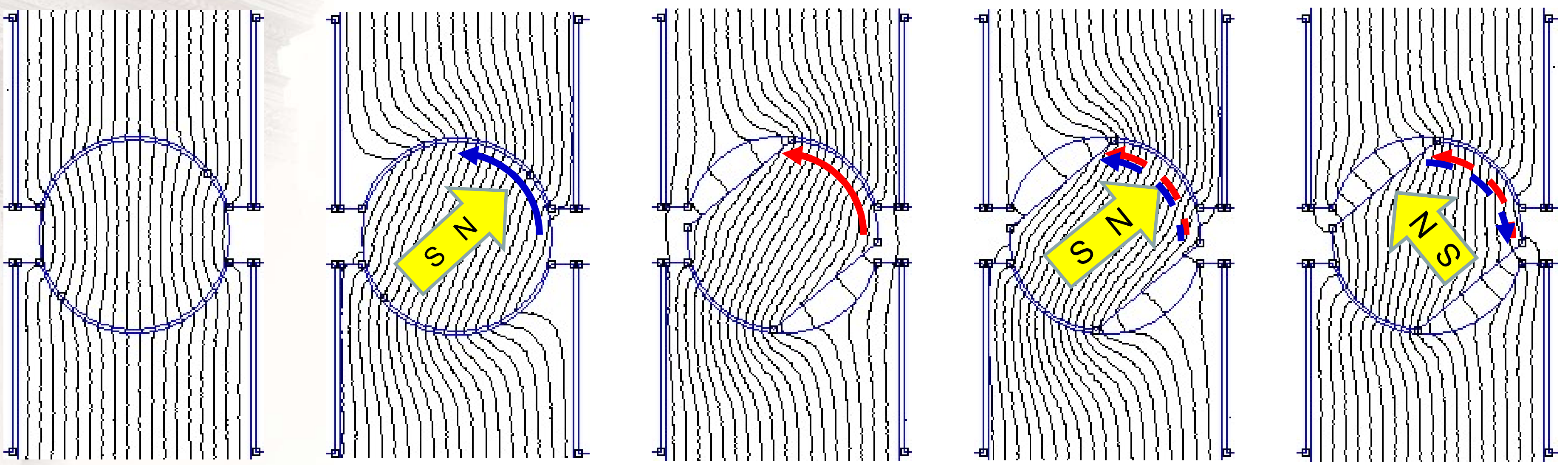
Torque Components

No Torque

Electrically or permanently magnetized = **Lorenz forces**

Only **Reluctance force**

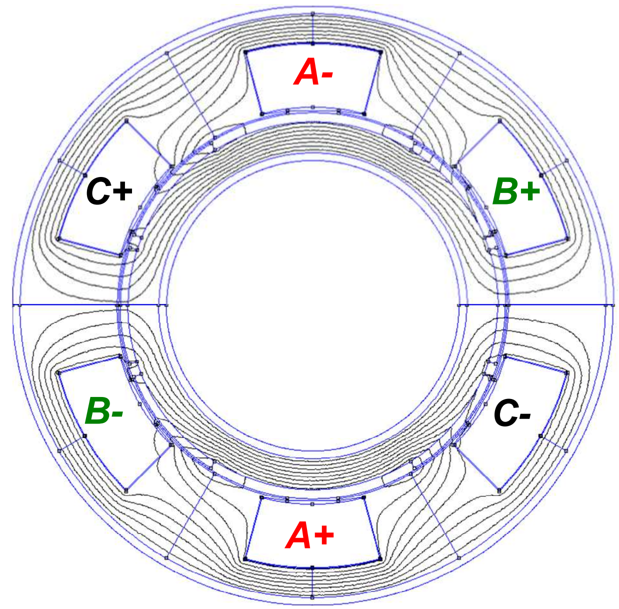
Lorentz AND **Reluctance** forces



External Magnetic Field

Stator windings …

- **Assume inner rotor**
- **Assume 3-phase stator winding**
  - ***The most common type in AC-machines***
- **Assume 2-pole**
  - ***To simplify understanding***
- **Assume 3-phase sinusoidal currents**
  - ***The normal case***



