



Control of Electrical Drives

Summary

PE-Summary

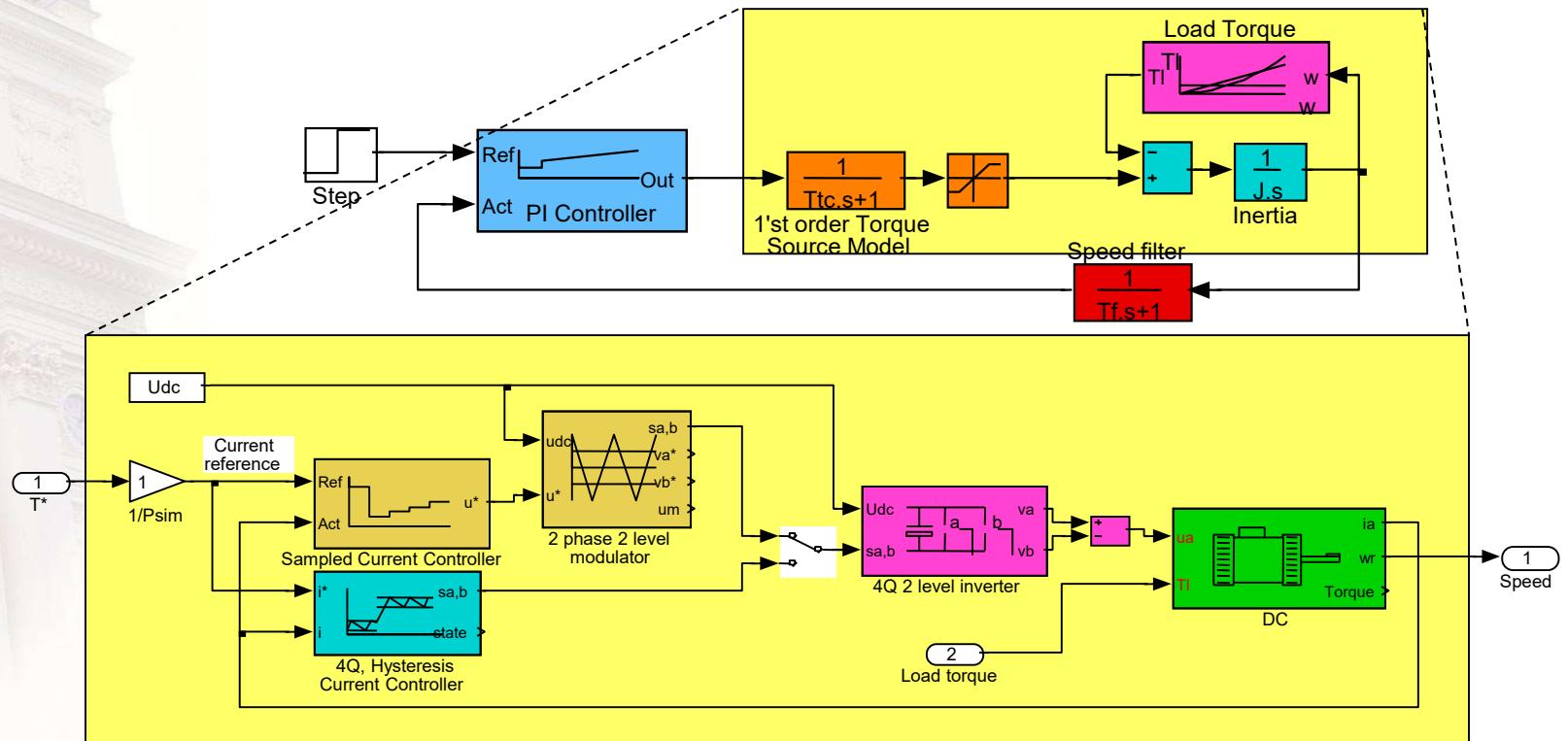




Important Contents

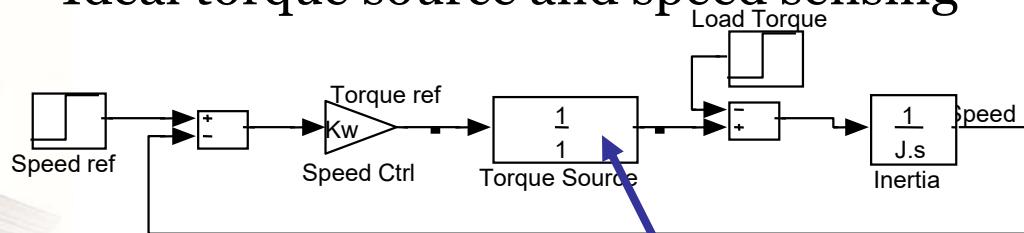
- Speed Control
 - *Ideal/Filtered torque source*
 - *P, PI, Symmetric Optimum*
- Modulation
 - *Principles*
 - *2 quadrant & 4 quadrant DC conv, 3-phase*
- Generic loads
 - *1-phase, 3-phase*
 - *Models, reference frames when applicable*
- Current Control
 - *Sampled, sampling instants vs. carrier wave period*
 - *DCC*
- Grid connected 3-phase
 - *Reference frame, current control*
 - *Active filtering*
 - *DC link voltage control*
- Synchronous Machines
 - *Torque control, including saliency (L_{sx} vs. L_{sy})*
 - *Field weakening*
- Losses & Cooling

Loops



Speed Control

Ideal torque source and speed sensing



Closed system:

$$\frac{\omega}{\omega^*} = \frac{K_w}{sJ + K_w} = \frac{1}{1 + s \frac{J}{K_w}}$$

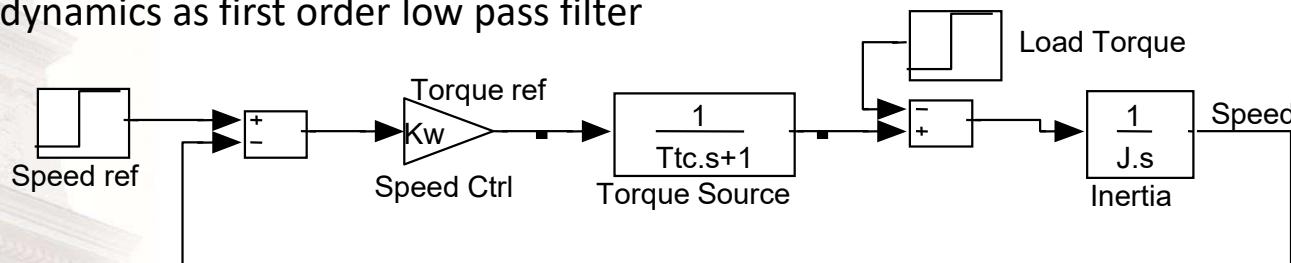
Roots:

$$s = -\frac{K_w}{J}$$

i.e. any bandwidth possible, ... but then is no longer true ...

Speed Control

Torque dynamics as first order low pass filter



Closed system:

$$\omega^* = \frac{\frac{K_w}{J \cdot T_{tc}}}{s^2 + s \cdot \frac{1}{T_{tc}} + \frac{K_w}{J \cdot T_{tc}}} - \frac{1}{2T_{tc}} \pm \sqrt{\frac{1}{4 \cdot T_{tc}^2} - \frac{K_w}{J \cdot T_{tc}}}$$

Roots:

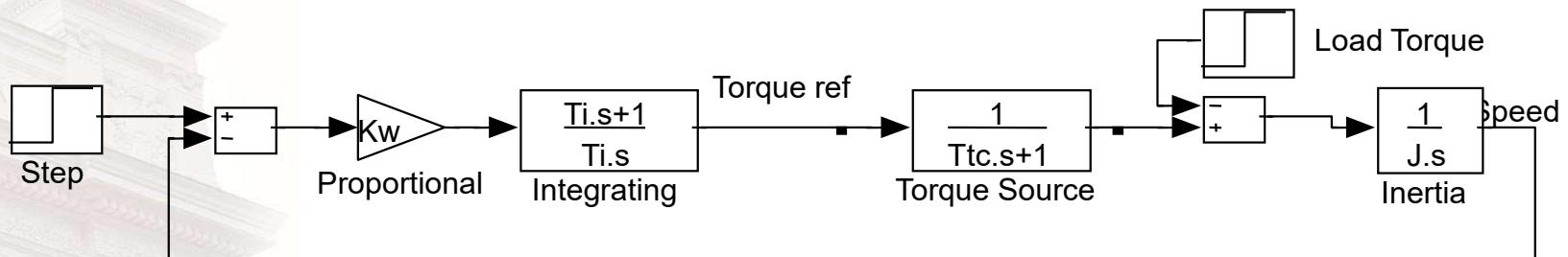
Non osc. roots ->

$$K_w = \frac{J}{4 \cdot T_{tc}}$$

Limited Kw gives
stationary error
with P-control!!

Speed Control

With PI speed controller and 1'st order torque source



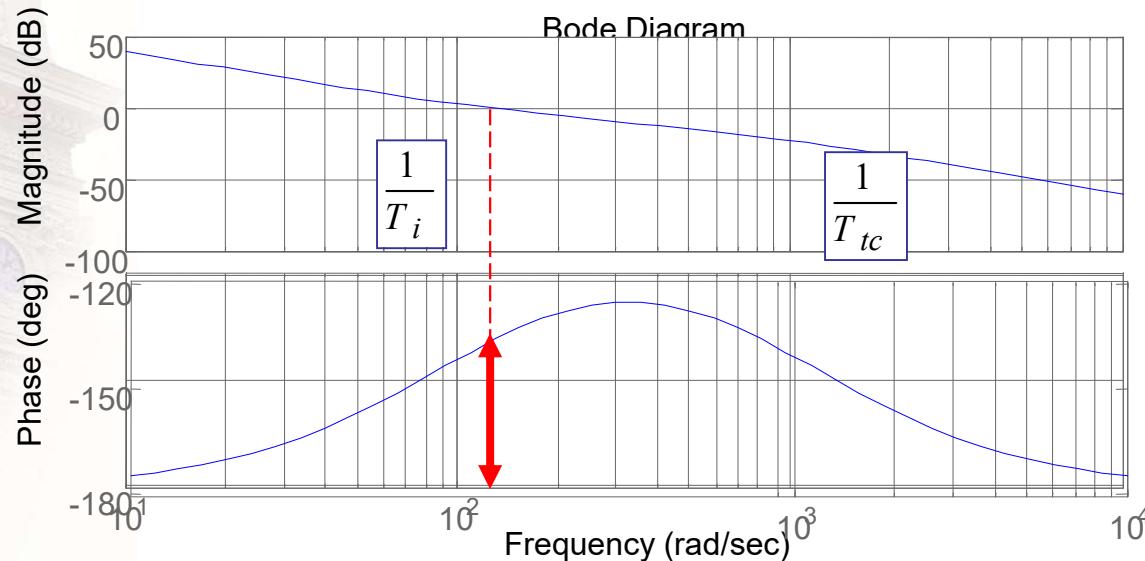
$$\frac{\omega(s)}{\omega^*(s)} = \frac{K_\omega (1 + s \cdot T_i)}{s^3 \cdot J \cdot T_i \cdot T_{tc} + s^2 \cdot J \cdot T_i \cdot T_{tc} + s \cdot K_\omega \cdot T_i + K_\omega}$$

3'rd order, how do we solve for the roots??

