Thermal modelling

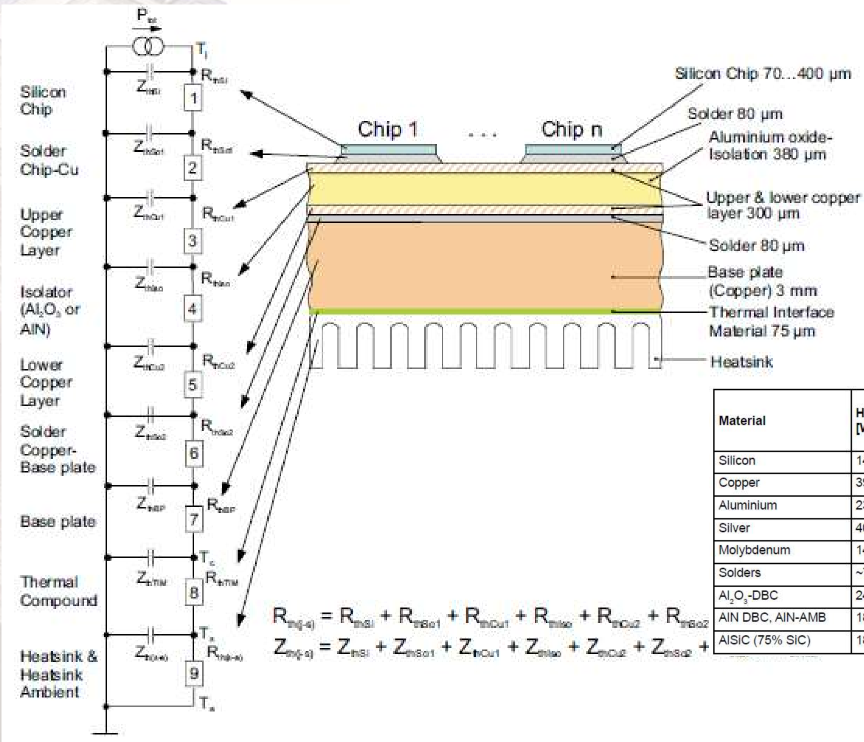
and

Position and Speed Control

Industrial Electrical Engineering and Automation
Lund University, Sweden

Thermal circuit

Semikron p 82-83



Material	Heat conductivity λ [W/(m*K)]	Heat storage charac-teristic s [kJ/(m³*K)]	Thermal expansion coefficient α [10⁻⁶/K]
Silicon	148	1650	4.1
Copper	394	3400	17.5
Aluminium	230	2480	22.5
Silver	407	2450	19
Molybdenum	145	2575	5
Solders	~70	1670	15 – 30
Al_2O_3-DBC	24	3025	8.3
AlN DBC, AlN-AMB	180	2435	5.7
AlSiC (75% SiC)	180	2223	7

- Chip temperature ϑj
- Heat sink temperature ϑ_s
- Power losses P
- Thermal resistance R_{th}
- Thermal capacitance C_{th}

$$R_{th(j-s)} = \sum R_{th(layers)} \qquad R_{th} = \frac{l}{\lambda A}\left[\frac{K}{W}\right]$$

$$C_{th(j-s)} = \sum C_{th(layers)} \qquad C_{th} = mc\left[\frac{J}{K}\right]$$

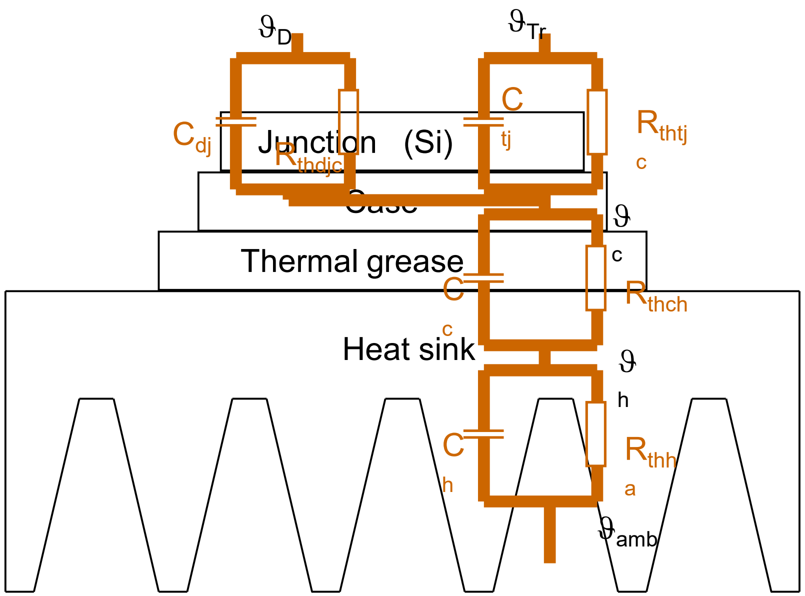
$$R_{conv} = \frac{1}{A_{cool}h}$$

$$\vartheta_j = P_j\left(R_{th} + R_{conv}\right) + \vartheta_{amb}$$

Thermal circuit

6.2 p 233-237

- Heat flow
  - *Heat sources: losses in diodes and transistors*
  - *Heat sink: natural or forced convection*
  - *Thermal resistance: components and thermal connections between them*
- Thermal nodes
  - *Junction*
  - *Case*
  - *Heat sink*
  - *Ambient*
- Solutions
  - *Steady state*
  - *Transient*

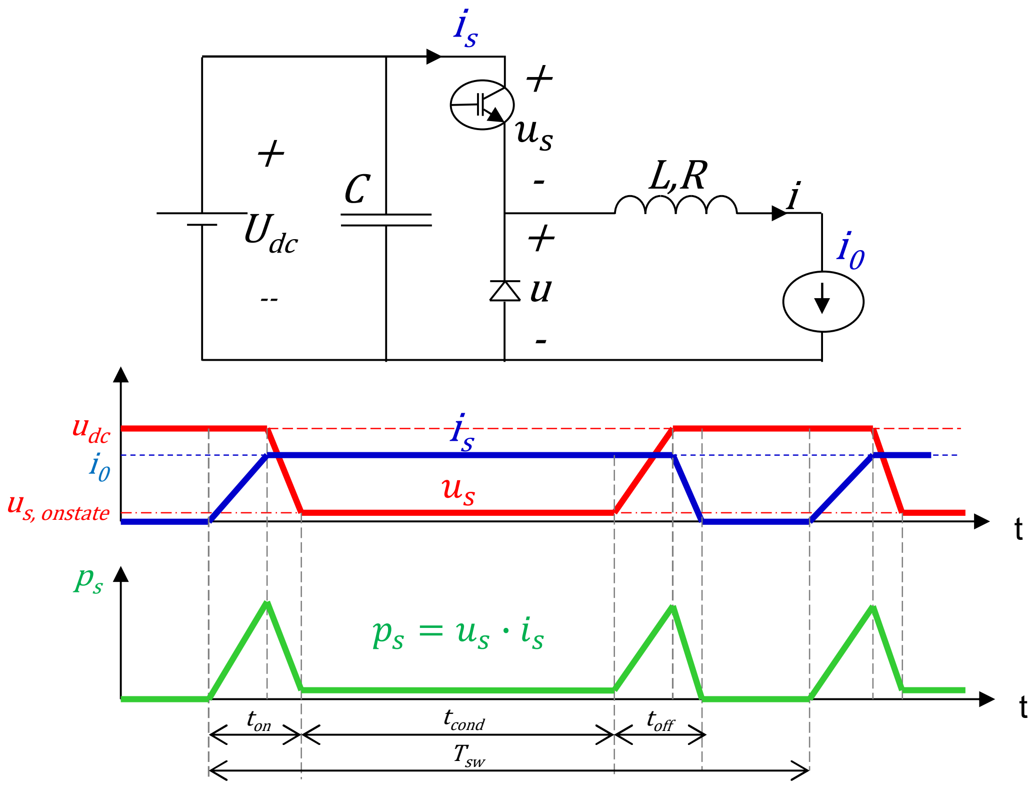


$$\vartheta_{1,end} = (R_{th} \cdot P + \vartheta_{amb}) \cdot (1 - e^{-\frac{t}{\tau}}) + \vartheta_{1,start} \cdot e^{-\frac{t}{\tau}}$$