Rotation

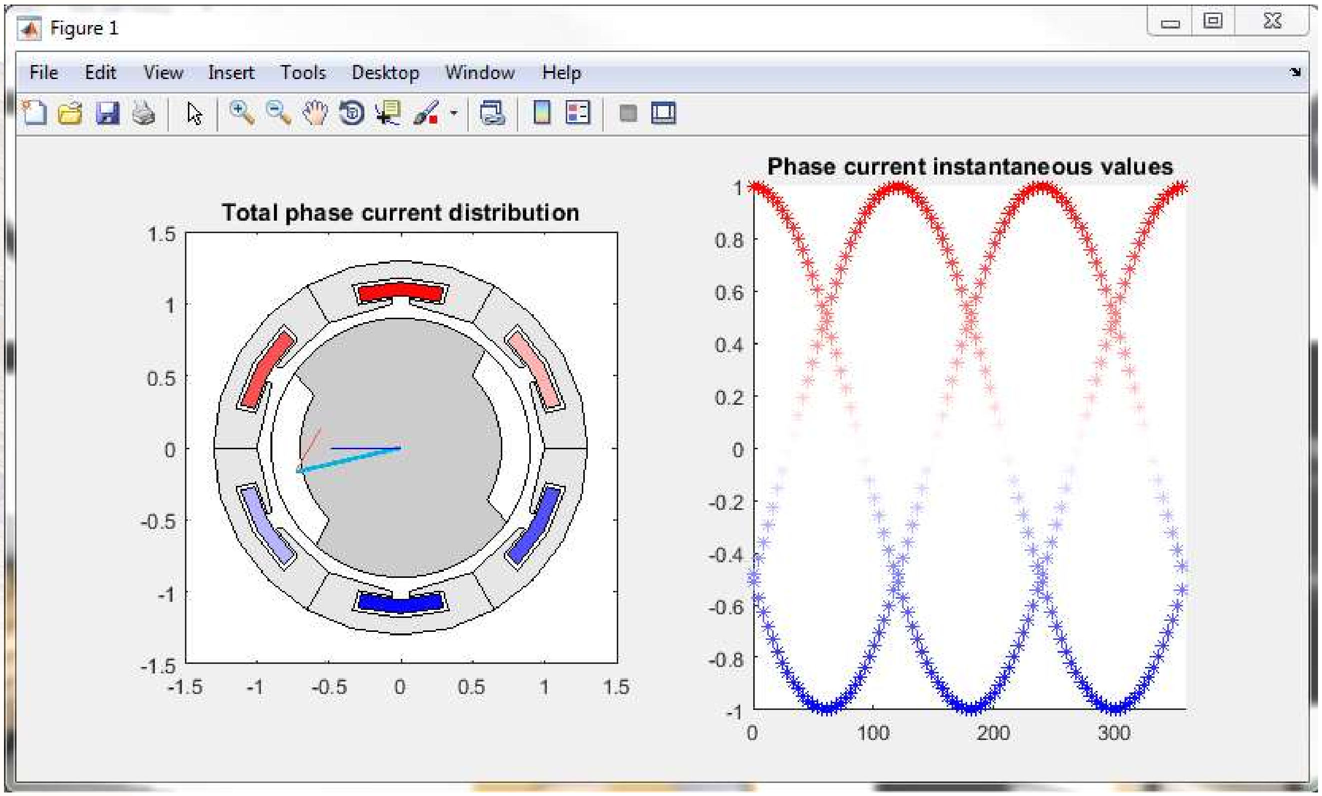


Figure 1
File Edit View Insert Tools Desktop Window Help
Total phase current distribution
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5 -1 -0.5 0 0.5 1 1.5
Phase current instantaneous values
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 100 200 300

Magnetomotive force, field intensity & flux

- **Amperes Law**

$F = mmf = N_t I = \oint \vec{H} dl$

- **Constitutive relation**

$B = \mu H = \mu_0 \mu_r H$

- **Magnetic circuit**

$$F = H_{Fe} l_{Fe} + H_\delta \delta = \frac{B_{Fe} l_{Fe}}{\mu_0 \mu_{Fe}} + \frac{B_\delta \delta}{\mu_0}$$

$$= \begin{Bmatrix} Assume \\ A_{fe} = A_\delta = A \end{Bmatrix} = \phi \cdot \left(\frac{l_{Fe}/\mu_0 \mu_{Fe}}{A} + \frac{\delta/\mu_0}{A} \right)$$

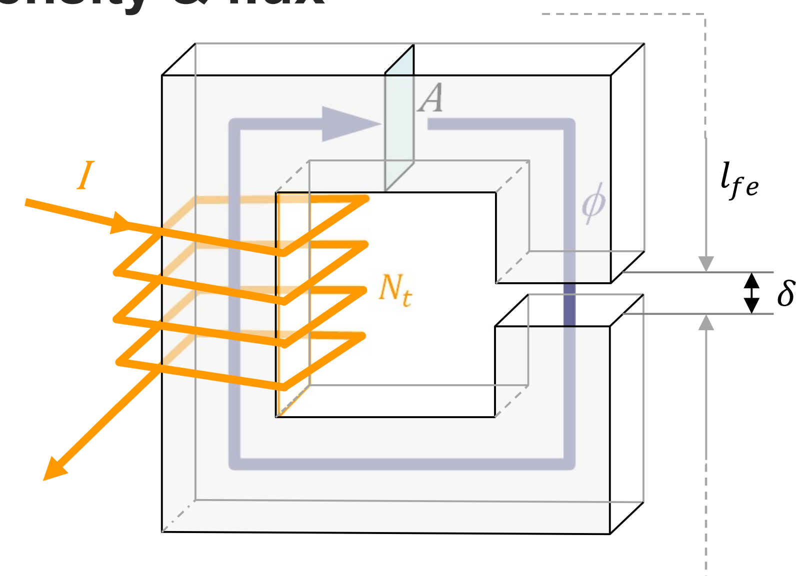
$$F = F_{Fe} + F_\delta = \phi \cdot R_{Fe} + \phi \cdot R_\delta$$