Symmetric optimum:1

Open loop transfer function

$$G(s) = K_{\omega} \frac{1 + s \cdot T_i}{s \cdot T_i} \cdot \frac{1}{1 + s \cdot T_{tc}} \cdot \frac{1}{s \cdot J}$$



Select K_{ω} to maximize phase margin

PE-Summary

Symmetric optimum:2

$$\omega_0 = \frac{1}{\sqrt{T_i \cdot T_{tc}}} \quad T_i = a^2 \cdot T_{tc}, \text{ where } a > 1$$

$$\omega_0 = \frac{1}{\sqrt{T_i \cdot T_{tc}}} = \frac{1}{a \cdot T_{tc}} = \frac{a}{T_i}$$

$$|G(j\omega_0)| = \left| K_\omega \frac{1 + j \cdot a}{j \cdot a} \cdot \frac{1}{1 + \frac{j}{a}} \cdot \frac{1}{\frac{j \cdot a}{T_i} \cdot J} \right| = K_\omega \cdot \frac{T_i}{a \cdot J} = 1$$

$$K_\omega = \frac{a \cdot J}{T_i} = \frac{J}{a \cdot T_{tc}}$$

Symmetric optimum:3

Close loop characteristic equation:

$$s^3 \cdot J \cdot \frac{T_i^2}{a^2} + s^2 \cdot T_i + s \cdot a_i + \frac{a}{T_i} = 0$$

One root:

$$s = -\omega_0 = -\frac{a}{T_i}$$

Polynomial division gives:

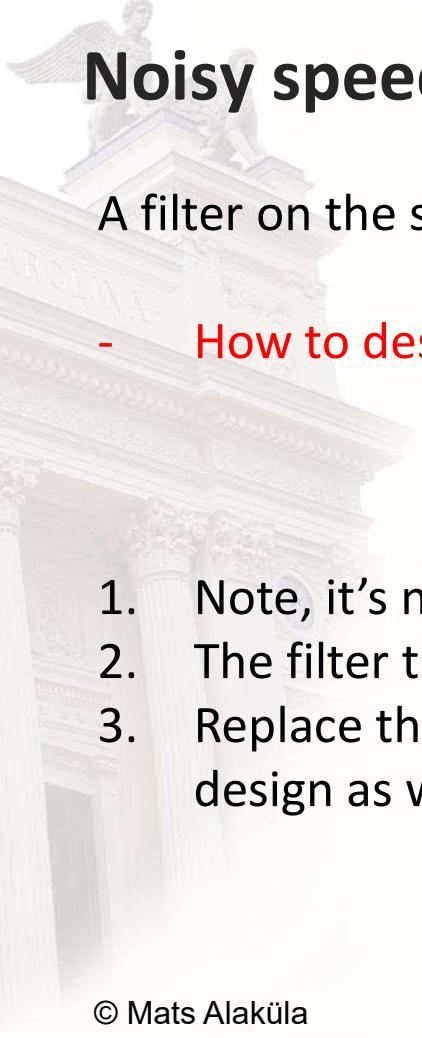
$$s^2 \cdot \frac{T_i^2}{a^2} + s \cdot T_i \cdot \frac{a-1}{a} + 1 = 0$$

Other roots:

$$s_{2,3} = -\omega_0 \left(\zeta \pm \sqrt{1 - \zeta^2} \right) \quad \zeta = \frac{a-1}{2}$$

Example $\zeta=1$, i.e. no complex poles:

$$\begin{aligned} a &= 3 \\ T_i &= 9 \cdot T_{tc} \\ K_w &= \frac{J}{3 \cdot T_{tc}} \end{aligned}$$



Noisy speed signal...

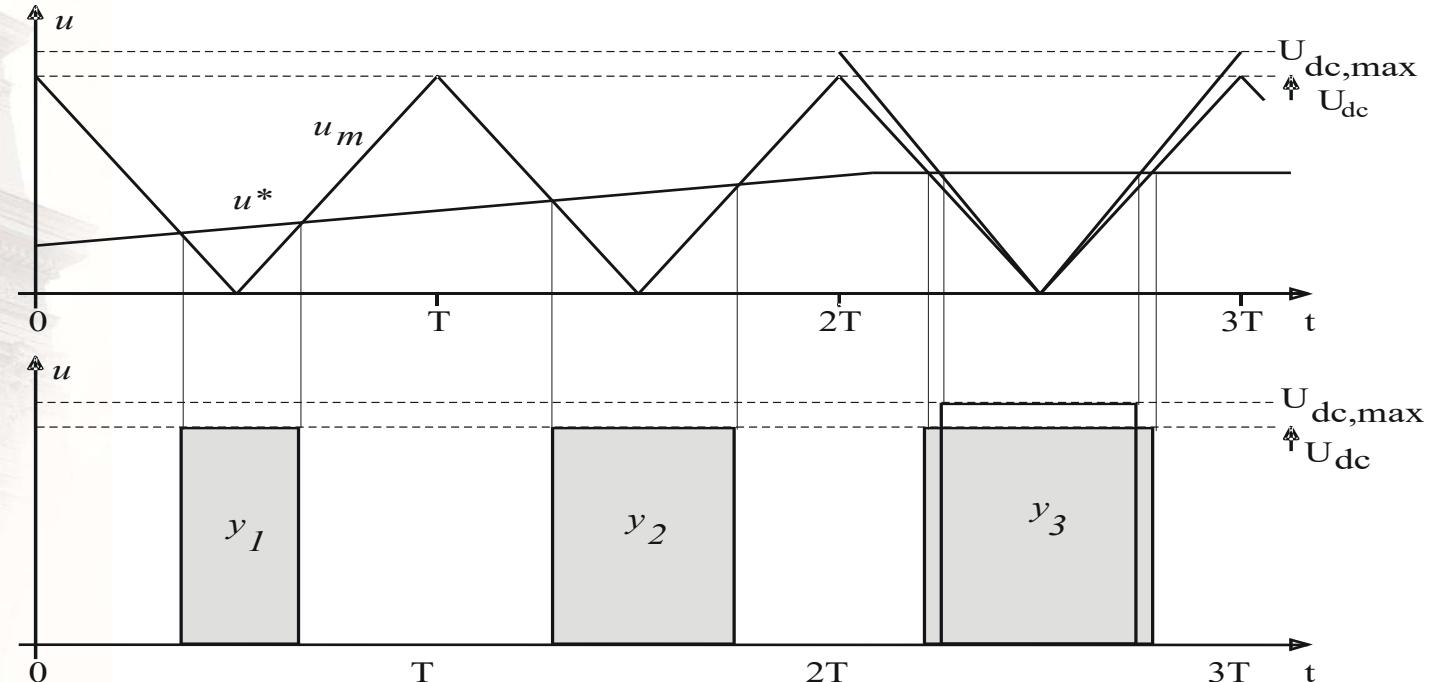
A filter on the speed signal gives a 4'th order system.

- How to design??

The engineering solution:

1. Note, it's not the speed, but the filtered speed that is controlled !
2. The filter time constant is usually much longer than T_{tc} !
3. Replace the fast torque dynamics with the slow filter dynamics and design as with symmetric optimum on a 3'rd order system.

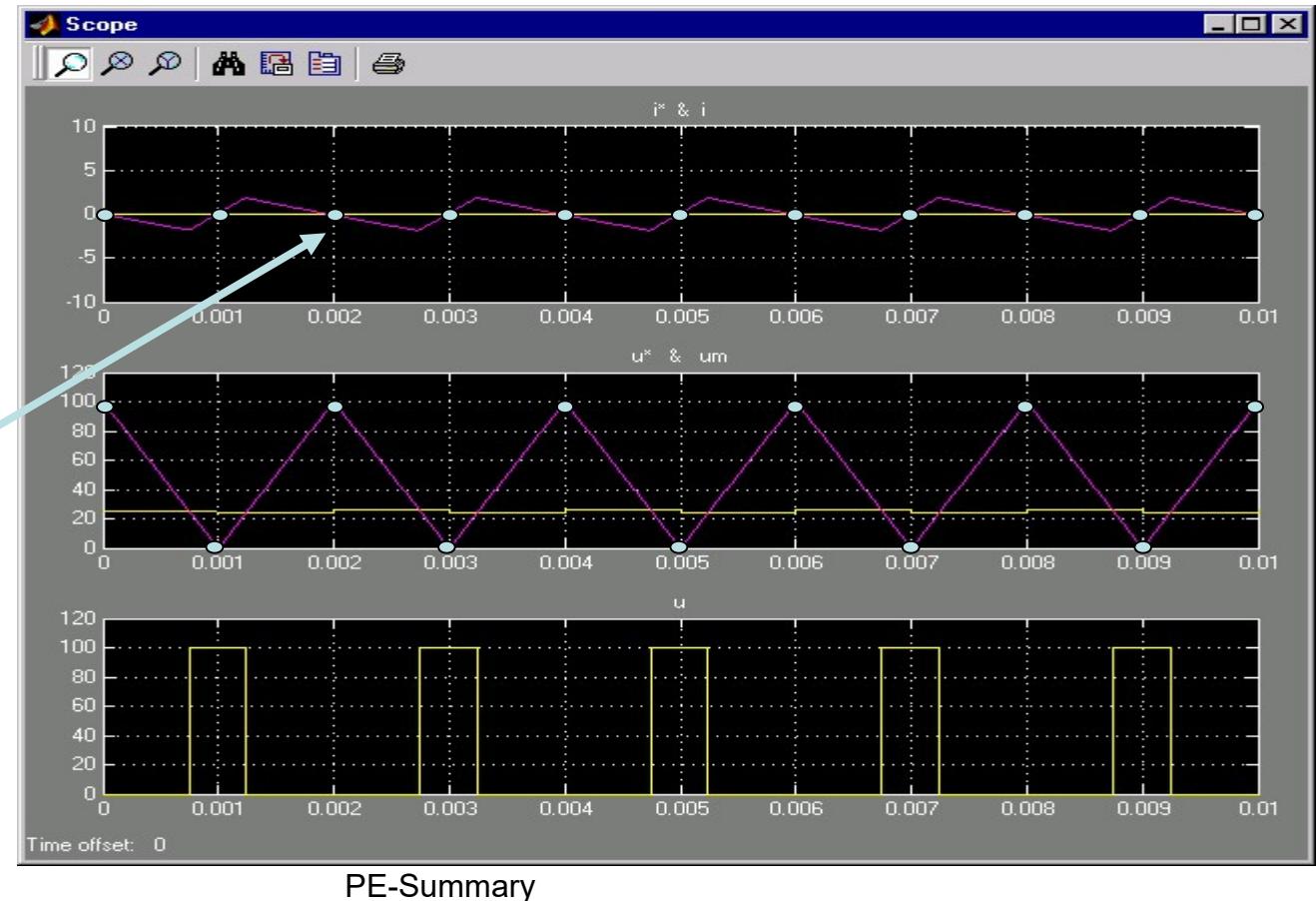
2-quadrant DC converters



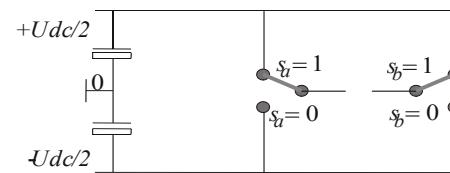
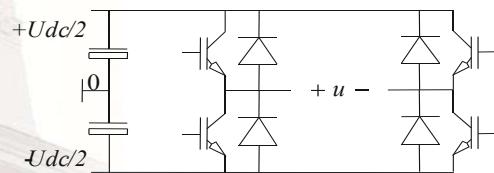
2-quadrant DC converters : III

Current sampling –
how often?

- When the carrier
turns, i.e. With
twice the switching
frequency!



4 – quadrant DC converters



Bridge connected

2 phase potentials, only 1 output voltage = 1 degree of freedom to be used for other purposes.