6.1 p 225-233

Simple converter loss model

- Switching waveforms, looking at turn-on, on-state and turn-off energy losses over switching sequence
- Considering temperature dependence
- Recalculate datasheet values to actual working point
- Pay attention if losses can be separated by components or they are provided as per integrated switch



Switching and conducting losses

Energy losses: $E_S(T_{sw})=\int_{T_{sw}} p_S(\tau)d\tau=E_{S,on}(T_{sw})+E_{S,cond}(T_{sw})+E_{S,off}(T_{sw})$

$$E_{S,on}(T_{sw})=\int_{t_{on}} p_S(\tau)d\tau=V_{DC}\cdot I_0\cdot\frac{t_{on}}{2}$$

$$E_{S,cond}(T_{sw})=\int_{t_{cond}} p_S(\tau)d\tau=V_{S(on)}\cdot I_0\cdot t_{cond}$$

Note $V_{S(on)}=V_{S0}+R_S\cdot I_0$

$$E_{S,off}(T_{sw})=\int_{t_{off}} p_S(\tau)d\tau=V_{DC}\cdot I_0\cdot\frac{t_{off}}{2}$$

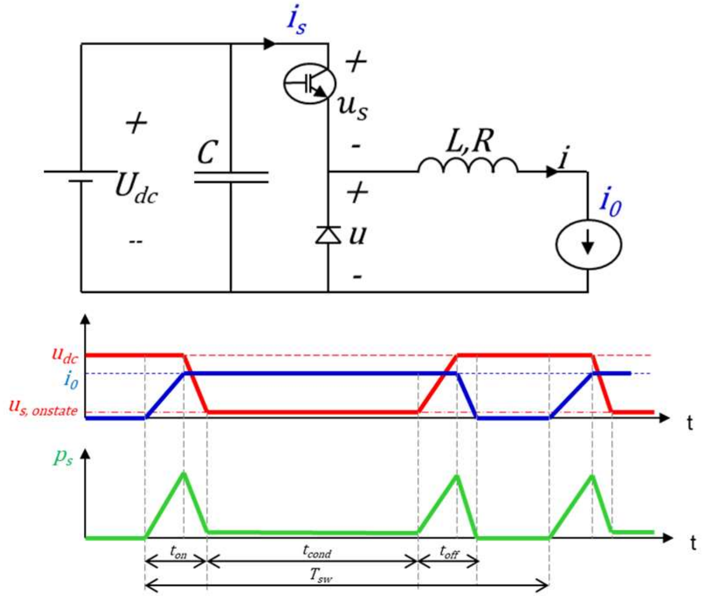
Power losses: $P_S(T_{sw})=\frac{E_S(T_{sw})}{T_{sw}}=P_{S,on}(T_{sw})+P_{S,cond}(T_{sw})+P_{S,off}(T_{sw})$

$$P_{S,on}(T_{sw})=\frac{E_{S,on}(T_{sw})}{T_{sw}}=E_{S,on}(T_{sw})\cdot f_{sw}=\frac{V_{DC}\cdot I_0\cdot t_{on}}{2}\cdot f_{sw}$$

$$P_{S,cond}(T_{sw})=\frac{E_{S,cond}(T_{sw})}{T_{sw}}=V_{S(on)}\cdot I_0\cdot\frac{t_{cond}}{T_{sw}}=V_{S(on)}\cdot I_0\cdot D_S$$

$$P_{S,off}(T_{sw})=\frac{E_{S,off}(T_{sw})}{T_{sw}}=E_{S,off}(T_{sw})\cdot f_{sw}=\frac{V_{DC}\cdot I_0\cdot t_{off}}{2}\cdot f_{sw}$$

$$P_{S,sw}(T_{sw})=P_{S,on}(T_{sw})+P_{S,off}(T_{sw})$$



Turn on t_{on}
On-state t_{cond}
Turn off t_{off}

4.2 p 145

Reverse recovery losses

Nominal data

Case data

If specified, use:

$$E_{S,on}(T_{sw})=\frac{E_{on,n}}{V_{DC,n}\cdot I_{0n}}\cdot V_{DC}\cdot I_0$$

$$E_{S,off}(T_{sw})=\frac{E_{off,n}}{V_{DC,n}\cdot I_{0n}}\cdot V_{DC}\cdot I_0$$

For the freewheeling diode:

$$P_{D,cond}(T_{sw})=V_{D(on)}\cdot I_0\cdot D_D \qquad V_{D(on)}=V_{D0}+R_D\cdot I_0$$

$$D_D\approx 1-D_S$$

$$P_{D,rr}=V_{DC}\cdot Q_f\cdot f_{sw} \qquad Q_f\approx\frac{1}{S+1}\cdot Q_{rr}\ \text{ where } S=\frac{t_{rr1}}{t_{rr2}}$$

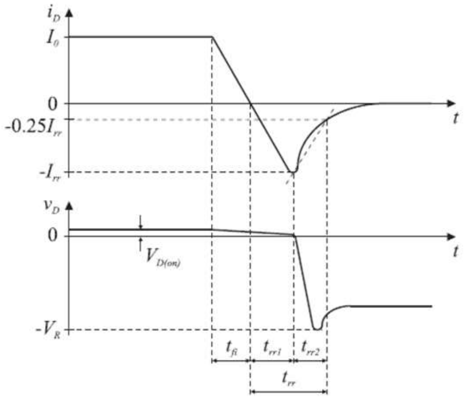


Figure 6.3: Diode turn-off.

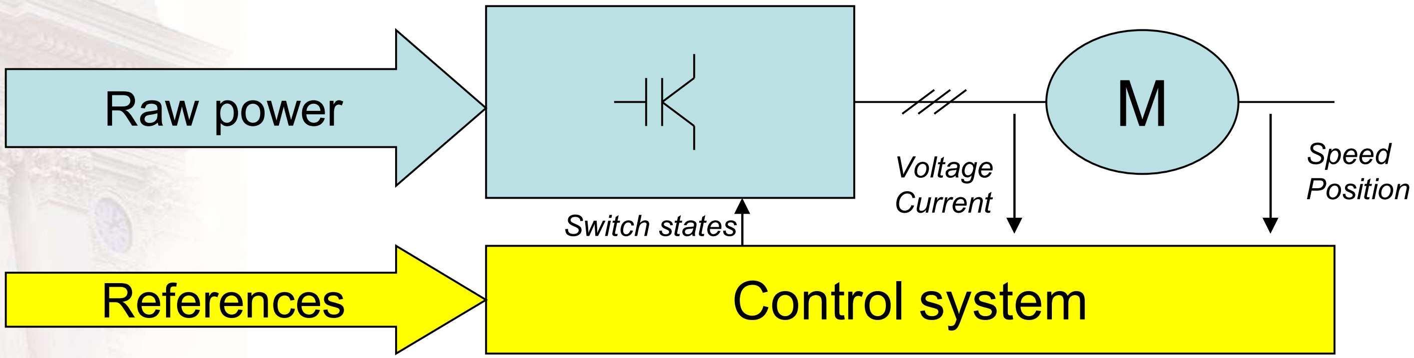
If specified, use:

$$P_{D,off}=E_{D,off}(T_{sw})\cdot f_{sw}\ ,\ E_{D,off}(T_{sw})=\frac{E_{off,n}}{V_{DC,n}\cdot I_{0n}}\cdot V_{DC}\cdot I_0 \qquad Q_f=\frac{Q_{f,n}}{I_{0n}}\cdot I_0$$