- **Magnetic flux**

$\phi = BA$

- **Magnetic reluctance**

$$R = \frac{l}{\mu A}$$



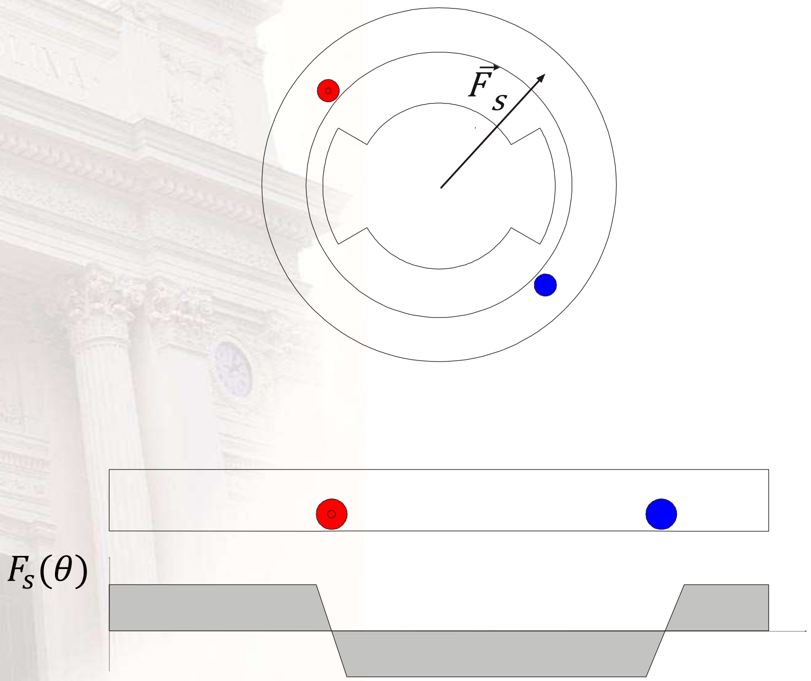
- **Magnetic energy**

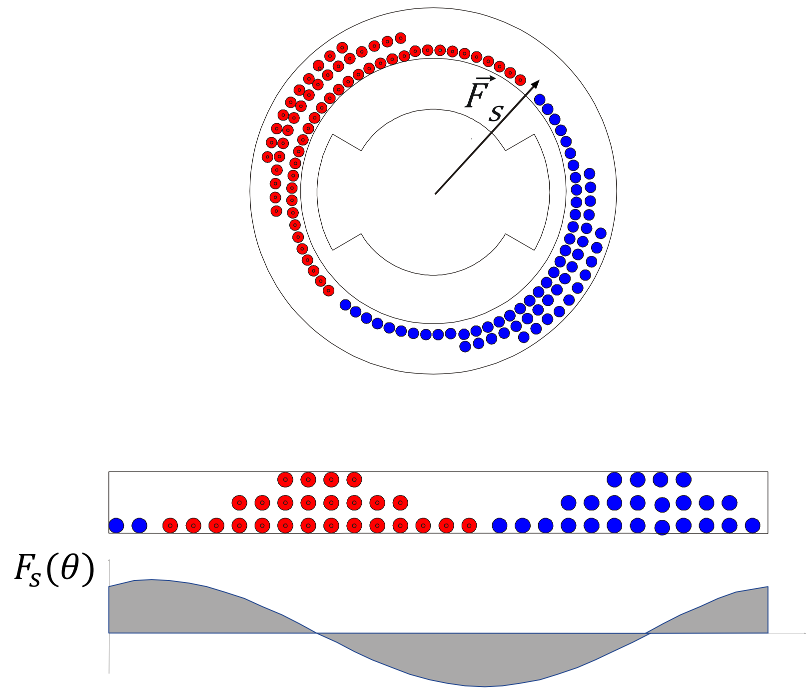
$$W_{mag} = \frac{\phi \cdot F}{2} = \frac{1}{2} \cdot \frac{{F_{Fe}}^2}{R_{Fe}} + \frac{1}{2} \cdot \frac{{F_\delta}^2}{R_\delta} =$$

$$= \frac{1}{2} \cdot \phi^2 \cdot R_{Fe} + \frac{1}{2} \cdot \phi^2 \cdot R_\delta$$