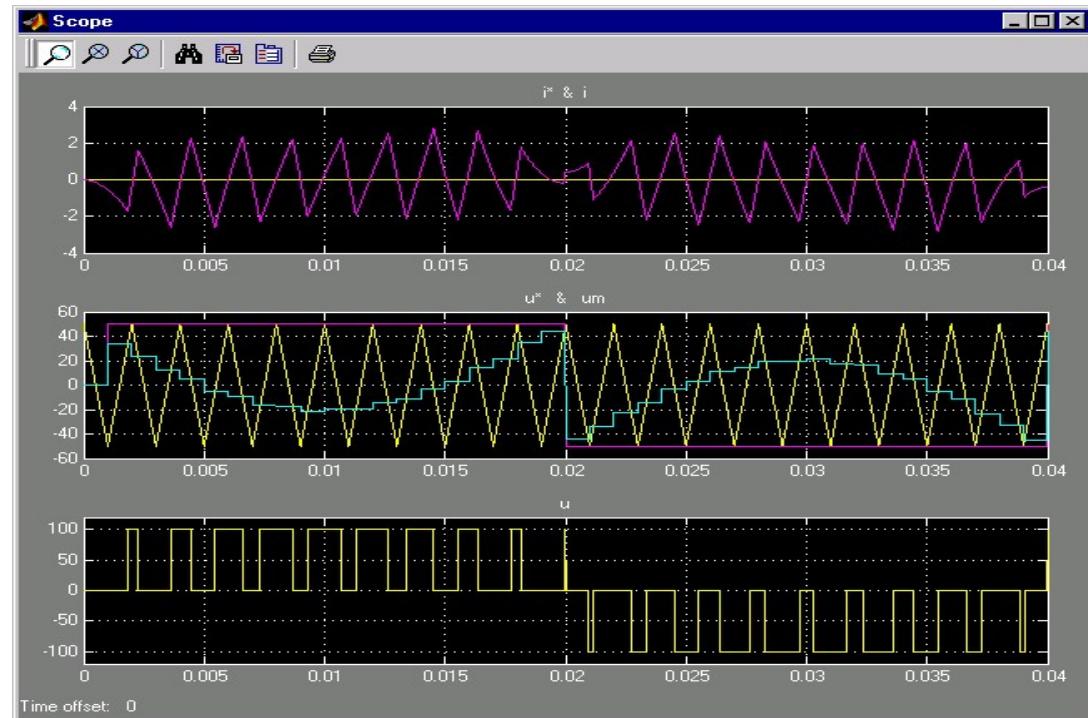
4-quadrant DC converters

$$u^* = v_a^* - v_b^*$$

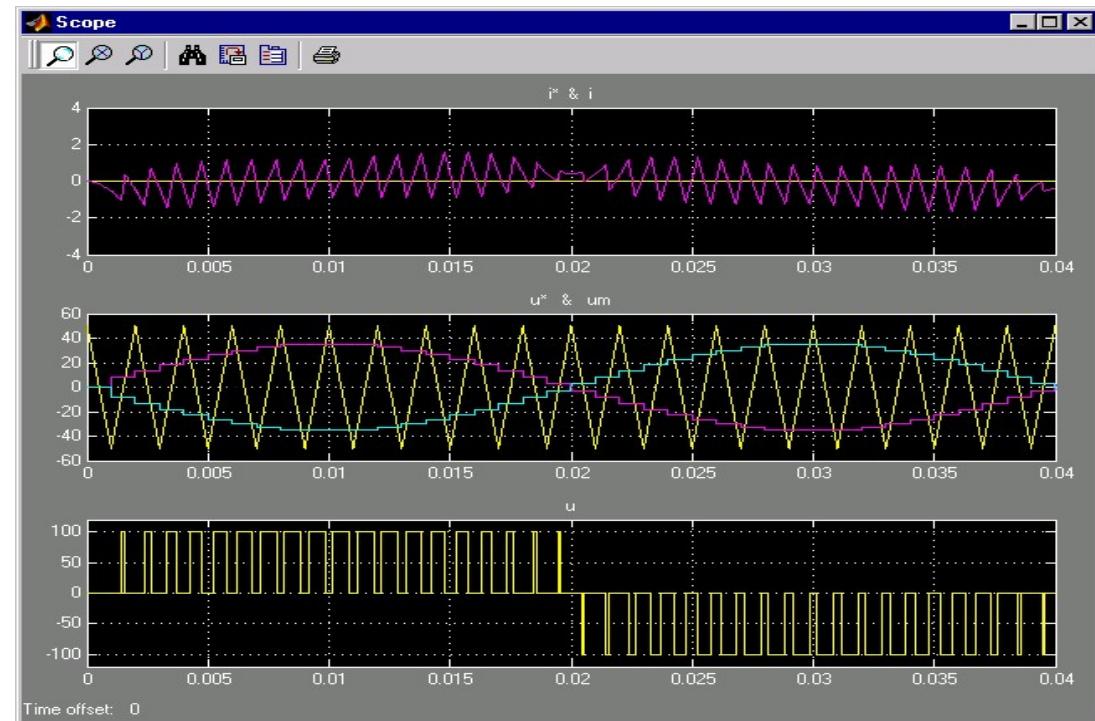
$$\text{alt 1 : } v_a^* = \text{sign}(u^*) \cdot \frac{U_{dc}}{2} \Rightarrow v_b^* = v_a^* - u^* = \text{sign}(u^*) \cdot \frac{U_{dc}}{2} - u^*$$

$$\text{alt 2 : } v_a^* = -v_b^* \Rightarrow v_a^* - v_b^* = 2 \cdot v_a^* \Rightarrow \begin{cases} v_a^* = \frac{u^*}{2} \\ v_b^* = -\frac{u^*}{2} \end{cases}$$

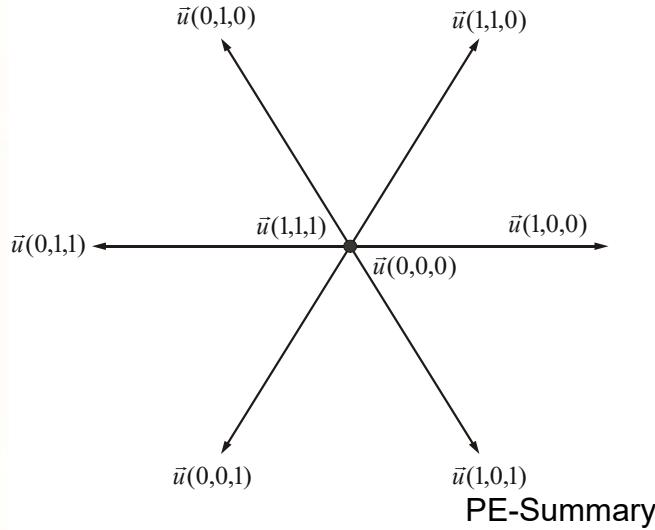
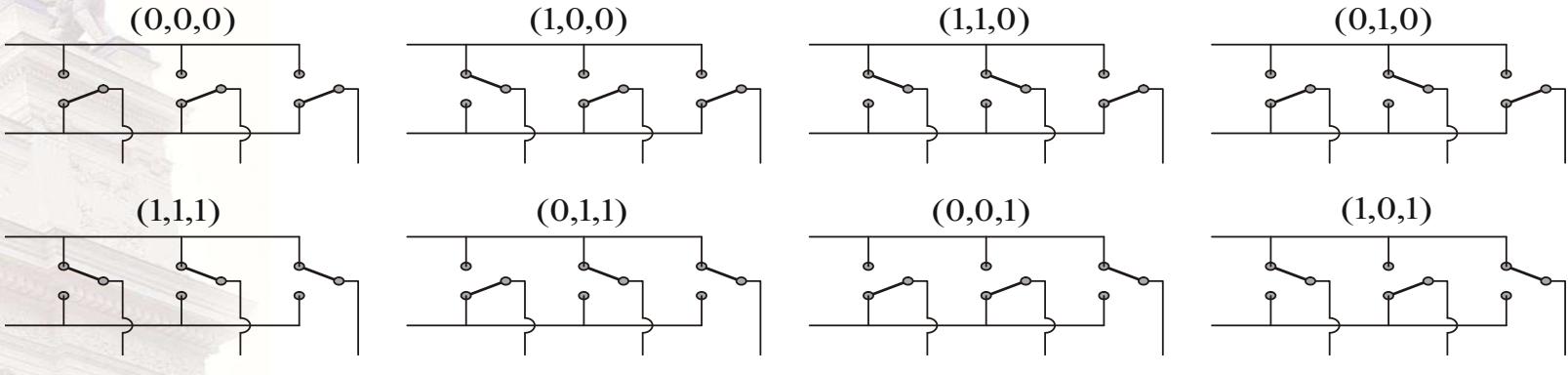
4-quadrant DC converters – alt 1



4-quadrant DC converters – alt 2



3-phase converters – 8 switch states



$$\vec{u}(1,0,0) = \sqrt{\frac{2}{3}}U_{dc} = -\vec{u}(0,1,1)$$

$$\vec{u}(0,1,0) = \sqrt{\frac{2}{3}}U_{dc} \cdot e^{j\frac{2\pi}{3}} = -\vec{u}(1,0,1)$$

$$\vec{u}(0,0,1) = \sqrt{\frac{2}{3}}U_{dc} \cdot e^{j\frac{4\pi}{3}} = -\vec{u}(1,1,0)$$

$$\vec{u}(0,0,0) = 0 = \vec{u}(1,1,1)$$

PE-Summary

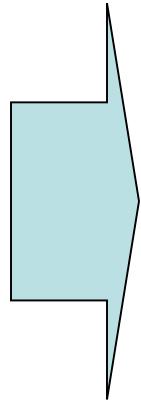
3-phase converters - sinusoidal references

$$\vec{u}^* = u^* \cdot e^{j\omega t} = u^* \cdot (\cos(\omega t) + j \cdot \sin(\omega t)) = u_\alpha^* + j \cdot u_\beta^*$$

$$u_a^* = \sqrt{\frac{2}{3}} u_\alpha^*$$

$$u_b^* = \frac{1}{\sqrt{2}} u_\beta^* - \frac{1}{\sqrt{6}} u_\alpha^*$$

$$u_c^* = -\frac{1}{\sqrt{2}} u_\beta^* - \frac{1}{\sqrt{6}} u_\alpha^*$$



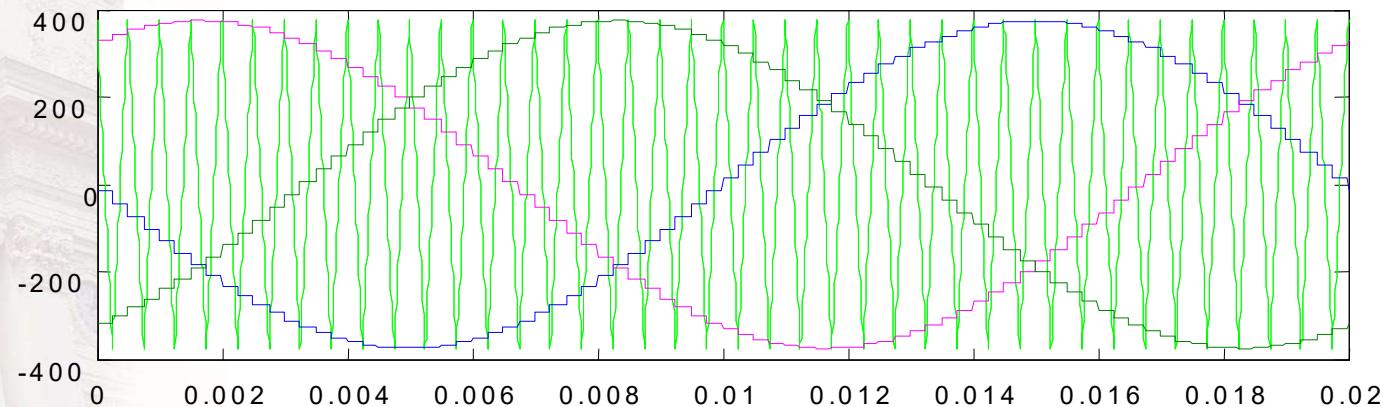
$$u_a^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t)$$

$$u_b^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t - \frac{2\pi}{3})$$

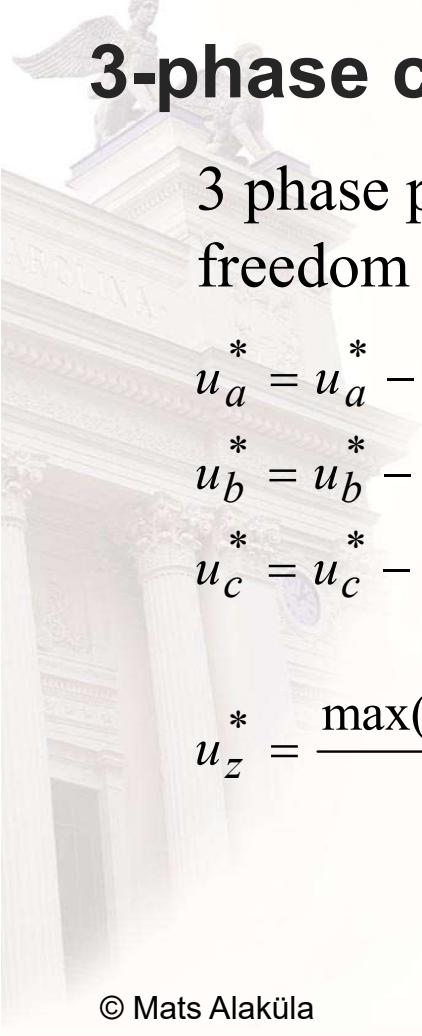
$$u_c^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t - \frac{4\pi}{3})$$

3-phase converters modulation

Simplest with sinusoidal references...



... but the DC link voltage is badly utilized.



3-phase converters – symmetrization

3 phase potentials, only 2 vector components. One degree of freedom to be used for other purposes.