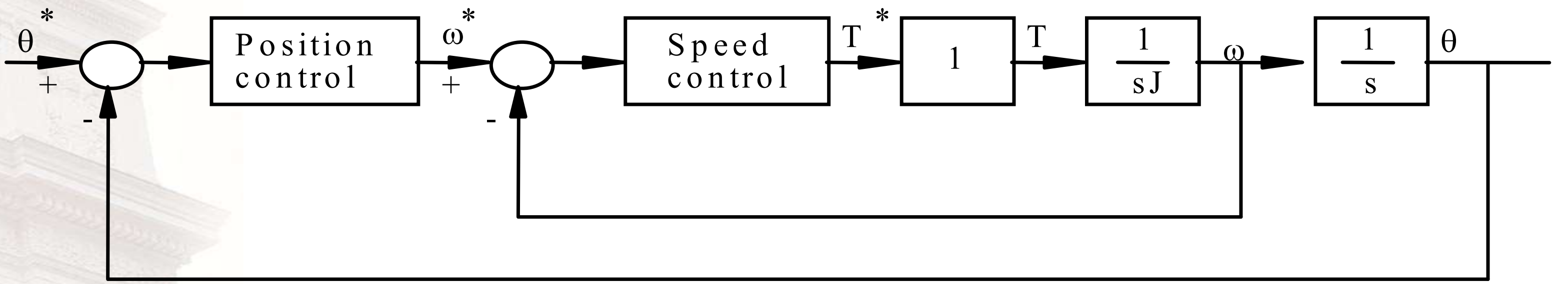
Fall t_{ft}
di/dt<0 t_{rr1}
di/dt>0 t_{rr2}

Position and Speed Control

Generic Structure



Cascade Control



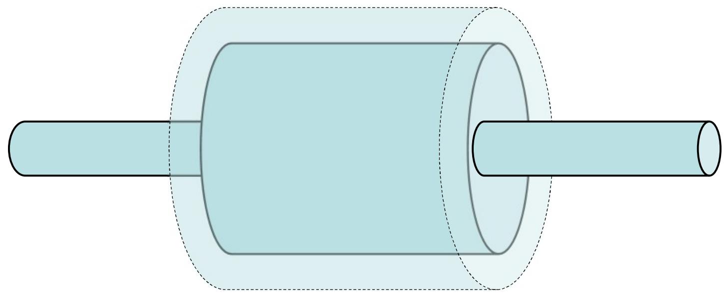
The system contains two integrations. This gives a hint about two properties of the system:

1 watch out for the stability margins;

2 integrators may help to eliminate remaining errors.

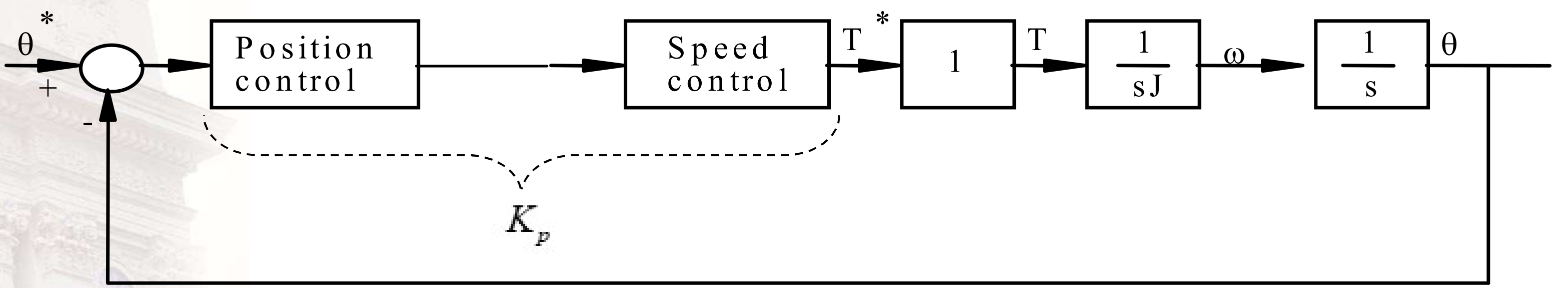
The mechanical system

$$\frac{d}{dt}(J \cdot \omega) = T_{el} - T_l$$



$$\frac{d}{dt}(J \cdot \omega) = J \cdot \frac{d\omega}{dt} + \omega \cdot \frac{dJ}{dt} = T_{el} - T_l$$

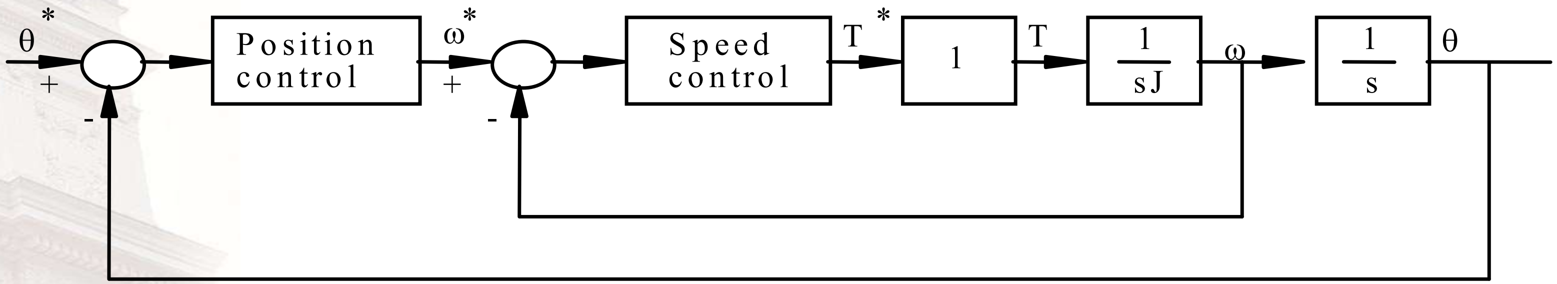
Position Control Without Speed Control



$$\frac{\theta(s)}{\theta^*(s)} = \frac{K_p}{J \cdot s^2 + K_p} \qquad s = \pm\sqrt{-\frac{K_p}{J}}$$

Only oscillatory poles!

Position Control with Speed Control



$$\frac{\theta(s)}{\theta^*(s)} = \frac{K_p \cdot K_\omega}{J \cdot s^2 + K_\omega \cdot s + K_p \cdot K_\omega}$$

$$poles = -\frac{K_\omega}{2J} \pm \sqrt{\frac{K_\omega^2}{4J^2} - \frac{K_p \cdot K_\omega}{J}}$$

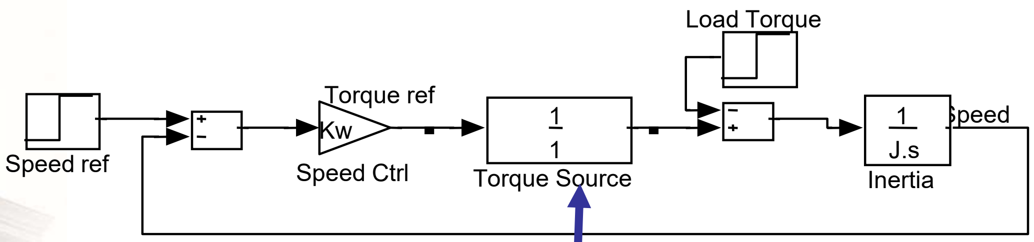
Limit for oscillatory poles:

$$\frac{K_\omega^2}{4J^2} - \frac{K_p \cdot K_\omega}{J} = 0$$

$$K_p = \frac{K_\omega}{4J}$$

Speed Control

Ideal torque source and speed sensing



Closed system: $$\frac{\omega}{\omega^*} = \frac{K_w}{sJ + K_w} = \frac{1}{1 + s\frac{J}{K_w}}$$