Sinusoidally distributed windings

8.5 p 262-263

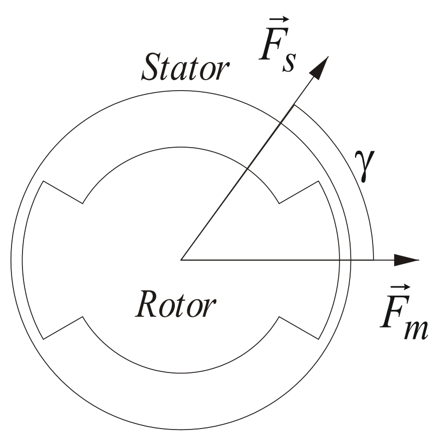


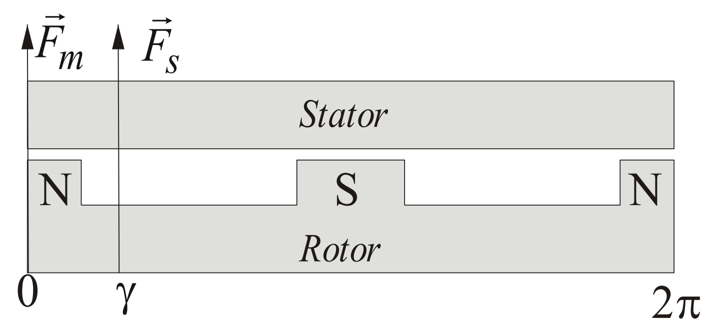


8.6 p 264-265

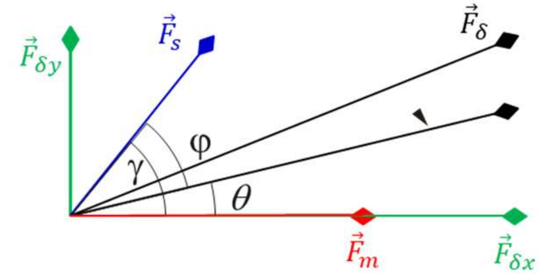
The PM and RM machine

- Salient poles in the rotor
- Rotor reference frame (*x/y*) (or (d/q))
- Rotor mmf $\vec{F}_m$ and stator mmf $\vec{F}_s$ (sinusiodal)
- Reluctances R_x and R_y
- Ideal iron (no magnetic saturation effects)





Air-gap magnetic energy

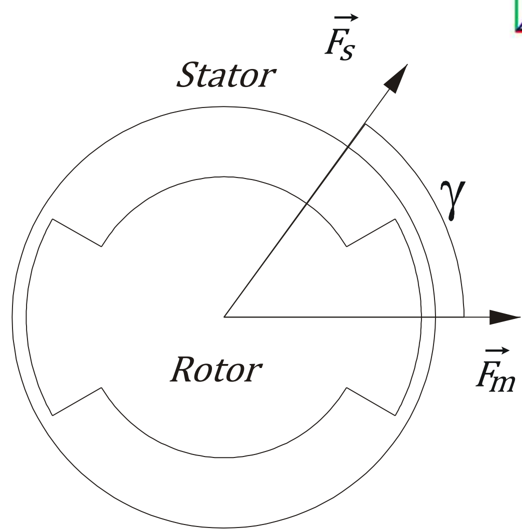


- **Air gap MMF**

$$\vec{F}_\delta = \vec{F}_m + \vec{F}_s = F_{\delta x} + F_{\delta y}$$

- **Split in orthogonal components**

$$\begin{cases} F_{\delta x} = \hat{F}_s \cdot \cos(\gamma) + \hat{F}_m = \hat{F}_{sx} + \hat{F}_m \\ F_{\delta y} = \hat{F}_s \cdot \sin(\gamma) = \hat{F}_{sy} \end{cases}$$



- **Air gap Energy**

$$W_{magn} = \frac{1}{2}\frac{\hat{F}_x^2}{R_x} + \frac{1}{2}\frac{\hat{F}_y^2}{R_y} = \frac{1}{2} \cdot \left(\frac{\hat{F}_s^2 \cdot \cos^2\gamma + 2 \cdot \hat{F}_s \cdot \hat{F}_m \cdot \cos\gamma + \hat{F}_m^2}{R_x}\right) + \frac{1}{2} \cdot \left(\frac{\hat{F}_s^2 \cdot \sin^2\gamma}{R_y}\right)$$

Torque: derivative of magnetic energy I

- **Torque = derivative of mechanical energy w.r.t. angle**

$$\frac{dW_{mec}}{d\gamma} = T$$

Compare to linear movement
(F=dW/dx or *W=F*x)*

- $\boldsymbol{W_{magn}}$ **+** $\boldsymbol{W_{mech} = Constant}$
  i.e. – no energy supplied

$$\frac{dW_{magn}}{d\gamma} + \frac{dW_{mec}}{d\gamma} = 0$$

Torque: derivative of magnetic energy II

$$W_{magn} = \frac{1}{2 \cdot R_x} \cdot \left(\hat{F}_s^2 \cdot \cos^2\gamma + 2 \cdot \hat{F}_s \cdot \hat{F}_m \cdot \cos\gamma + \hat{F}_m^2\right) + \frac{1}{2 \cdot R_y} \cdot \hat{F}_s^2 \cdot \sin^2\gamma$$

$$\frac{d\left(-W_{magn}\right)}{d\gamma} = -\left(\frac{\hat{F}_s^2 \cdot 2 \cdot \cos\gamma \cdot (-\sin\gamma) + 2 \cdot \hat{F}_s \cdot \hat{F}_m \cdot (-\sin\gamma)}{2 \cdot R_x}\right) - \frac{\hat{F}_s^2 \cdot 2 \cdot \sin\gamma \cdot \cos\gamma}{2 \cdot R_y} =$$

$$= \frac{1}{R_x} \cdot \left(\hat{F}_s \cdot \cos\gamma \cdot \hat{F}_s \cdot \sin\gamma + \hat{F}_s \cdot \hat{F}_m \cdot \sin\gamma\right) - \frac{1}{R_y} \cdot \hat{F}_s \cdot \sin\gamma \cdot \hat{F}_s \cdot \cos\gamma =$$

$$= \frac{1}{R_x} \cdot \left(\hat{F}_{sx} \cdot \hat{F}_{sy} + \hat{F}_{sy} \cdot \hat{F}_m\right) - \frac{1}{R_y} \cdot \hat{F}_{sy} \cdot \hat{F}_{sx} = \frac{\hat{F}_{sy} \cdot \hat{F}_m}{R_x} + \hat{F}_{sx} \cdot \hat{F}_{sy} \cdot \left(\frac{1}{R_x} - \frac{1}{R_y}\right)$$

$$T = -\frac{dW_{magn}}{d\gamma} = \ldots = \frac{\hat{F}_{sy} \cdot \hat{F}_m}{R_x} + \hat{F}_{sx} \cdot \hat{F}_{sy} \cdot \left(\frac{1}{R_x} - \frac{1}{R_y}\right)$$