$$u_a^* = u_a^* - u_z^*$$

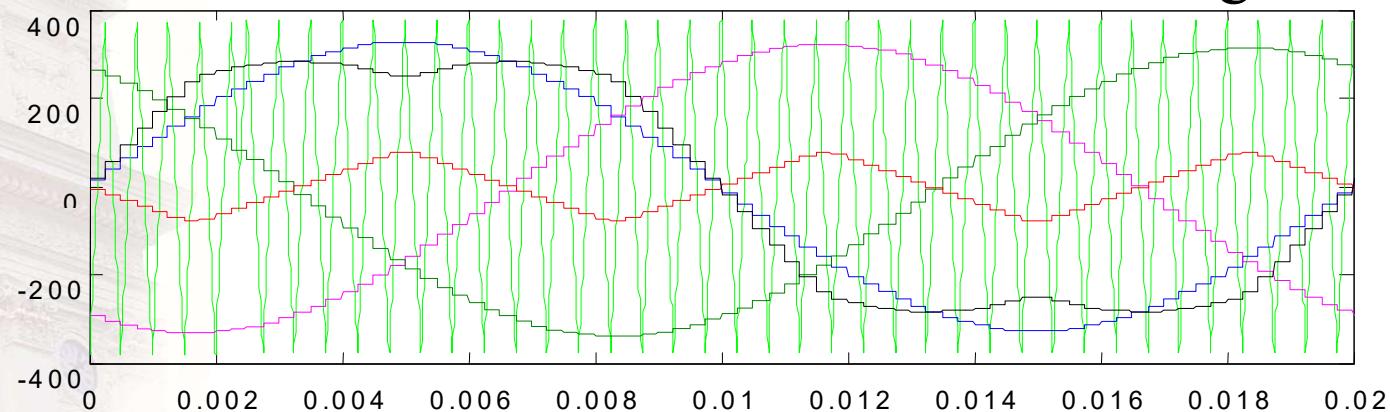
$$u_b^* = u_b^* - u_z^*$$

$$u_c^* = u_c^* - u_z^*$$

$$u_z^* = \frac{\max(u_a^*, u_b^*, u_c^*) + \min(u_a^*, u_b^*, u_c^*)}{2}$$

3-phase symmetrized modulation

Symmetrized modulation utilize the DC link voltage better

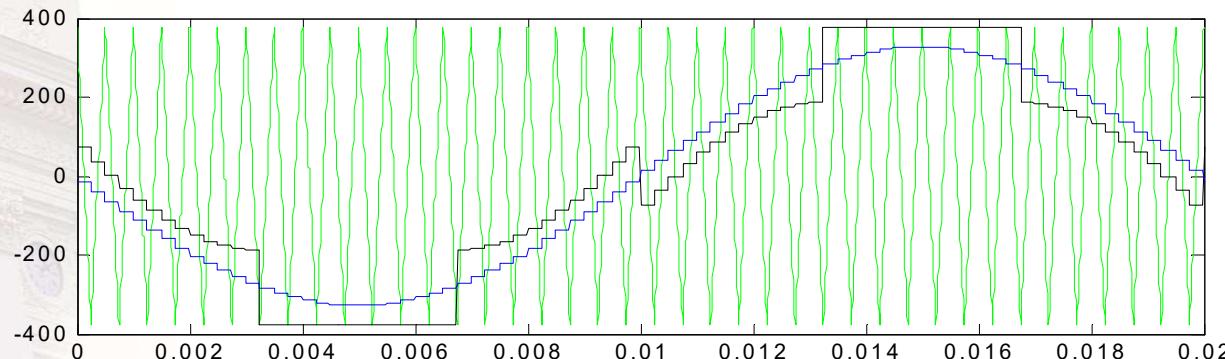


Maximum phase voltage with sinusoidal modulation : $U_{dc}/2$

Maximum phase-to phase voltage with symmetrized modulation : $U_{dc} \rightarrow$ Phase voltage
 $U_{dc}/\sqrt{3}$, i.e. $2/\sqrt{3}=1.15$ times larger than with sinusoidal modulation.

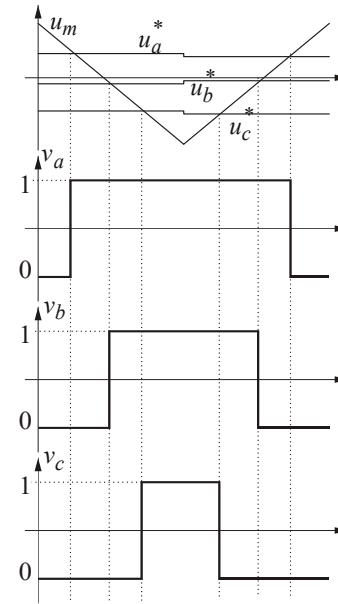
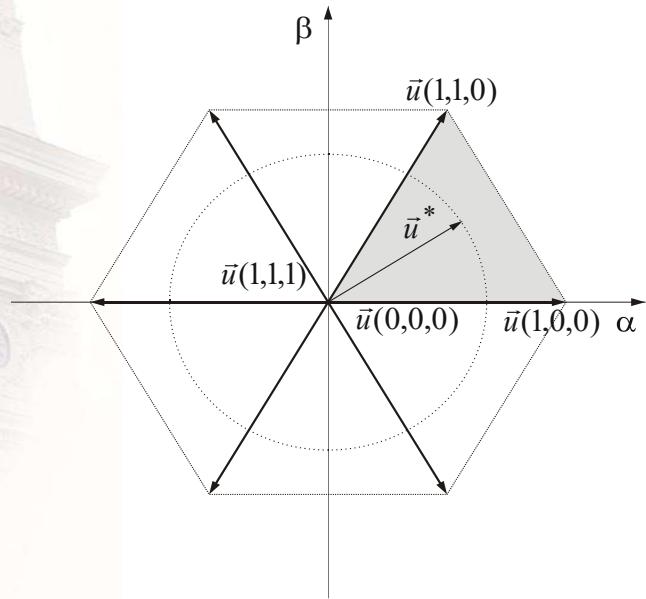
3-phase minimum switching modulation

$$u_z^* = -\min \left(\frac{U_{dc}}{2} - \max(u_a^*, u_b^*, u_c^*) \right) - \frac{U_{dc}}{2} - \min(u_a^*, u_b^*, u_c^*)$$

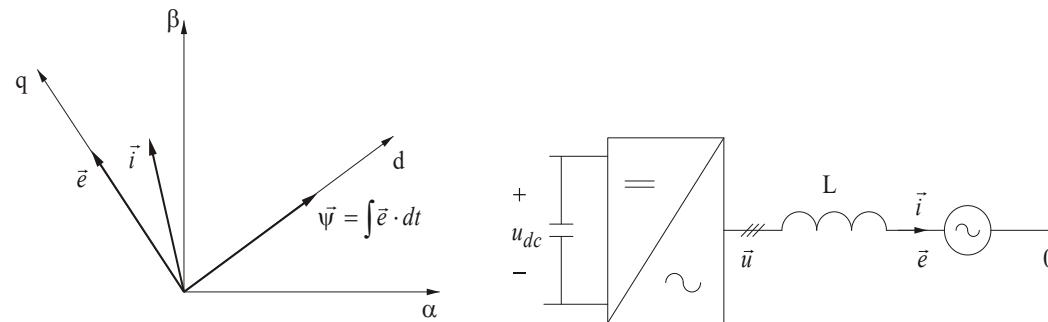


One phase is not switching for 2 60 degree intervals ...

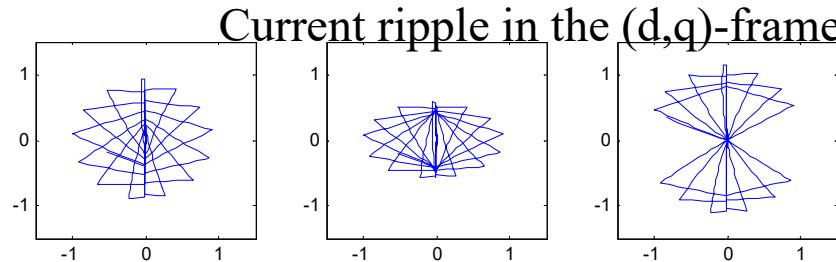
Modulation sequence vs. ripple



Modulation sequence vs. ripple

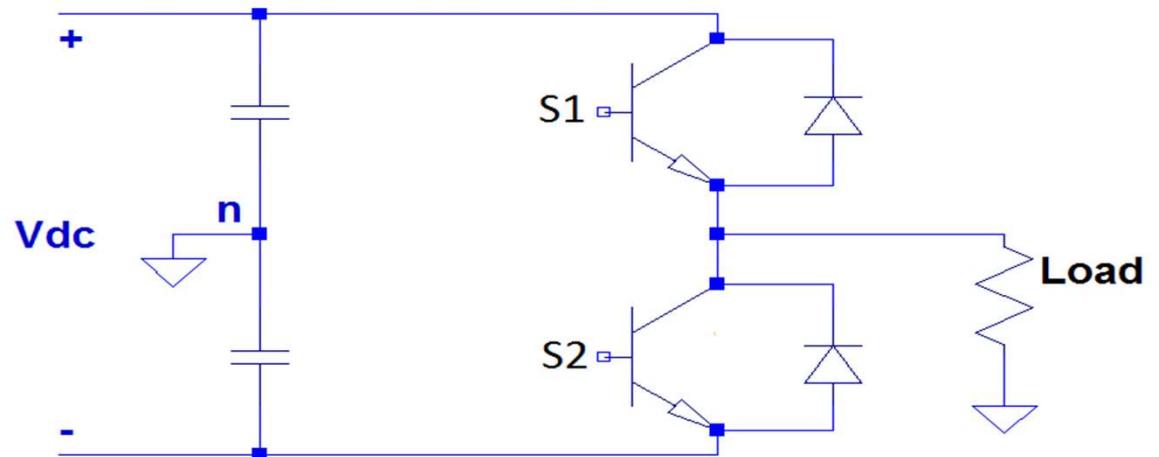


$$\frac{d\vec{i}}{dt} = \frac{\vec{u} - \vec{e}}{L}$$



Conventional 2-level Converter

- Topology reference
- Two level output: $\pm V_{dc}/2$
- High dv/dt ($= \frac{V_{dc}}{t_{sw}}$)
- Few components
- Easy to control
- EMC reducing implementation required



One phase leg of a 2-level inverter

Multilevel Converters

Introduction:

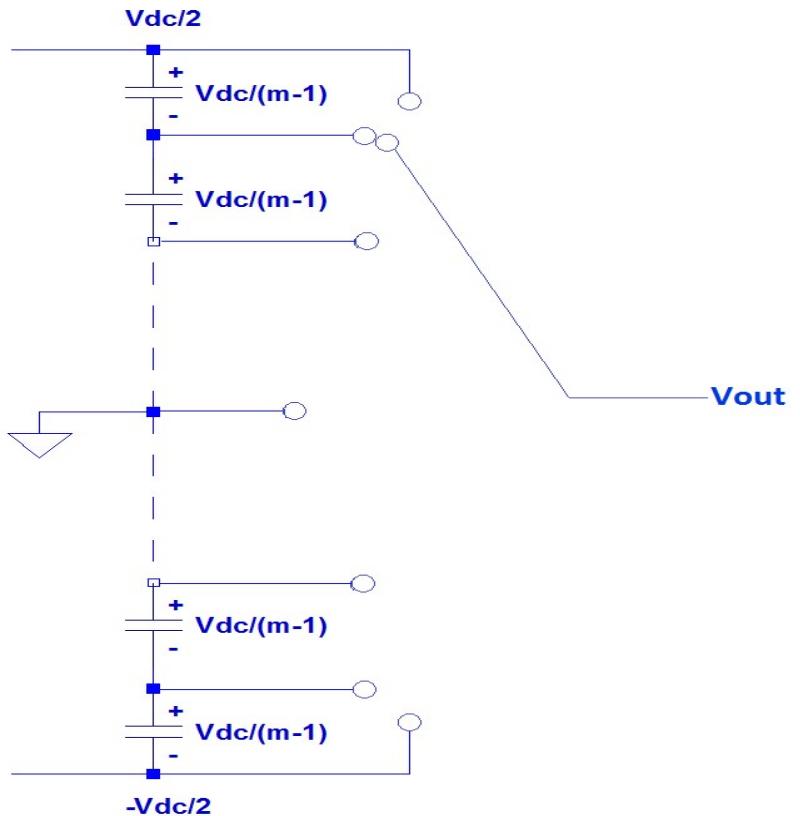
- Inverters with 3+ voltage levels are called multilevel inverters
- $m-1$ capacitors split the DC voltage into m levels ($m-1$ levels in the line voltage)
- The switches select the correct level
- The output only changes 1 level up/down at a time
$$\left(\frac{dv}{dt} = \frac{V_{dc}/(m-1)}{t_{sw}}\right)$$

Example in figure:

Assume $m = 5$ which means 4 capacitors are used to split up the DC voltage.

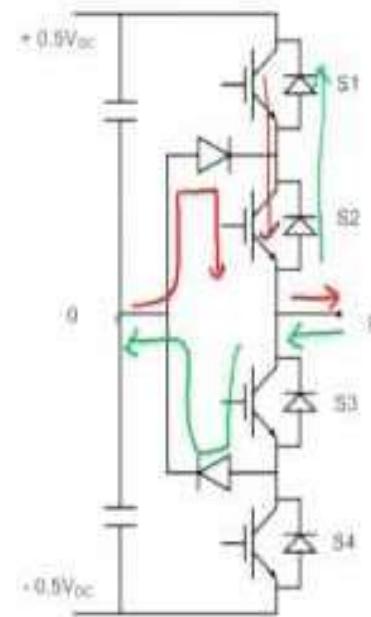
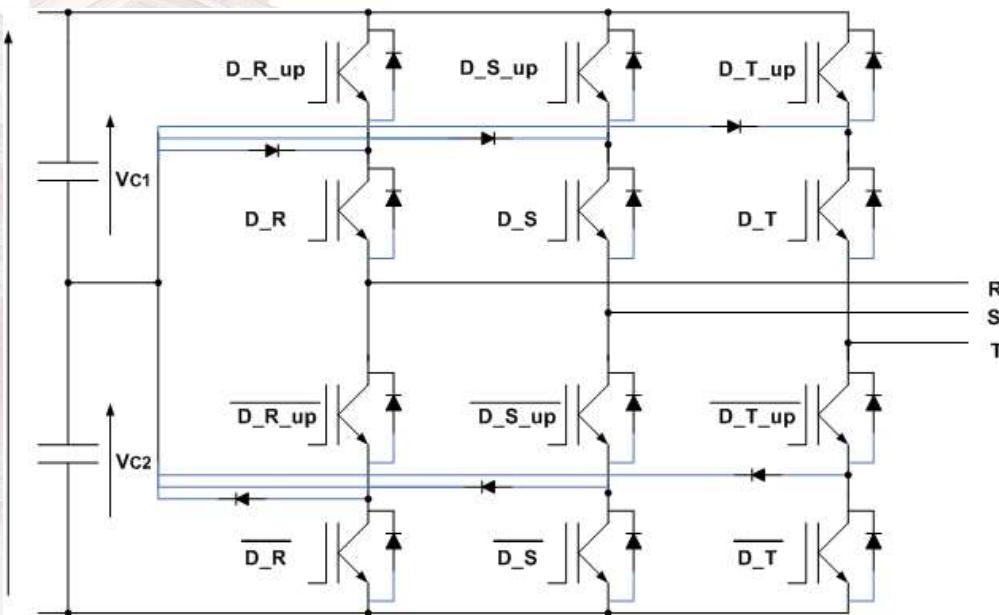
Then the output shown in the figure is:

$$V_{out} = \frac{V_{dc}}{2} - \frac{V_{dc}}{4} = \frac{V_{dc}}{4}$$



Simplified m -level inverter

3-level in(con)verter

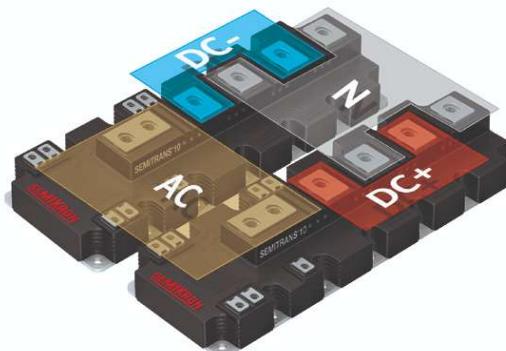


S1 and S3 are complementary
S2 and S4 are complementary

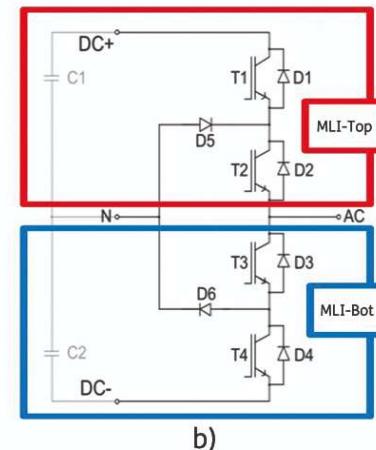
S_1	S_2	S_3	S_4	$\frac{\sqrt{R_D}}{0.5V_{dc}}$
1	1	0	0	$0.5V_{dc}$
0	1	1	0	0
0	0	1	1	$-0.5V_{dc}$

Example from Semikron

- SEMITRANS 10 MLI
- ...for these type of inverters SEMIKRON introduced the SEMITRANS 10 MLI modules where the NPC topology is split to two halves. With current rating of 1200A and the use of 1200V medium power IGBT chips in combination with SEMIKRON CAL4F diodes SEMITRANS 10 MLI enables air cooled power blocks up to 750kW without paralleling of modules.



a)



b)

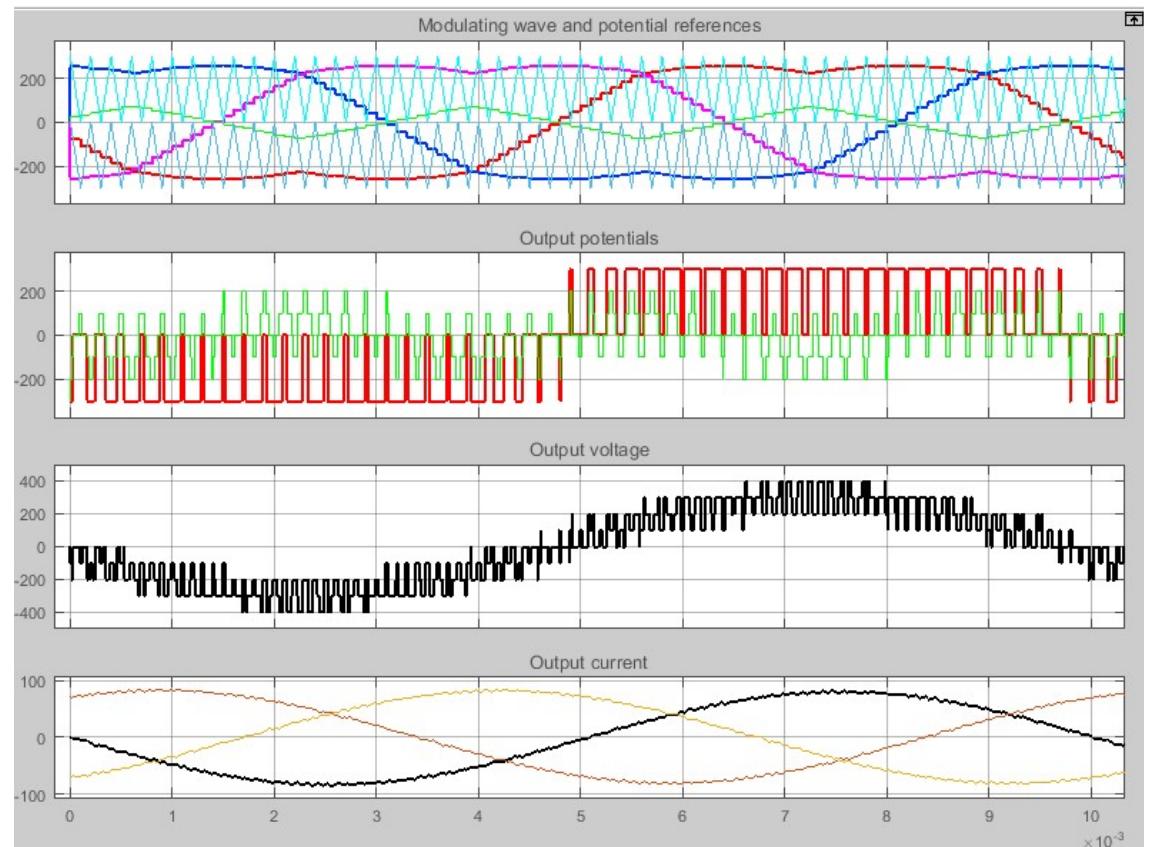
Product range

Half-bridges 1200V / 1400A and 1700V 1000A/1400A
MLI 1200V/1200A

<https://www.semikron.com/zh/about-semikron/technical-articles/module-solutions-for-1500v-solar-inverters.html>

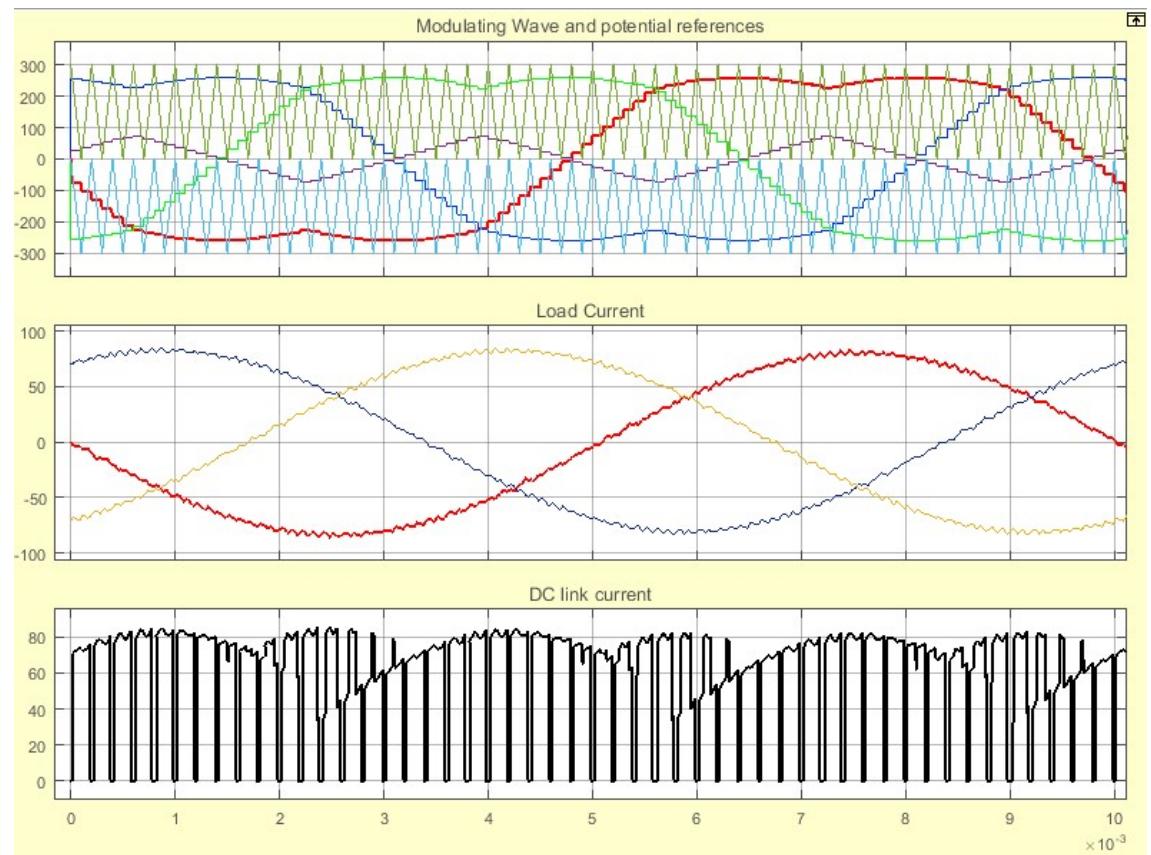
Example: 3-level converter

- Two modulating waves:
 - One between upper and mid
 - One between mid and lower
- The rest is the same!

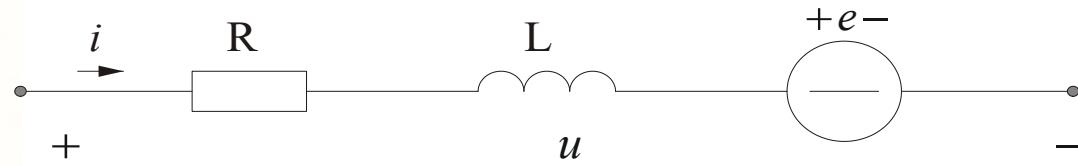


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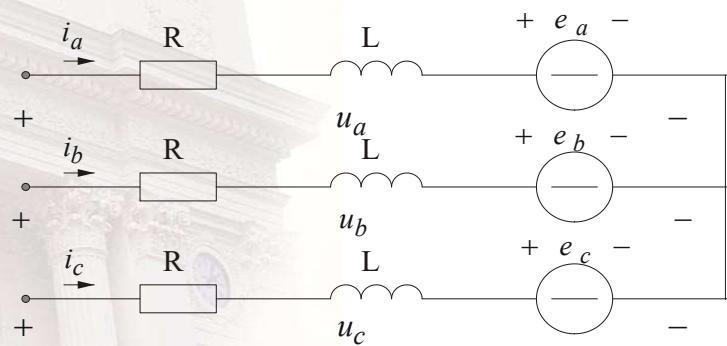


Generic 1 phase load



$$u = R \cdot i + L \cdot \frac{di}{dt} + e$$

Generic 3-phase load



$$\begin{aligned} & \sqrt{\frac{2}{3}} \left(u_a = R \cdot i_a + L \cdot \frac{di_a}{dt} + e_a \right) \\ & \sqrt{\frac{2}{3}} \cdot e^{j\frac{2\pi}{3}} \cdot \left(u_b = R \cdot i_b + L \cdot \frac{di_b}{dt} + e_b \right) \\ & + \sqrt{\frac{2}{3}} \cdot e^{-j\frac{2\pi}{3}} \cdot \left(u_c = R \cdot i_c + L \cdot \frac{di_c}{dt} + e_c \right) \end{aligned}$$

$$\vec{u} = R \cdot \vec{i} + L \cdot \frac{d\vec{i}}{dt} + \vec{e}$$

Current Controller with a fast computer – II

$$\begin{aligned} u^*(k) &= R \cdot \frac{i^*(k) + i(k)}{2} + L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \\ &= R \cdot \frac{i^*(k) - i(k)}{2} + R \cdot i(k) + L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \\ &= \left(\frac{L}{T_s} + \frac{R}{2} \right) (i^*(k) - i(k)) + R \cdot \sum_{n=0}^{n=k-1} (i^*(n) - i(n)) + e(k) = \\ &= \left(\frac{L}{T_s} + \frac{R}{2} \right) \cdot \underbrace{\left(\underbrace{(i^*(k) - i(k))}_{\text{Proportional}} + \underbrace{\left(\frac{T_s}{R + \frac{T_s}{2}} \cdot \sum_{n=0}^{n=k-1} (i^*(n) - i(n)) \right)}_{\text{Integral}} \right)}_{\text{Feed forward}} + \underbrace{e(k)}_{\text{Feed forward}} \end{aligned}$$