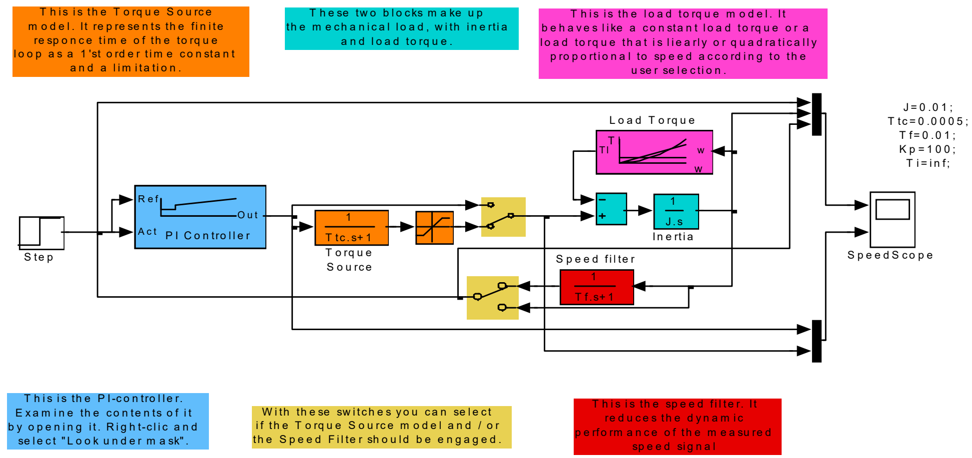
Roots: $$s = -\frac{K_w}{J}$$

i.e. any bandwidth possible, ... but then is no longer true ...

Example 1

- $T_{tc} = 0.001;$
- $J = 0.038;$
- Select $k_\omega = 100;$

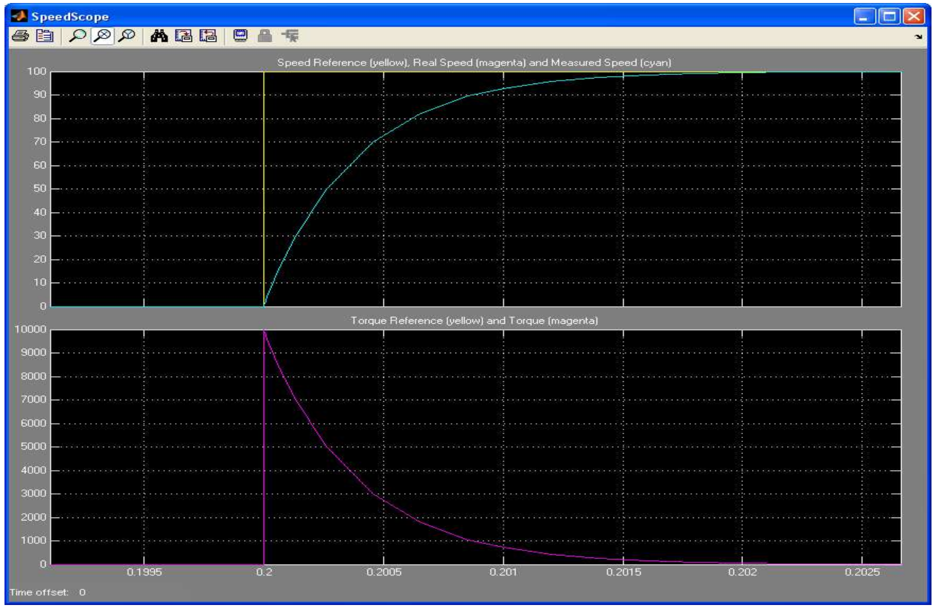
- High gain + Unlimited and infinitely fast torque source



Conclusion 1

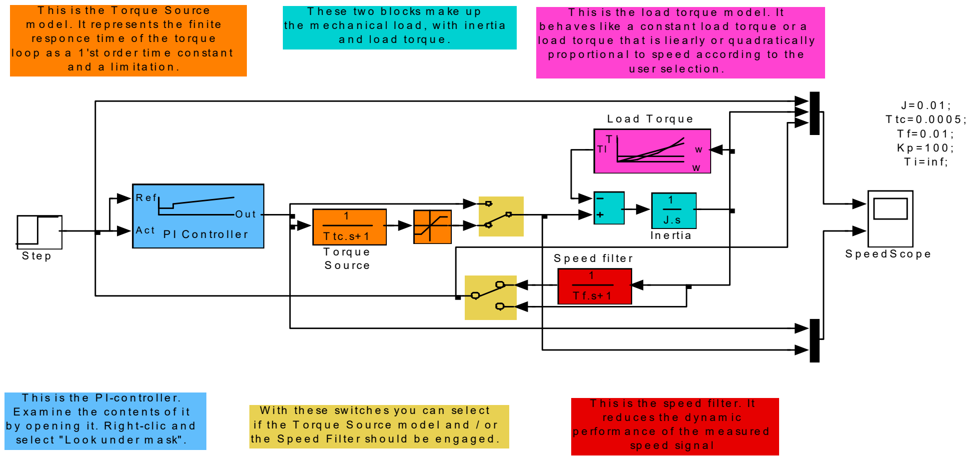
- Ideal conditions leads to a too fast speed controller. The torque source is not as fast in reality as it seems in the simulations.

- Try limiting the torque source in amplitude and dynamic responce time.
  – *Simplest: LIMITED 1'st order time constant!*



Example 2

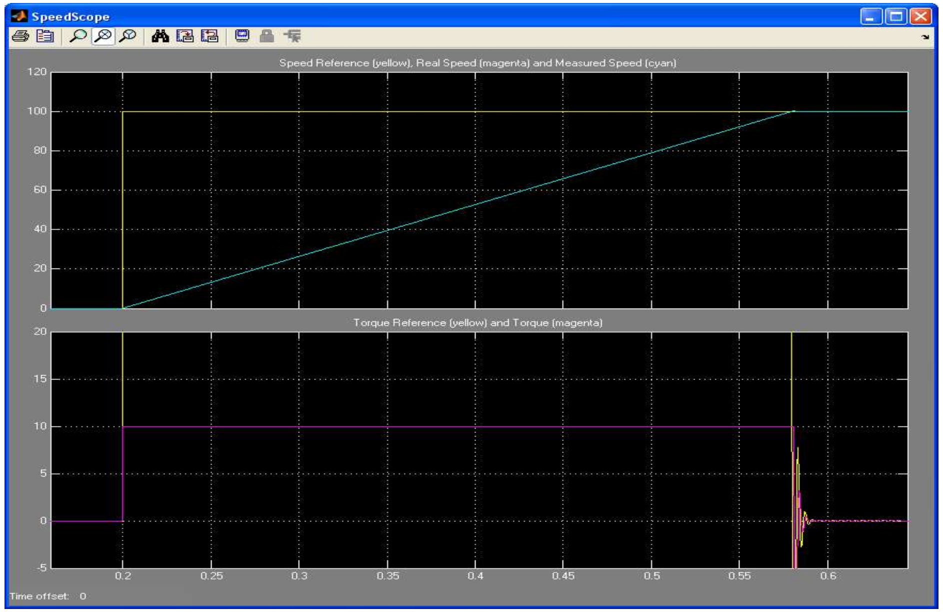
- Introduce a torque source, LIMITED in dynamic response time AND amplitude!