Torque Components

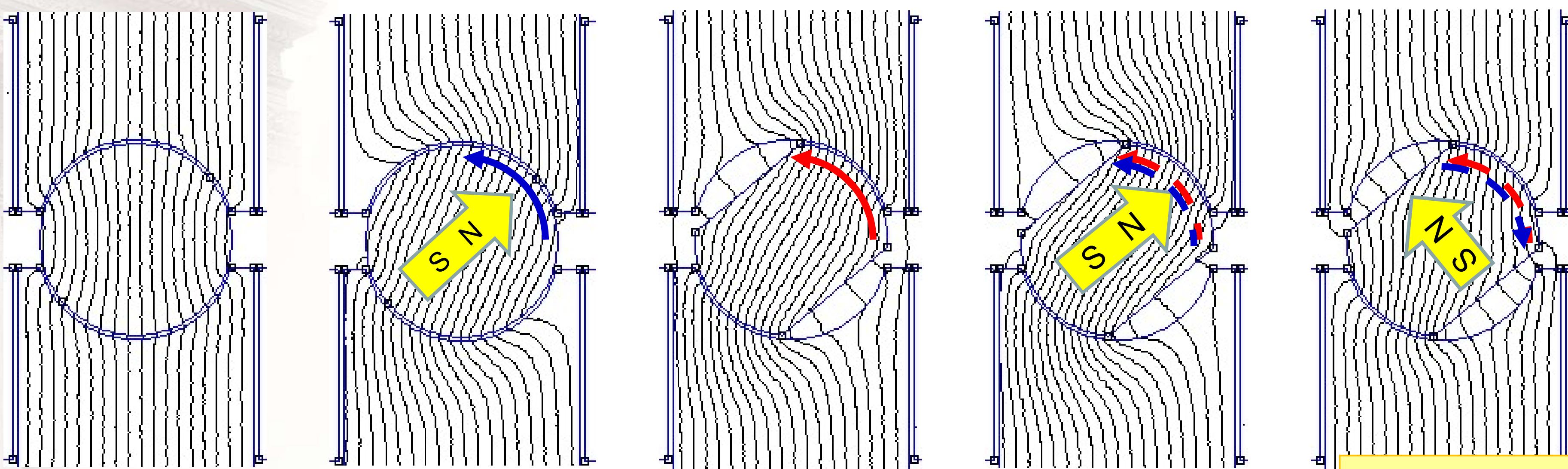
$$T = -\frac{dW_{magn}}{d\gamma} = \ldots = \frac{\hat{F}_{sy} \cdot \hat{F}_m}{R_x} + \hat{F}_{sx} \cdot \hat{F}_{sy} \cdot \left(\frac{1}{R_x} - \frac{1}{R_y}\right)$$

No Torque

Electrically or permanently magnetized = **Lorenz forces**

Only **Reluctance force**

Lorentz AND **Reluctance** forces

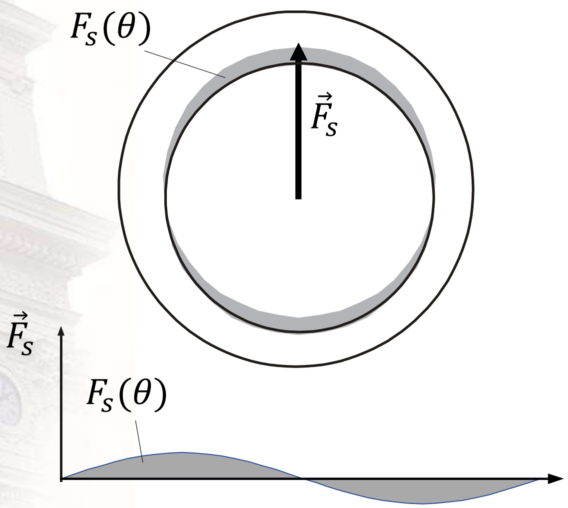


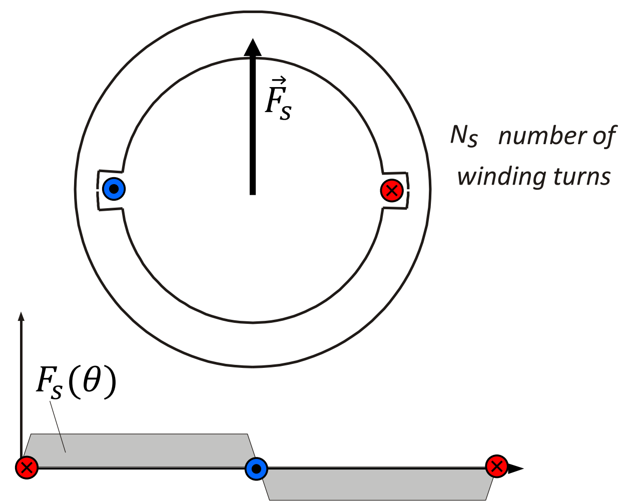
External Magnetic Field

8.5 p 266-267

Electrically magnetized stator

8.7 p 267-271





Fundamental mmf

$$\hat{F}_{s1} = \frac{2}{\pi} \cdot N_s \cdot i_s$$

Torque expressed in flux and mmf

$$T = \frac{1}{R_x} \cdot \left(\hat{F}_m + \hat{F}_{sx}\right) \cdot \hat{F}_{sy} - \frac{1}{R_y} \cdot \hat{F}_{sy} \cdot \hat{F}_{sx} =$$