3-phase sampled vector control : 2

Assume sampled control @ [...] Ts

Calculate voltage average over one sample period

$$\frac{\int_{k \cdot T_s}^{(k+1)T_s} \vec{u} \cdot dt}{T_s} = \frac{R \cdot \int_{k \cdot T_s}^{(k+1)T_s} \vec{i} \cdot dt + L \cdot \int_{k \cdot T_s}^{(k+1)T_s} \frac{d\vec{i}}{dt} \cdot dt + j \cdot \omega \cdot L \int_{k \cdot T_s}^{(k+1)T_s} \vec{i} \cdot dt + \int_{k \cdot T_s}^{(k+1)T_s} \vec{e} \cdot dt}{T_s} = \\ = \bar{\vec{u}}(k, k+1) = (R + j \cdot \omega \cdot L) \cdot \bar{\vec{i}}(k, k+1) + L \cdot \frac{\bar{\vec{i}}(k+1) - \bar{\vec{i}}(k)}{T_s} + \bar{\vec{e}}(k, k+1)$$

3-phase sampled vector control : 3

Assume:

$$\vec{u}(k, k+1) = \vec{u}^*(k) \quad (a)$$

$$\vec{i}(k+1) = \vec{i}^*(k) \quad (b)$$

$$\vec{i}(k, k+1) = \frac{\vec{i}^*(k) + \vec{i}(k)}{2} \quad (c)$$

$$\vec{e}(k, k+1) = \vec{e}(k) \quad (d)$$

$$\vec{i}(k) = \sum_{n=0}^{n=k-1} (\vec{i}^*(n) - \vec{i}(n)) \quad (e)$$

Gives:

$$\begin{aligned}
 \vec{u}^*(k) &= (R + j \cdot \omega \cdot L) \cdot \frac{\vec{i}^*(k) + \vec{i}(k)}{2} + L \cdot \frac{\vec{i}^*(k) - \vec{i}(k)}{T_s} + \vec{e}(k) = \\
 &= R \cdot \frac{\vec{i}^*(k) - \vec{i}(k)}{2} + R \cdot \vec{i}(k) + L \cdot \frac{\vec{i}^*(k) - \vec{i}(k)}{T_s} + j \cdot \omega \cdot L \cdot \frac{\vec{i}^*(k) + \vec{i}(k)}{2} + \vec{e}(k) \approx \\
 &\approx \left(\frac{L}{T_s} + \frac{R}{2} \right) (\vec{i}^*(k) - \vec{i}(k)) + R \cdot \sum_{n=0}^{n=k-1} (\vec{i}^*(n) - \vec{i}(n)) + j \cdot \omega \cdot L \cdot \vec{i}(k) + \vec{e}(k) = \\
 &= \left(\frac{L}{T_s} + \frac{R}{2} \right) \cdot \underbrace{\left(\vec{i}^*(k) - \vec{i}(k) \right)}_{\text{Proportion al}} + \underbrace{\left(\frac{L}{R} + \frac{T_s}{2} \right) \cdot \sum_{n=0}^{n=k-1} (\vec{i}^*(n) - \vec{i}(n))}_{\text{Integral}} + \underbrace{j \cdot \omega \cdot L \cdot \vec{i}(k) + \vec{e}(k)}_{\text{Feed forward}}
 \end{aligned}$$

3-phase sampled vector control : 4

Components:

$$u_d^*(k) = \left(\frac{L}{T_s} + \frac{R}{2} \right) \cdot \left((i_d^*(k) - i_d(k)) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2} \right)} \cdot \sum_{n=0}^{n=k-1} (i_d^*(n) - i_d(n)) \right) - \omega \cdot L \cdot i_q(k)$$

$$u_q^*(k) = \left(\frac{L}{T_s} + \frac{R}{2} \right) \cdot \left((i_q^*(k) - i_q(k)) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2} \right)} \cdot \sum_{n=0}^{n=k-1} (i_q^*(n) - i_q(n)) \right) + \omega \cdot L \cdot i_d(k) + e_q(k)$$

2-Quadrant Direct Current Controller

$$s = \begin{cases} 1 & \text{if } i < i^* - \frac{\Delta i}{2} \\ -1 & \text{if } i > i^* + \frac{\Delta i}{2} \\ s & \text{if } i^* - \frac{\Delta i}{2} < i < i^* + \frac{\Delta i}{2} \end{cases}$$

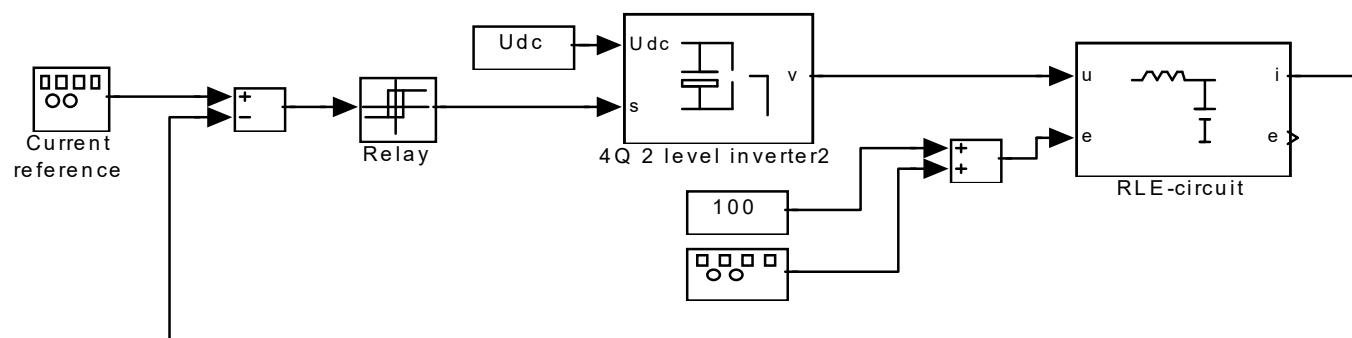
$$L = 10 \text{ mH}$$

$$R = 1 \Omega$$

$$T_s = 0.5 \text{ ms}$$

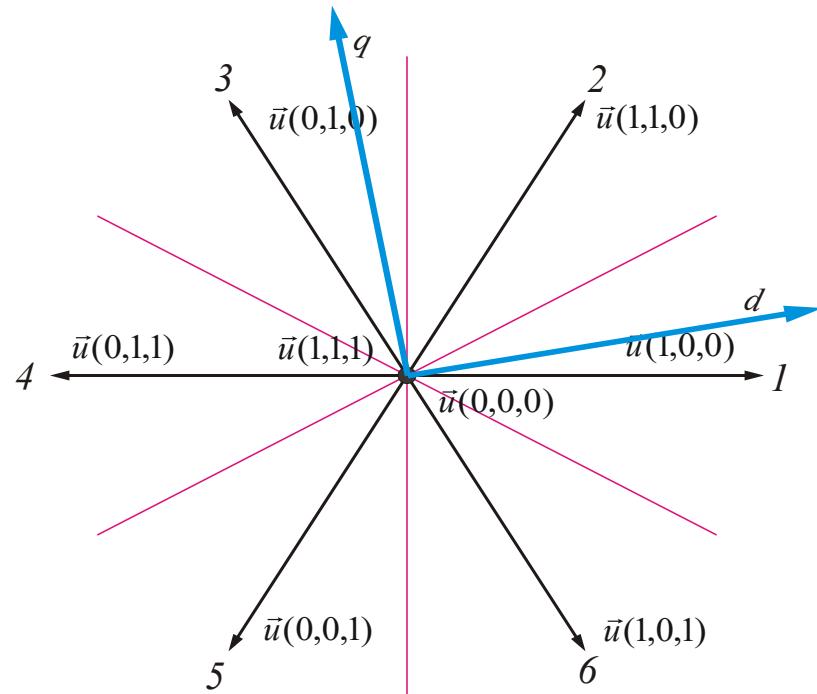
$$U_{dc} = 100 \text{ V}$$

$$\Delta i = 3 \text{ A}$$

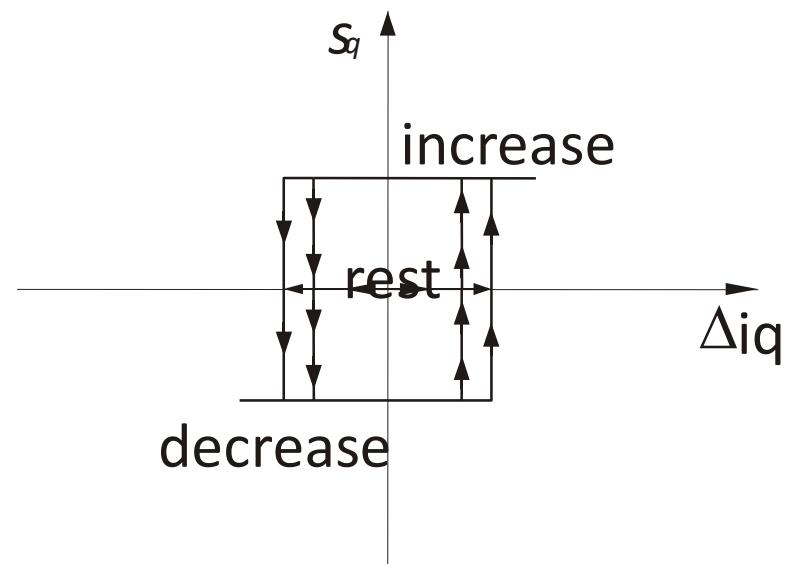
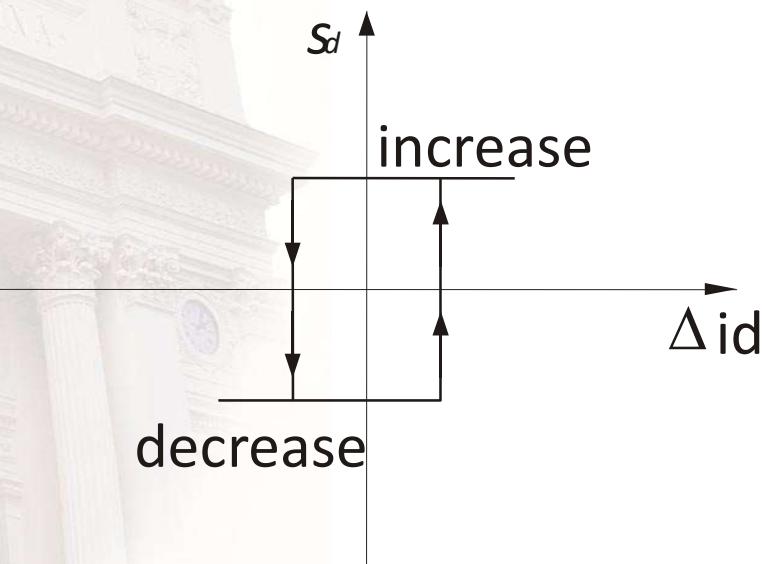