Conclusion 2

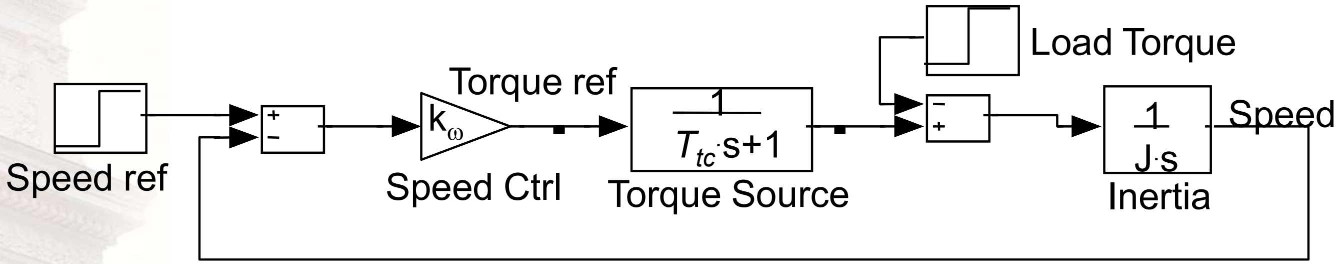
- Now the responce time is much longer, and oscillatory when out of torque limitation ...

- What is a suitable gain?
  - *Derive the gain based on the assumption that the torque source reacts like a limited 1'st order time constant*



Speed Control

Torque dynamics as first order low pass filter



Closed system:

$$\frac{\omega}{\omega^*} = \frac{\frac{k_w}{J \cdot T_{tc}}}{s^2 + s \cdot \frac{1}{T_{tc}} + \frac{k_w}{J \cdot T_{tc}}}$$

Roots:

$$-\frac{1}{2T_{tc}} \pm \sqrt{\frac{1}{4 \cdot T_{tc}^2} - \frac{k_w}{J \cdot T_{tc}}}$$

Non osc. roots ->

$$k_w = \frac{J}{4 \cdot T_{tc}}$$

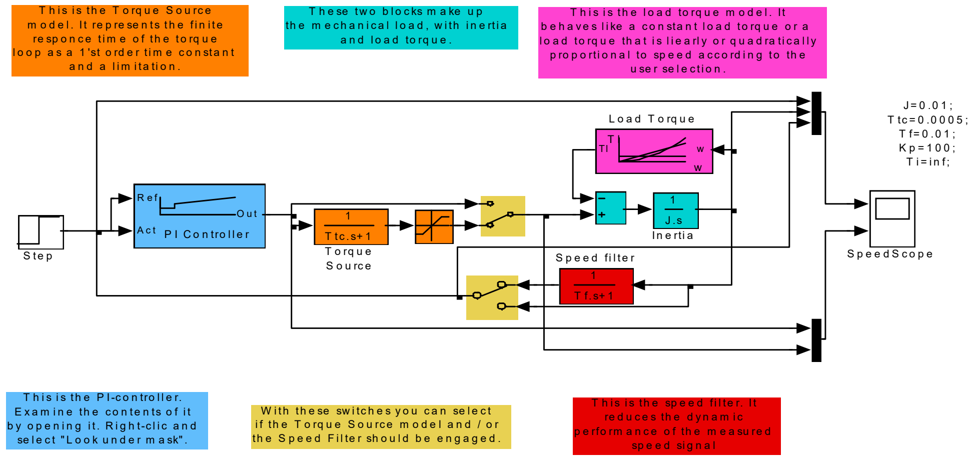
Limited k_ω gives stationary error with P-control!!

Example 3

T_{tc} = 0.001

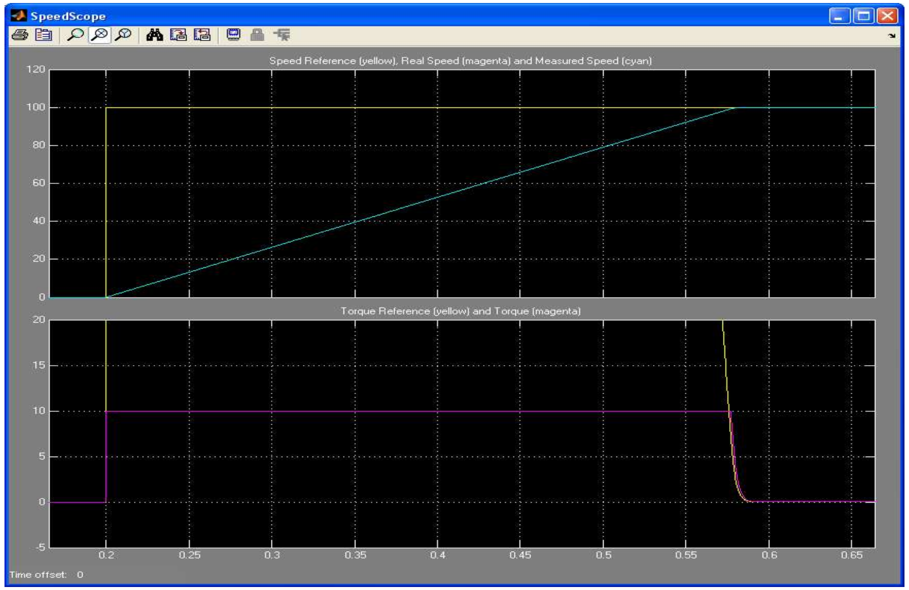
J = 0.034

Select k_ω = J/4/T_{tc}= 9.5;



Conclusion 3

- Dynamics is realistic and the control system stable.
- What about stationary errors?



Toad torque can be:

- Constant
- Linear to speed
- Quadratic to speed.
  - *Try with constant 5 Nm*
- What happens?

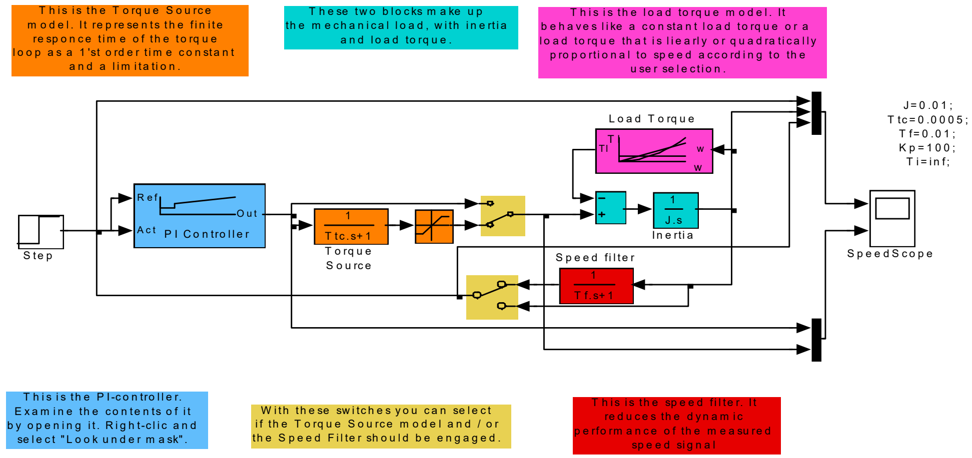
Example 4

T_{tc} = 0.001;

J = 0.034;