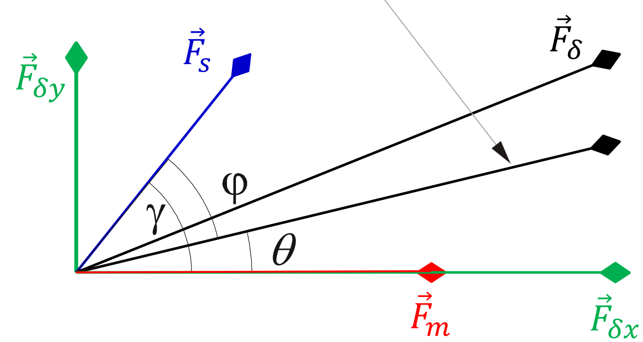
$$= \frac{\pi}{2} \cdot \left(\varphi_{\delta x} \cdot \hat{F}_{sy} - \varphi_{\delta y} \cdot \hat{F}_{sx}\right) =$$

$$= \frac{\pi}{2} \cdot \varphi_\delta \cdot \hat{F}_s \cdot (\cos\theta \sin\gamma - \sin\theta \cos\gamma) =$$

$$= \frac{\pi}{2} \cdot \varphi_\delta \cdot \hat{F}_s \cdot \sin(\gamma - \theta) = \frac{\pi}{2} \cdot \varphi_\delta \cdot \hat{F}_s \cdot \sin(\phi) =$$

$$= \frac{\pi}{2} \cdot \vec{\varphi}_\delta \times \vec{F}_s$$

$$\vec{\varphi}_\delta = \frac{\frac{2}{p} \cdot \hat{F}_{\delta x}}{R_x} + j \cdot \frac{\frac{2}{p} \cdot \hat{F}_{\delta y}}{R_y}$$



Important conclusion

Remember: Flux=MMF/Reluctance

Torque expressed in flux linkage

- **Effective number of turns**

$N_{s,eff} = N_s \cdot k_{r1}$

- **Express MMF in "Ampere-turns"**

$$\vec{F}_s = \hat{F}_{sx} + j\hat{F}_{sy} = \frac{2}{\pi} \cdot N_{s,eff} \cdot \vec{i}_s = \frac{2}{\pi} \cdot N_{s,eff} \cdot \left(i_{sx} + j i_{sy}\right)$$

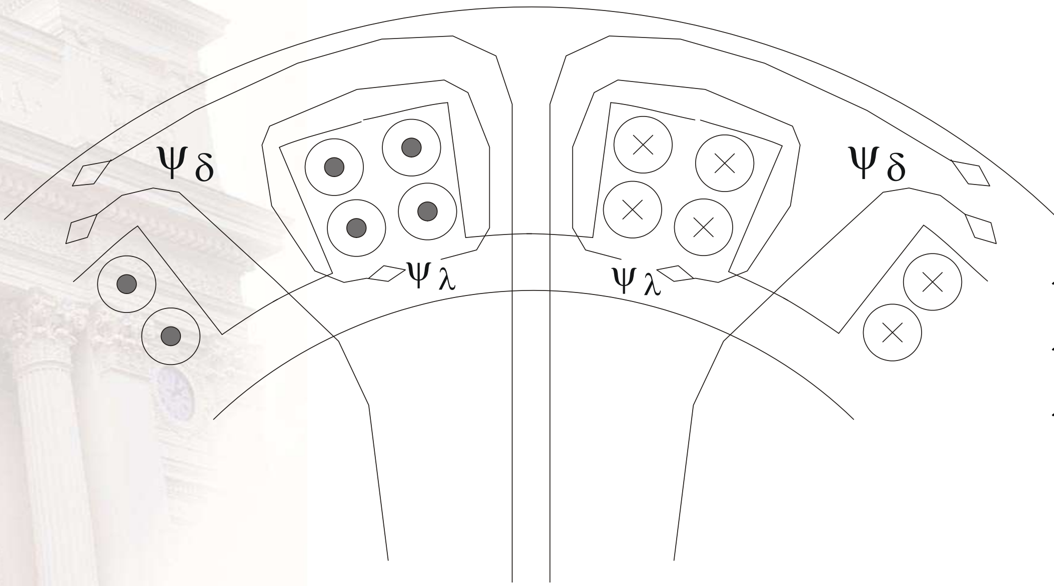
- **Insert in the Torque-equation**

$$T = \frac{\pi}{2} \cdot \left(\varphi_{\delta x} \cdot \hat{F}_{sy} - \varphi_{\delta y} \cdot \hat{F}_{sx}\right) = \frac{\pi}{2} \cdot \left(\varphi_{\delta x} \cdot \frac{2}{\pi} \cdot N_{s,eff} \cdot i_{sy} - \varphi_{\delta y} \cdot \frac{2}{\pi} \cdot N_{s,eff} \cdot i_{sx}\right) =$$

$$= \varphi_{\delta x} \cdot N_{s,eff} \cdot i_{sy} - \varphi_{\delta y} \cdot N_{s,eff} \cdot i_{sx} = \psi_{\delta x} \cdot i_{sy} - \psi_{\delta y} \cdot i_{sx} = \vec{\psi}_\delta \times \vec{i}_s$$