3-phase DCC

$$\frac{d\vec{i}}{dt} = \frac{\vec{u} - R \cdot \vec{i} - \vec{e}}{L}$$



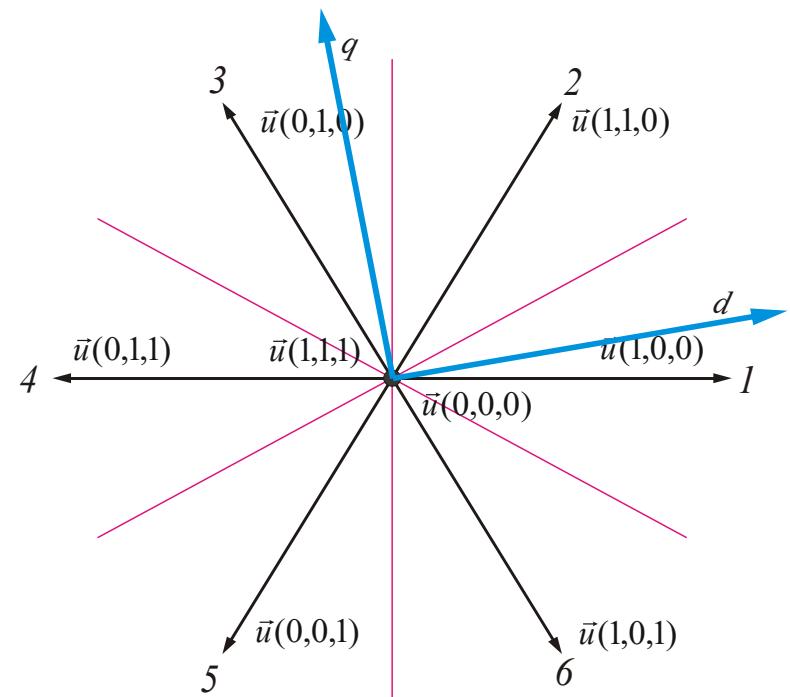
Tolerance bands in d - and q -



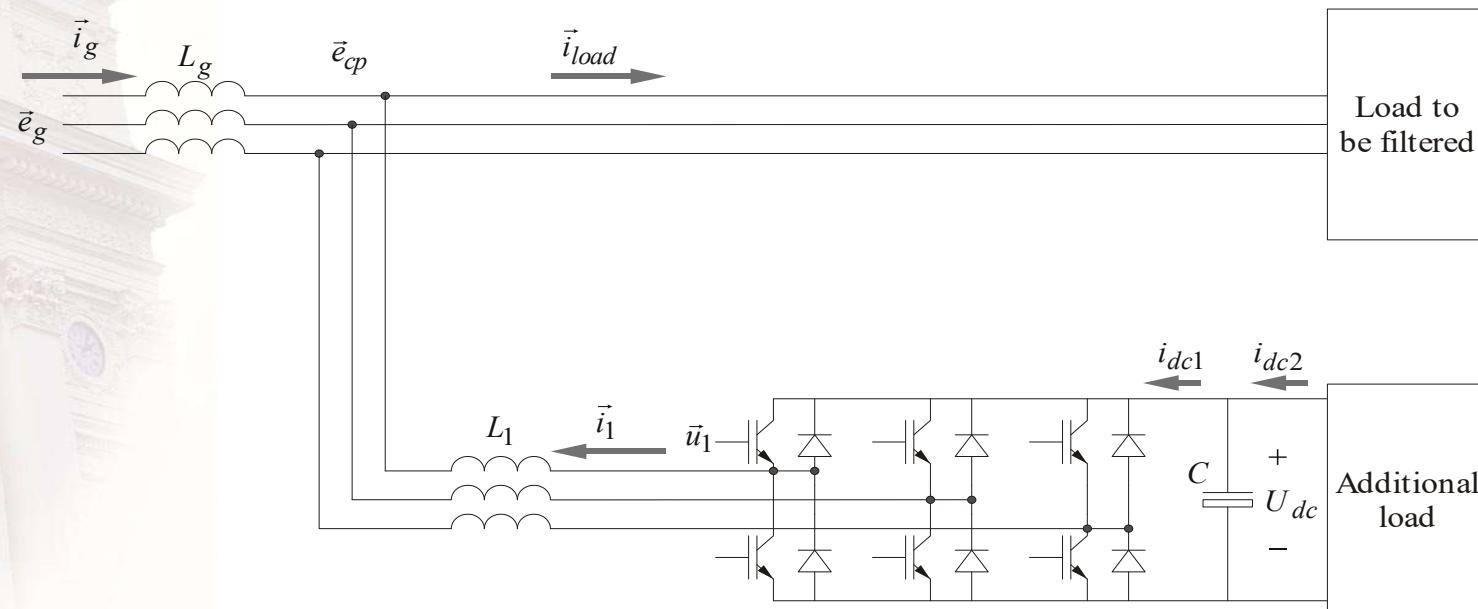
Selecting the right vector

$$vector = sector + s_{offset}$$

s_{offset}	Decrease iq	Increase iq
Decrease id	4	2
Increase id	5	1



Active Filter

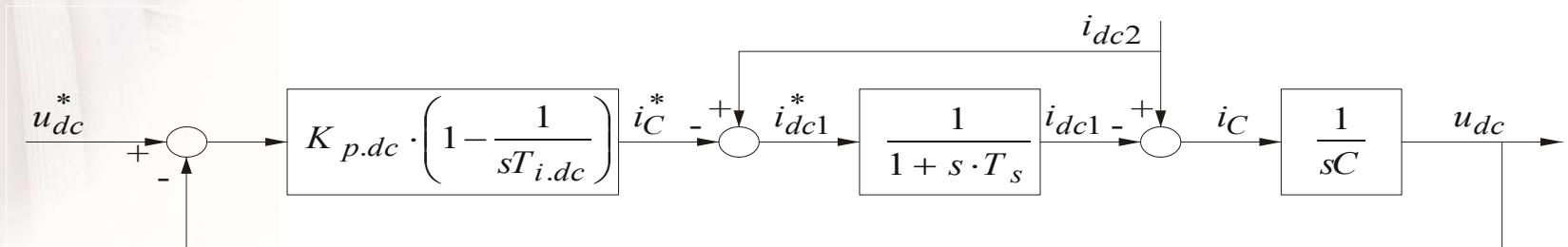
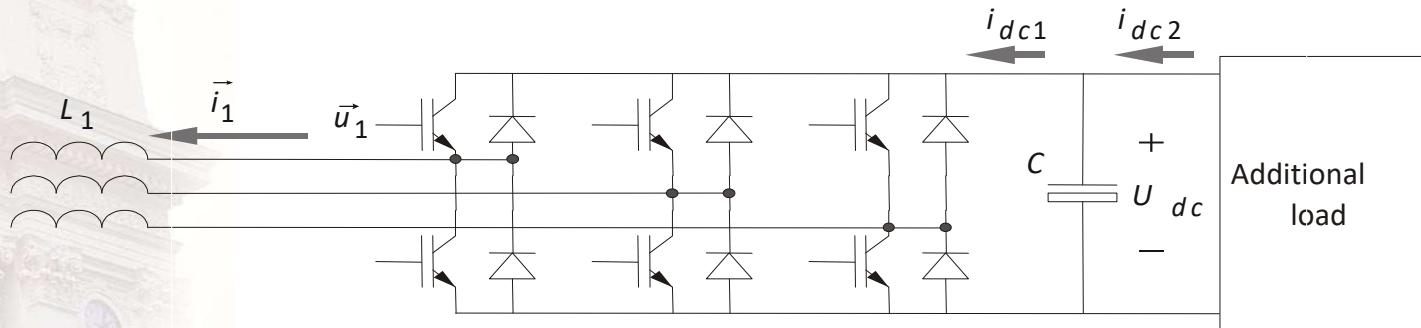


AC side Current Control

- **Vector Control with Field Orientation**

$$\vec{u}_1^*(k) = \left(\frac{L_1}{T_s} + \frac{R_1}{2} \right) \cdot \left(\left(\vec{i}_1^*(k) - \hat{\vec{i}}_1(k) \right) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2} \right)} \cdot \sum_{n=0}^{n=k-1} \left(\vec{i}_1^*(n) - \hat{\vec{i}}_1(n) \right) \right) + \hat{\vec{e}}_{cp}(k)$$

DC link Voltage Control System





Controller Parameters ...

- Use Symmetric Optimum

$$\zeta = \frac{a-1}{2}$$

$$T_{i.dc} = a^2 \cdot T_s, \text{ where } a > 1$$

$$K_{p.dc} = \frac{a \cdot C}{T_{i.dc}}$$

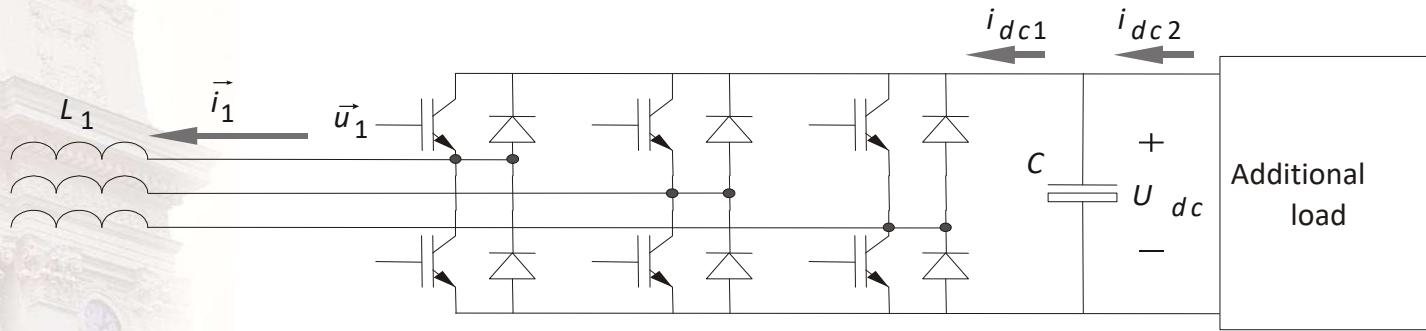
Convert DC to AC current references

$$p(t) = R i_{1d}^2 + R i_{1q}^2 + L \frac{di_{1d}}{dt} i_{1d} + L \frac{di_{1q}}{dt} i_{1q} + e_{cp.q} i_{1q} = u_{dc} \cdot i_{dc1} \approx e_{cp.q} i_{1q}$$

↓

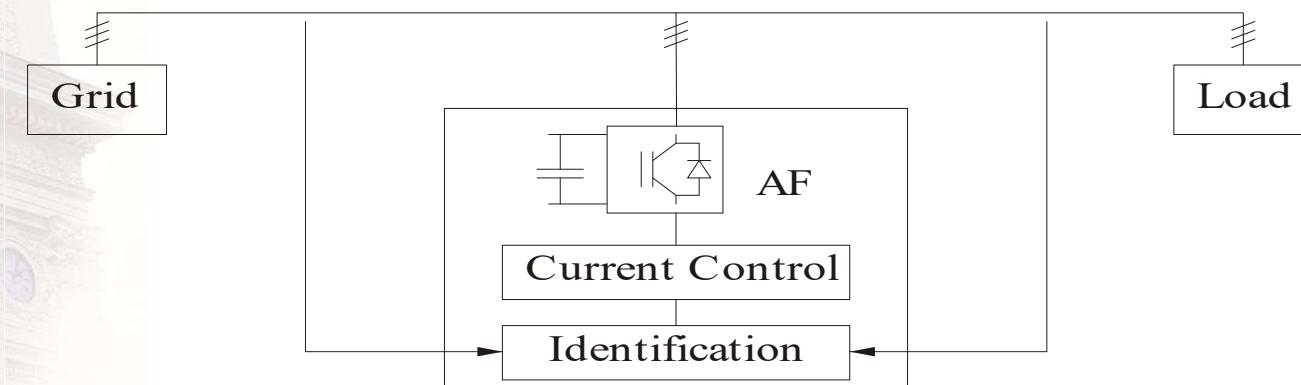
$$i_{dc1} = \frac{e_{cp.q}}{u_{dc}} \cdot i_{1q}$$

DC link voltage controller



$$i_{1q}^* = \frac{u_{dc}}{e_{cp}} \left(i_{dc2} - K_{p.dc} \cdot \left(1 - \frac{1}{sT_{i.dc}} \right) \cdot \left(u_{dc}^* - u_{dc} \right) \right)$$

Active filter control

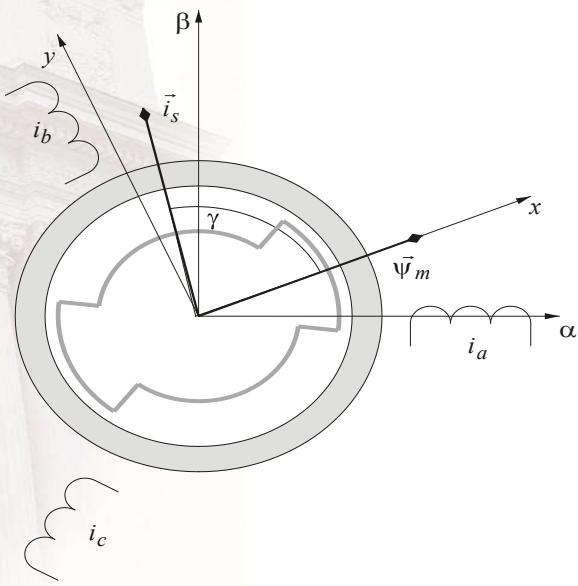


Filter Current References

$$i_{d,ActiveFilter}^* = i_{d,load}$$

$$i_{q,ActiveFilter}^* = i_{q,load} \cdot \frac{s \cdot T_f}{1 + s \cdot T_f} + \frac{u_{dc}}{e_{cp}} \left(i_{dc2} - K_{p.dc} \cdot \left(1 - \frac{1}{sT_{i.dc}} \right) \cdot (u_{dc}^* - u_{dc}) \right) \cdot \frac{1}{1 + s \cdot T_f}$$

PMSM - Mathematical Model



$$\vec{u}_s^{\alpha\beta} = R_s \cdot \vec{i}_s^{\alpha\beta} + \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = R_s \cdot \vec{i}_s^{\alpha\beta} + \frac{d}{dt} (\vec{\psi}_\delta^{\alpha\beta} + L_{s\lambda} \cdot \vec{i}_s^{\alpha\beta})$$

$$\vec{u}_s^{xy} = R_s \cdot \vec{i}_s^{xy} + \frac{d}{dt} (\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}) + j\omega_r \cdot (\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy})$$

$$\psi_{sx} = \psi_m + L_{sx} \cdot i_{sx} = \psi_m + (L_{mx} + L_{s\lambda}) \cdot i_{sx}$$

$$\psi_{sy} = L_{sy} \cdot i_{sy} = (L_{my} + L_{s\lambda}) \cdot i_{sy}$$

$$\begin{aligned} T &= \vec{\psi}_s \times \vec{i}_s = \psi_{sx} \cdot i_{sy} - \psi_{sy} \cdot i_{sx} = \\ &= (\psi_m + (L_{mx} + L_{s\lambda}) \cdot i_{sx}) \cdot i_{sy} - (L_{my} + L_{s\lambda}) \cdot i_{sy} \cdot i_{sx} = \\ &= \psi_m \cdot i_{sy} + (L_{mx} - L_{my}) \cdot i_{sx} \cdot i_{sy} \end{aligned}$$

Torque Control

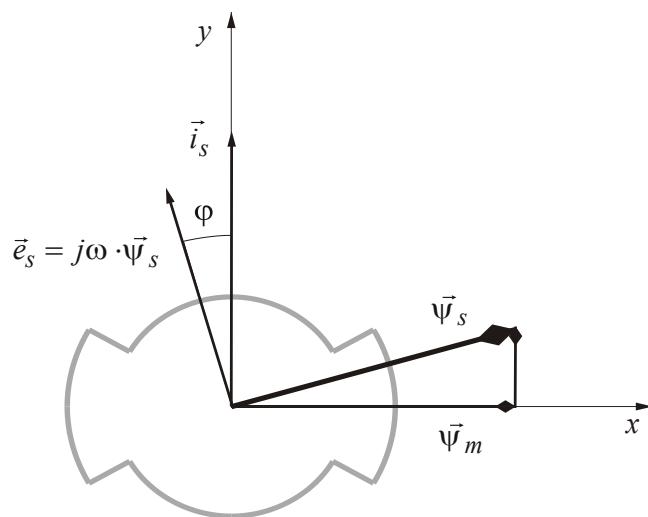
Priorities:

- *Speed*
- *Accuracy*
- *Power Factor*
- *Stator Flux level*
- ...