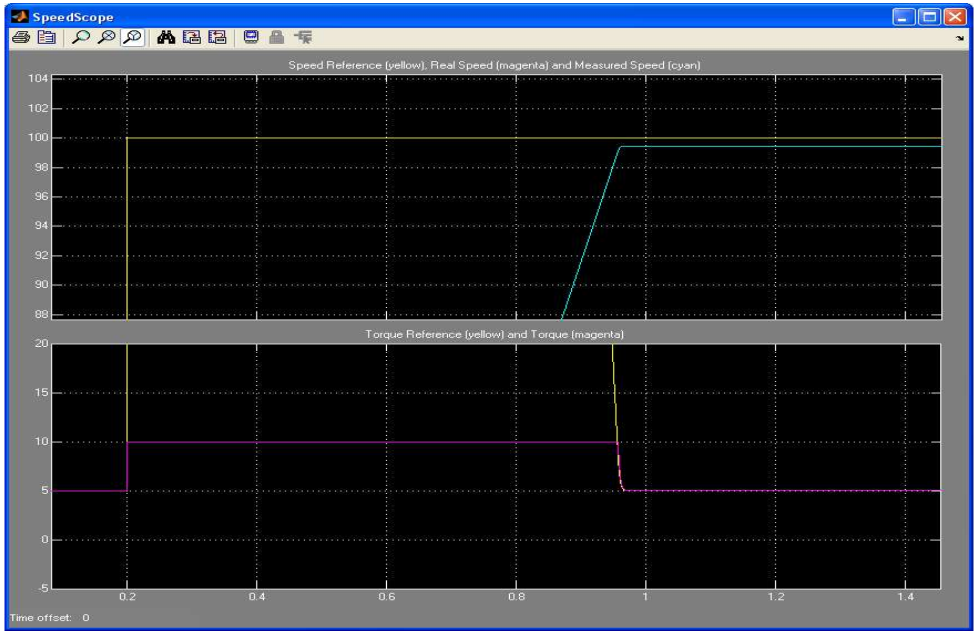
Constant Load Torque 5 Nm

Select k_ω =J/4/T_{tc} = 9.5



Conclusion 4

- With P-control and a ”non-zero” load torque, the will be a stationary error.



Solution to stationary error, PI-control

Control error $e = y^*(t) - y(t)$

$$u(t) = K_p \cdot \left(e(t) + \frac{1}{\tau_i} \int e(t)dt \right)$$

$$u(s) = K_p \frac{1 + s\tau_i}{s\tau_i} e(s)$$

Digital PI controller

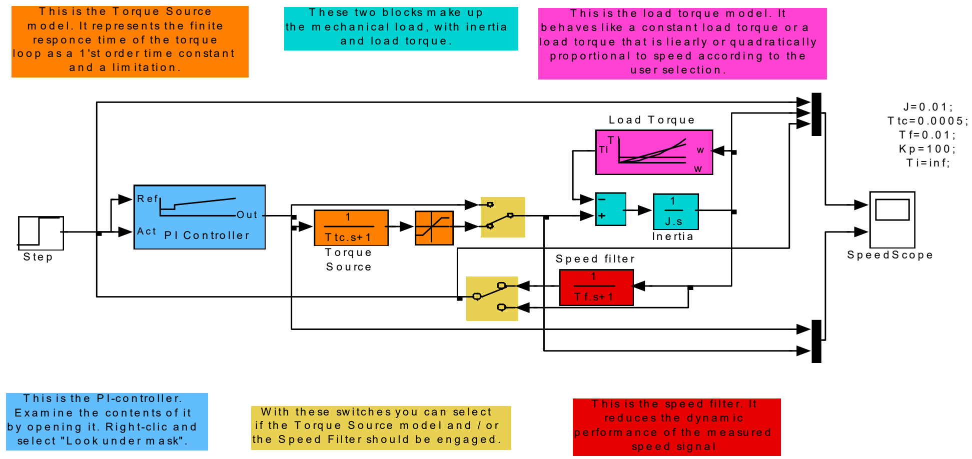
$$u(k) = K_p \cdot \left(y^*(k) - y(k) + \frac{T_s}{T_i} \sum_{n=0}^{n=k} \big(y^*(n) - y(n) \big) \right)$$

$$K_p \cdot \frac{T_s}{T_i} \sum_{n=0}^{n=k} \big(y^*(n) - y(n) \big) = u_{\text{int}}(k)$$

$$u(k) = K_p \cdot \big(y^*(k) - y(k) \big) + u_{\text{int}}(k)$$

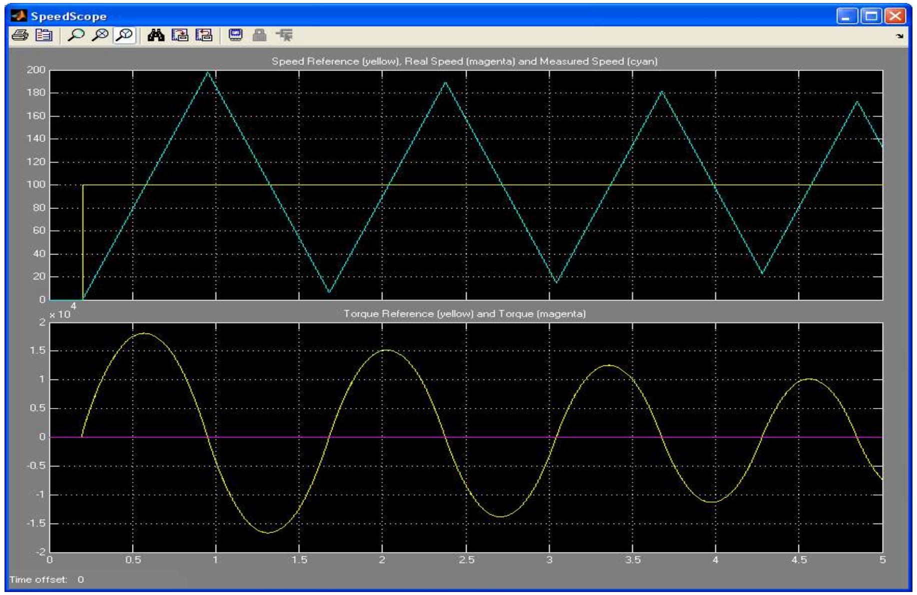
Example 5

- How does the integrator react to the torque source limitation?
- Try with integration time T_i = 0.1 second?



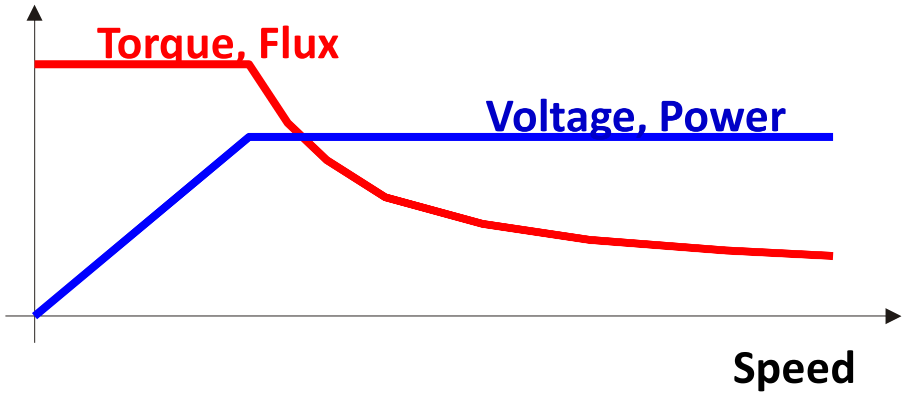
Conclusion 5

- With the torque source limitation, the integrator integates ”in vain”. The result is unstable!
- The problem is called “windup” of the integrator
- The solution is called “Anti Windup”

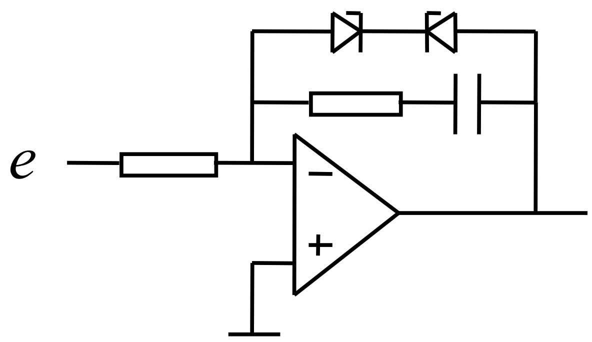


The torque source is limited

- **Often speed dependent, e.g. "field weakening"**



Analog anti windup (= history)