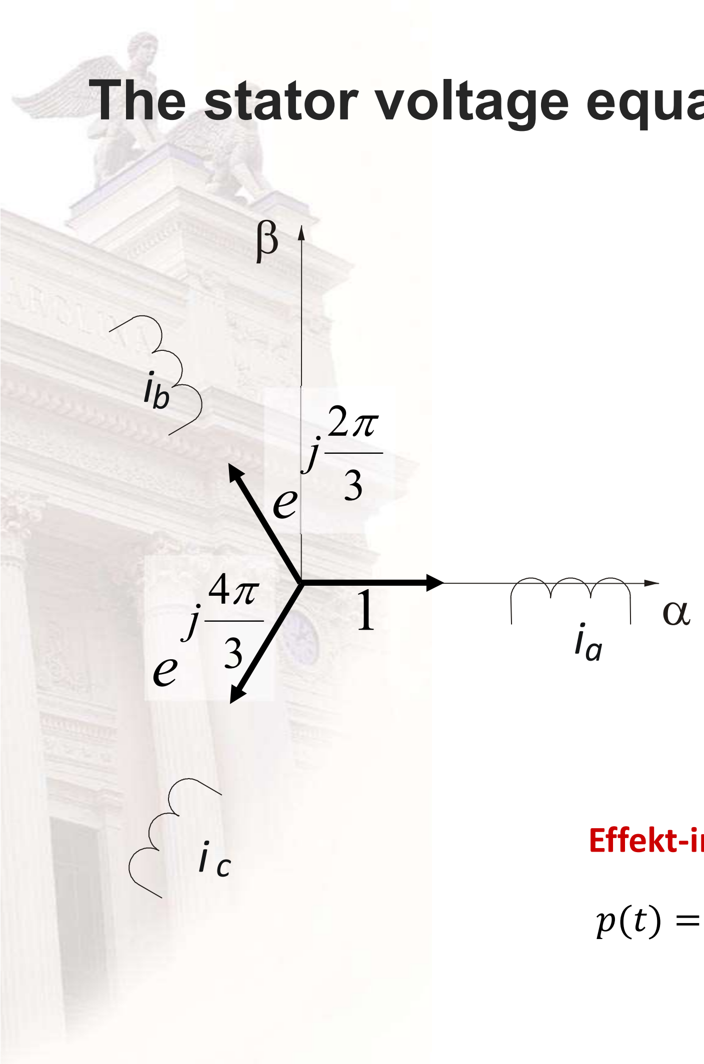
Important conclusion

Leakage inductances



$$\psi_{a\lambda} = N_{s,eff} \cdot \varphi_{a\lambda} = L_{s\lambda} \cdot i_a$$
$$\psi_{b\lambda} = N_{s,eff} \cdot \varphi_{b\lambda} = L_{s\lambda} \cdot i_b$$
$$\psi_{c\lambda} = N_{s,eff} \cdot \varphi_{c\lambda} = L_{s\lambda} \cdot i_c$$

The stator voltage equation in the stator reference frame



$$\sqrt{\frac{2}{3}}\cdot\left(u_a = R_s\cdot i_a + \frac{d\psi_a}{dt} = R_s\cdot i_a + \frac{d}{dt}(\psi_{\delta a} + L_{s\lambda}\cdot i_a)\right)$$

$$e^{j\cdot\frac{2\pi}{3}}\cdot\sqrt{\frac{2}{3}}\cdot\left(u_b = R_s\cdot i_b + \frac{d\psi_b}{dt} = R_s\cdot i_b + \frac{d}{dt}(\psi_{\delta b} + L_{s\lambda}\cdot i_b)\right)$$

$$+\quad e^{j\cdot\frac{4\pi}{3}}\cdot\sqrt{\frac{2}{3}}\cdot\left(u_c = R_s\cdot i_c + \frac{d\psi_c}{dt} = R_s\cdot i_c + \frac{d}{dt}(\psi_{\delta c} + L_{s\lambda}\cdot i_c)\right)$$

$$\vec{u}_s^{\alpha\beta} = R_s\cdot\vec{i}_s^{\alpha\beta} + \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = R_s\cdot\vec{i}_s^{\alpha\beta} + \frac{d}{dt}\left(\vec{\psi}_\delta^{\alpha\beta} + L_{s\lambda}\cdot\vec{i}_s^{\alpha\beta}\right)$$

Effekt-invarians

$$p(t) = u_a\cdot i_a + u_b\cdot i_b + u_c\cdot i_c = u_\alpha\cdot i_\alpha + u_\beta\cdot i_\beta$$

Rotation and multi-phase winding

$$\vec{i}_s^{xy} = i_{sx} + ji_{sy} = i_s e^{j\gamma}$$

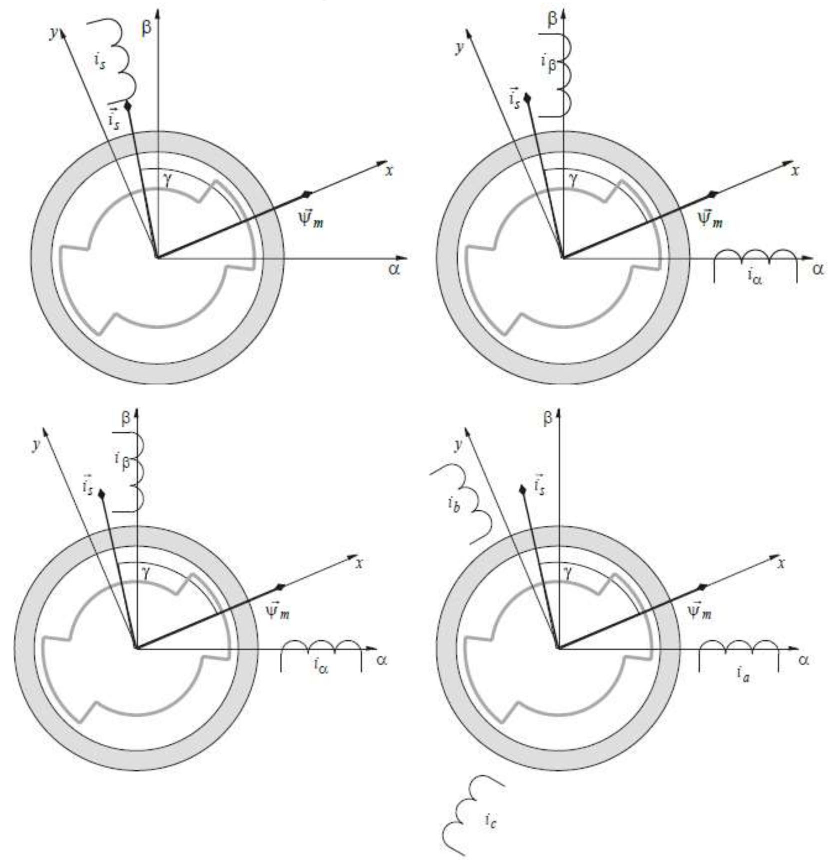
$$\vec{i}_s^{\alpha\beta} = \vec{i}_s^{xy} e^{j\theta_r} = i_s e^{j(\gamma+\theta_r)} = i_{s\alpha} + ji_{s\beta}$$

$$i_{s\alpha} + ji_{s\beta} = i_s \cos(\omega_r t + \gamma) + ji_s \sin(\omega_r t + \gamma)$$

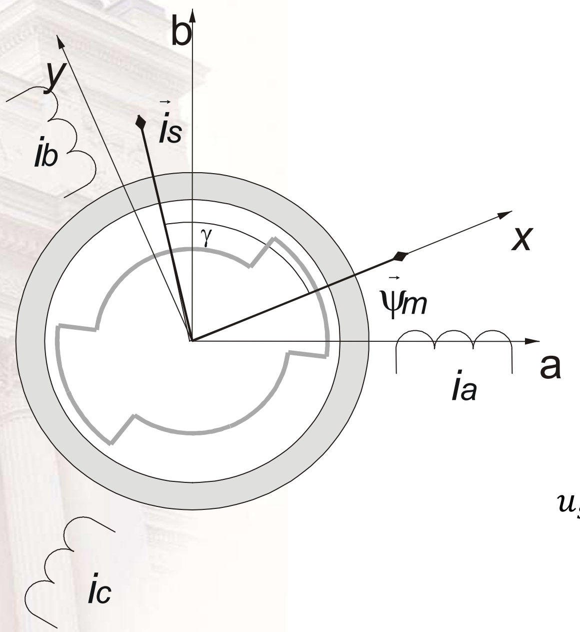
$$i_a = \sqrt{\frac{2}{3}} i_{s\alpha}$$

$$i_b = \sqrt{\frac{2}{3}}\left(-\frac{1}{2} i_{s\alpha} + \frac{\sqrt{3}}{2} i_{s\beta}\right)$$

$$i_c = \sqrt{\frac{2}{3}}\left(-\frac{1}{2} i_{s\alpha} - \frac{\sqrt{3}}{2} i_{s\beta}\right)$$



The stator voltage in the rotor reference frame



$$\vec{u}_s^{xy} = R_s \cdot \vec{i}_s^{xy} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right) + j\omega_r \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)$$

$$u_{sx} = R_s \cdot i_{sx} + \frac{d}{dt}(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}) - \omega_r \cdot \left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) =$$
$$= R_s \cdot i_{sx} + \frac{d}{dt}(\psi_m + L_{sx} \cdot i_{sx}) - \omega_r \cdot L_{sy} \cdot i_{sy}$$
$$u_{sy} = R_s \cdot i_{sy} + \frac{d}{dt}\left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) + \omega_r \cdot (\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}) =$$
$$= R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot (\psi_m + L_{sx} \cdot i_{sx})$$

Exercises on MMF distribution (4)

- **PE ExercisesWithSolutions2019b vers 190206**
- **Draw cross-section: EMSM (4.1) & DCM (4.2)**
- **Sine wave MMF: Single (4.3) Dual (4.9) wave**
- **Torque expression (4.10)**
- **Flux vector (4.11) (4.12) + armature current (4.13)**
- **Rotation problem (4.14)**
- **Voltage equation (4.15)**