Digital Anti-Windup

$$u_{\text{int}}(k) = K_p \cdot \frac{T_s}{T_i} \sum_{n=0}^{n=k} \left(y^*(n) - y(n) \right)$$

$$u(k) = K_p \cdot \left(y^*(k) - y(k) \right) + u_{\text{int}}(k)$$

if $u(k) > u_{\max}$ *or* $u(k) < u_{\min}$ then

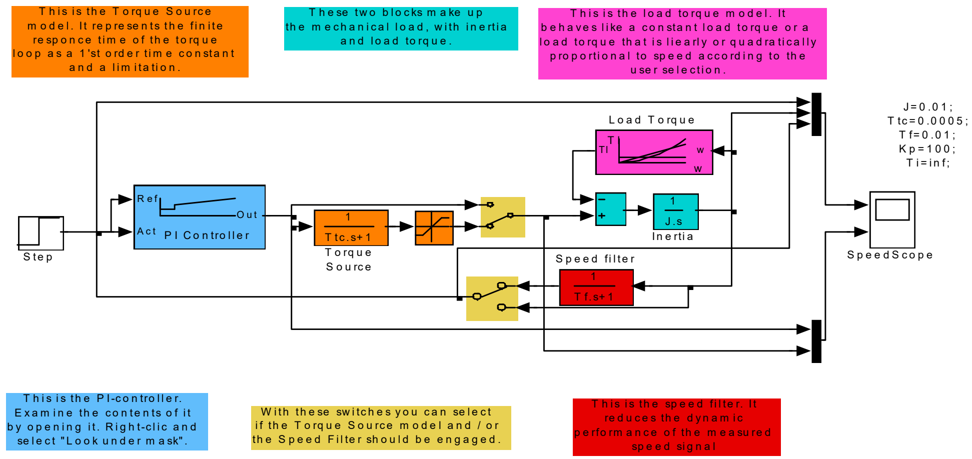
$u(k) = u_{\max}$ *or* $u(k) = u_{\min}$

$u_{\text{int}}(k) = u_{\text{int}}(k-1)$

end

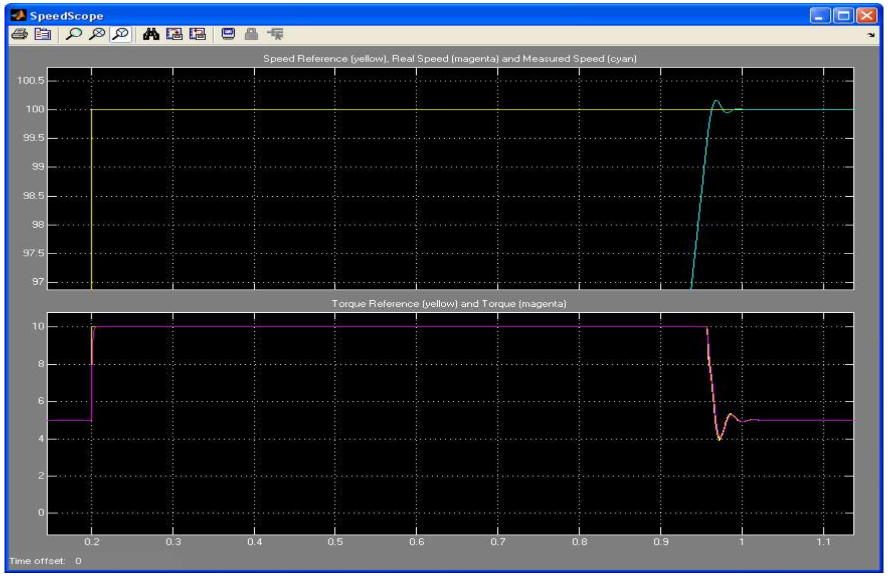
Example 6

- Introduce Anti-Windup and run again!



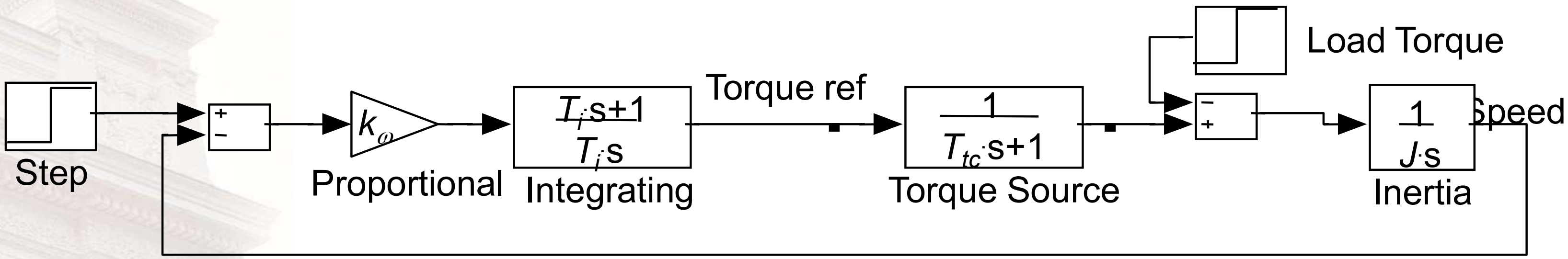
Conclusion 6

- With integrator ”Anti Windup”, the result is stable.
- Note that the torque settles at the load torque, 5 Nm.
- BUT, how do we set the control parameters?



Speed Control

With PI speed controller and 1'st order torque source



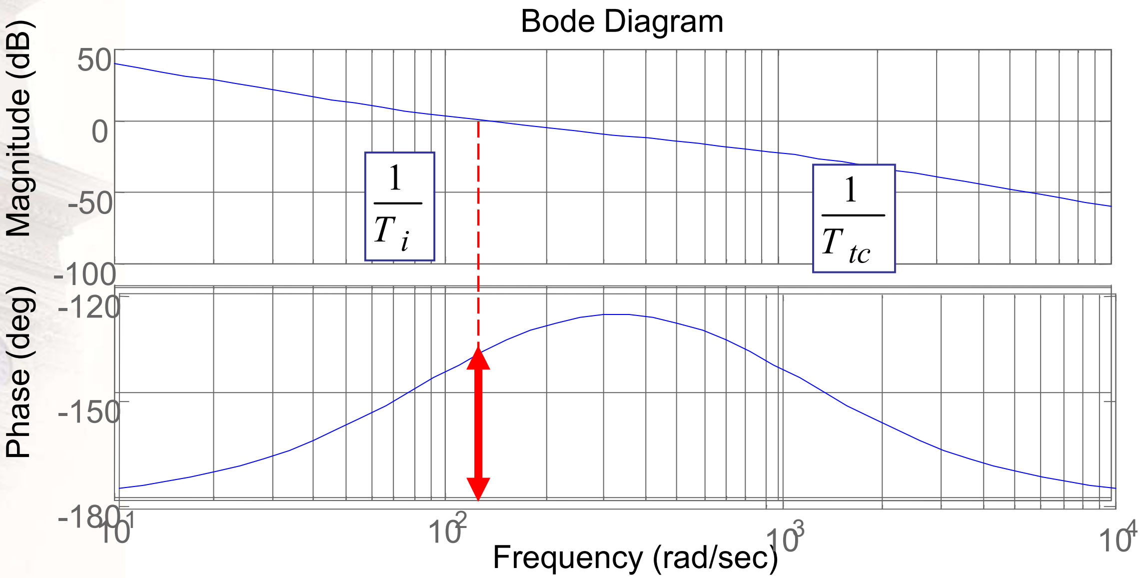
$$\frac{\omega(s)}{\omega^*(s)} = \frac{K_\omega (1 + s \cdot T_i)}{s^3 \cdot J \cdot T_i \cdot T_{tc} + s^2 \cdot J \cdot T_i + s \cdot K_\omega \cdot T_i + K_\omega}$$

3'rd order, how do we solve for the roots??

Symmetric optimum:1

Open loop transfer function

$$G(s) = K_\omega \frac{1 + s \cdot T_i}{s \cdot T_i} \cdot \frac{1}{1 + s \cdot T_{tc}} \cdot \frac{1}{s \cdot J}$$



Select k_ω to maximize phase margin

Symmetric optmum:2

$$\omega_0 = \frac{1}{\sqrt{T_i \cdot T_{tc}}} \qquad T_i = a^2 \cdot T_{tc}\text{, where } a > 1$$

$$\omega_0 = \frac{1}{\sqrt{T_i \cdot T_{tc}}} = \frac{1}{a \cdot T_{tc}} = \frac{a}{T_i}$$

$$\left|G(j\omega_0)\right| = \left|K_\omega \frac{1 + j \cdot a}{j \cdot a} \cdot \frac{1}{1 + \frac{j}{a}} \cdot \frac{1}{\frac{j \cdot a}{T_i} \cdot J}\right| = K_\omega \cdot \frac{T_i}{a \cdot J} = 1$$

$$K_\omega = \frac{a \cdot J}{T_i} = \frac{J}{a \cdot T_{tc}}$$

Symmetric optimum:3

Close loop characteristic equation: $s^3 \cdot J \cdot \frac{T_i^2}{a^2} + s^2 \cdot T_i + s \cdot a_i + \frac{a}{T_i} = 0$

One root: $s = -\omega_0 = -\frac{a}{T_i}$

Polynomial division gives: $s^2 \cdot \frac{T_i^2}{a^2} + s \cdot T_i \cdot \frac{a-1}{a} + 1 = 0$

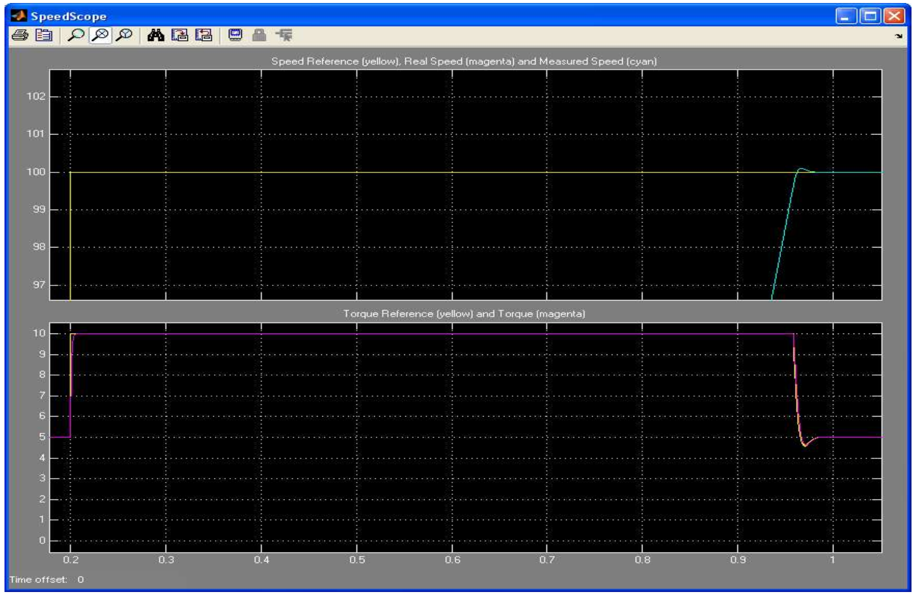
Other roots: $s_{2,3} = -\omega_0 \left(\xi \pm \sqrt{\xi^2 - 1} \right) \qquad \zeta = \frac{a-1}{2}$

Example *z* = 1, i.e. no complex poles:

$$a = 3$$
$$T_i = 9 \cdot T_{tc}$$
$$K_w = \frac{J}{3 \cdot T_{tc}}$$

Conclusion 7

- With the right control parameters, the result is again stable, with no stationary errors



Example 7

- **Try out oscillatory roots**

$$s_{2,3} = -\omega_0\left(\xi \pm \sqrt{\xi^2 - 1}\right)$$

$$\zeta = \frac{a-1}{2}$$

$$a = 2$$

$$T_i = 4 \cdot T_{tc}$$

$$K_w = \frac{J}{2 \cdot T_{tc}}$$

$$\xi = \frac{2-1}{2} = 0.5$$

Noisy speed signal...

A filter on the speed signal gives a 4’th order system.

- How to design??

The engineering solution:

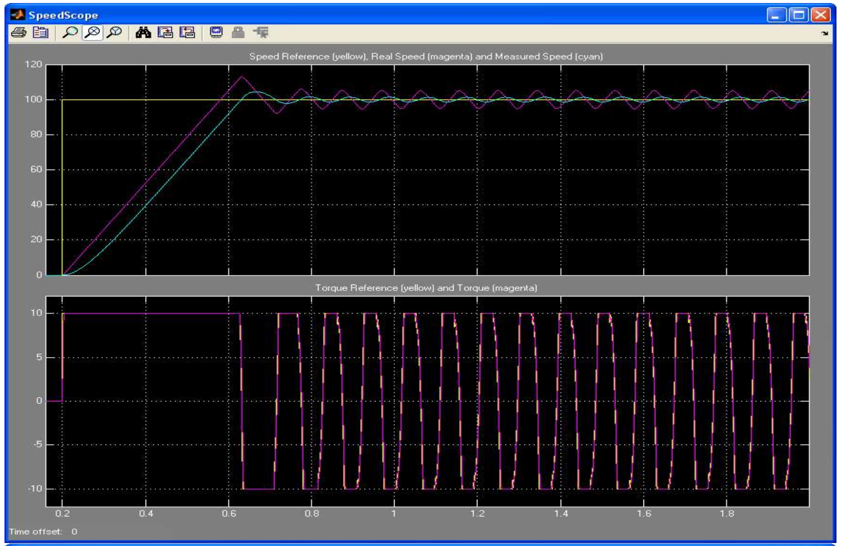
1. Note, it’s not the speed, but the filtered speed that is controlled !
2. The filter time constant is usually much longer than Ttc !
3. Replace the fast torque dynamics with the slow filter dynamics and design as with symmetric optimum on a 3’rd order system.

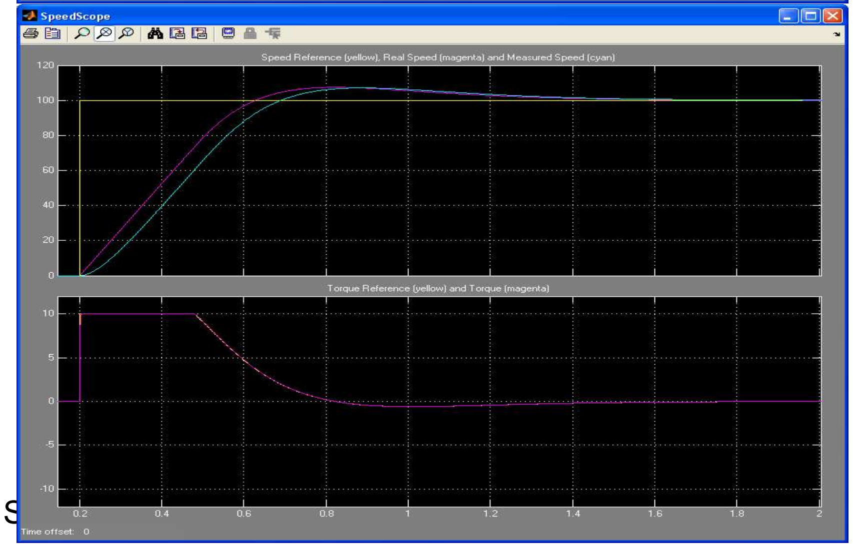
Example 8

- Filtered speed signal, T_f = 50 ms
  - *Non-adjusted parameters*

- Adjust parameters:

$$a = 3$$
$$T_i = 9 \cdot T_f$$
$$K_w = \frac{J}{3 \cdot T_f}$$





That’s all folks...