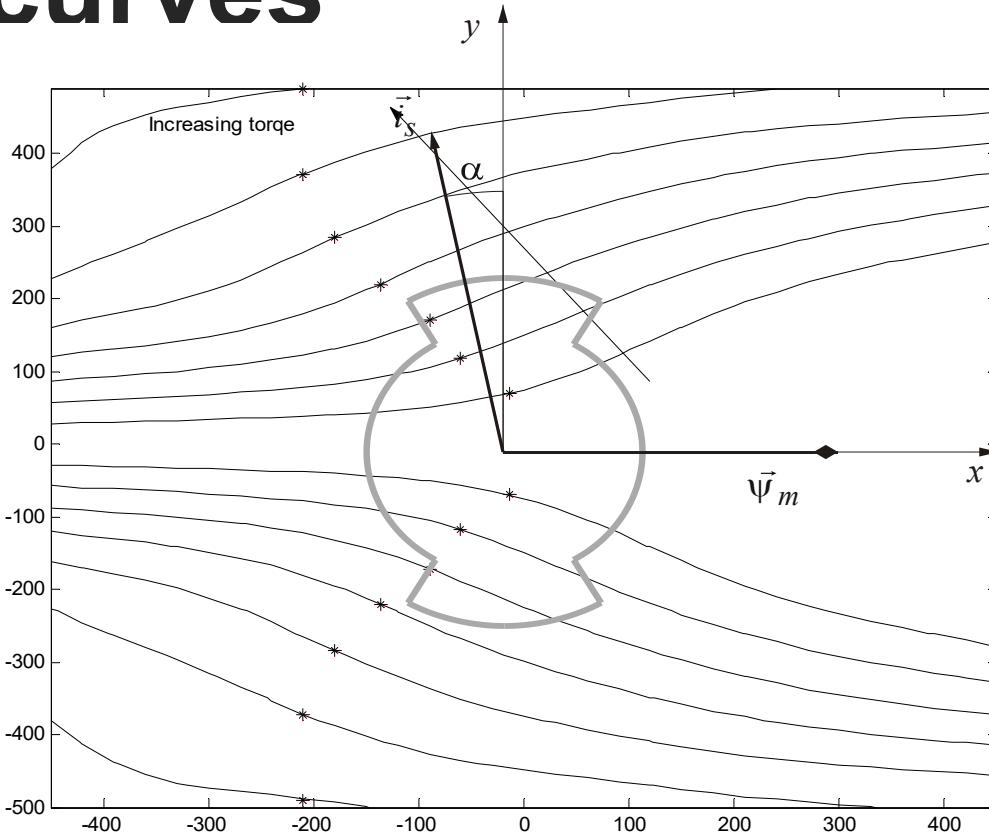
Only y-axis current control

$$T = \psi_m \cdot i_{sy}$$

$$\begin{cases} i_{sx}^* = 0 \\ i_{sy}^* = \frac{T^*}{\psi_m} \end{cases}$$

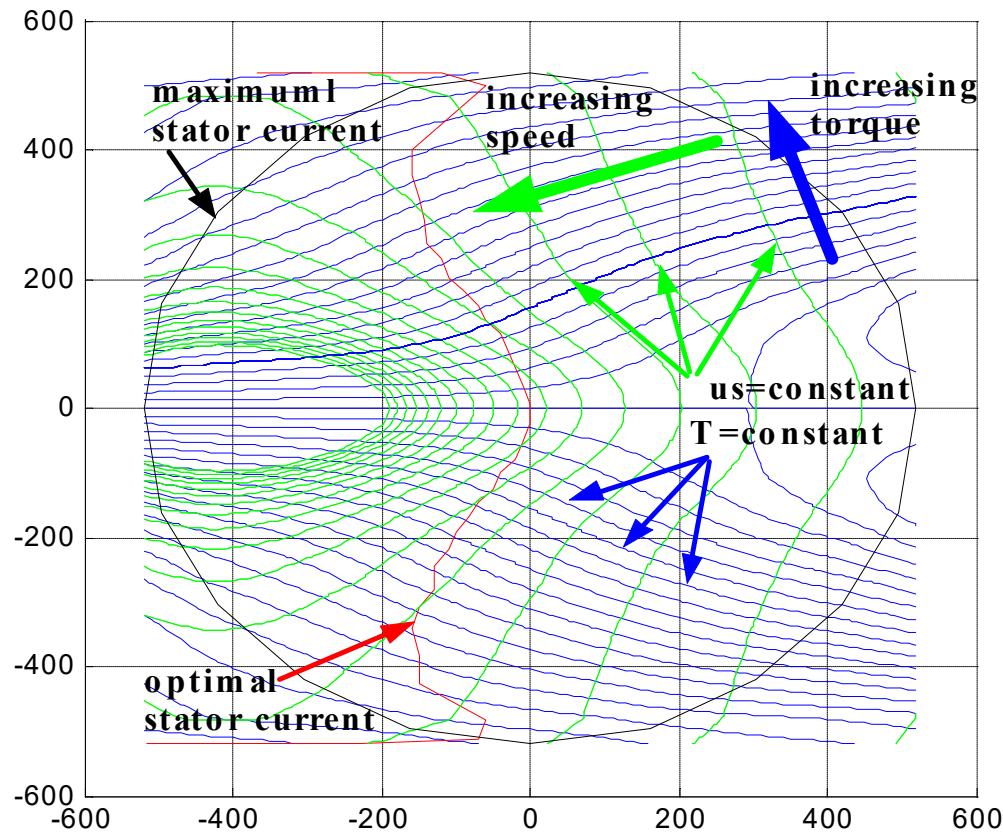


"Iso"-torque curves



Field Weakening

$$|\vec{u}_s| = \sqrt{(R_s \cdot i_{sx} - \omega_r \cdot L_{sy}(i_{sx}, i_{sy}) \cdot i_{sy})^2 + (R_s \cdot i_{sy} - \omega_r \cdot (\psi_m + L_{sx}(i_{sx}, i_{sy}) \cdot i_{sx}))^2}$$



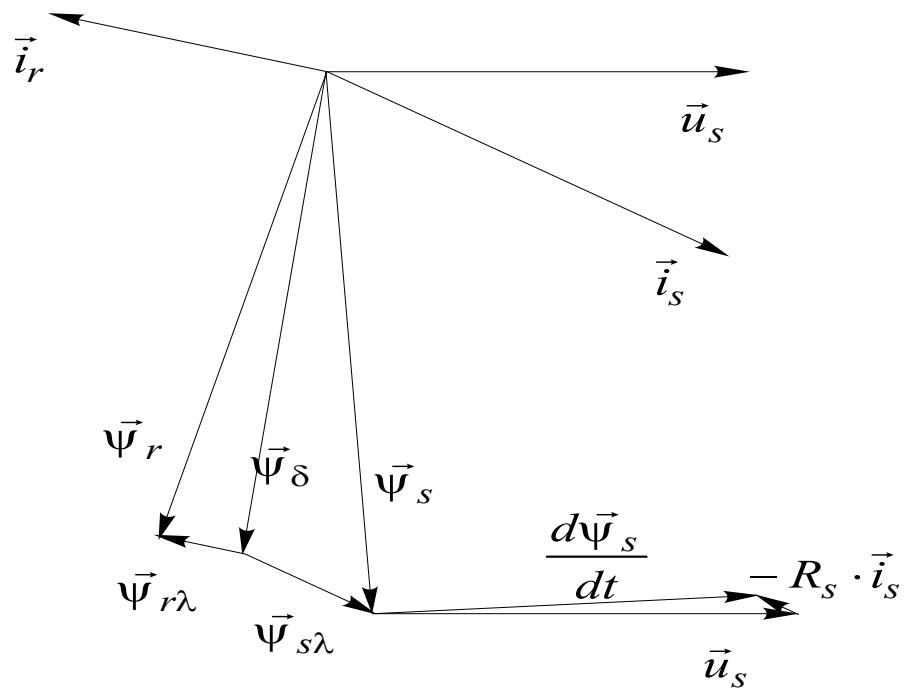
Induction Machine - Mathematical model

- Stator

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}$$

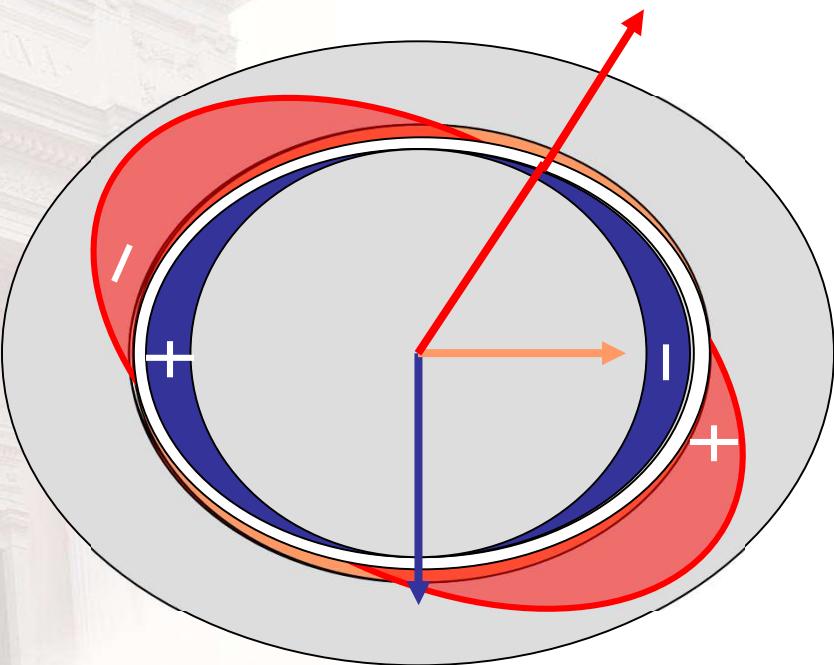
- Rotor

$$\vec{u}_r = R_r \cdot \vec{i}_r + \frac{d\vec{\psi}_r}{dt}$$



How does it work?

Imagine the following sequence :



- 1 A magnetizing stator current is stationary vs. the rotor.
- 2 The current is instantly moved and increased.
- 3 The rotor conserves the (rotor-)flux and thus the stator flux

Torque

$$T = \vec{\psi}_s \times \vec{i}_s = \vec{\psi}_r \times \vec{i}_r = -\frac{L_m}{L_s} \cdot \vec{\psi}_s \times \vec{i}_r = \frac{L_m}{L_s} \cdot \vec{\psi}_r \times \vec{i}_s = \vec{\psi}_\delta \times \vec{i}_s$$

Stator flux orientation

Rotor flux orientation

Full Flux Observer 1

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}$$

$$0 = R_r \cdot \vec{i}_r + \frac{d\vec{\psi}_r}{dt} - j \cdot \omega_r \cdot \vec{\psi}_r$$

↓

$$\begin{bmatrix} \vec{u}_s \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_s \\ \vec{i}_r \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -j \cdot \omega_r \end{bmatrix} \cdot \begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix}}_{u=R \cdot i + \frac{d\psi}{dt} + \Omega \cdot \psi}$$

Split equation solving

$$\begin{bmatrix} \frac{d \vec{\psi}_s}{dt} \\ \frac{d \vec{\psi}_r}{dt} \end{bmatrix} = \begin{bmatrix} A_{obs,11} & A_{obs,12} \\ A_{obs,21} & A_{obs,22} \end{bmatrix} \cdot \begin{bmatrix} \hat{\vec{\psi}}_s \\ \hat{\vec{\psi}}_r \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \vec{u}_s + \begin{bmatrix} R_s \cdot k_1 \cdot \vec{i}_s \\ R_r \cdot k_2 \cdot \vec{i}_s \end{bmatrix}$$

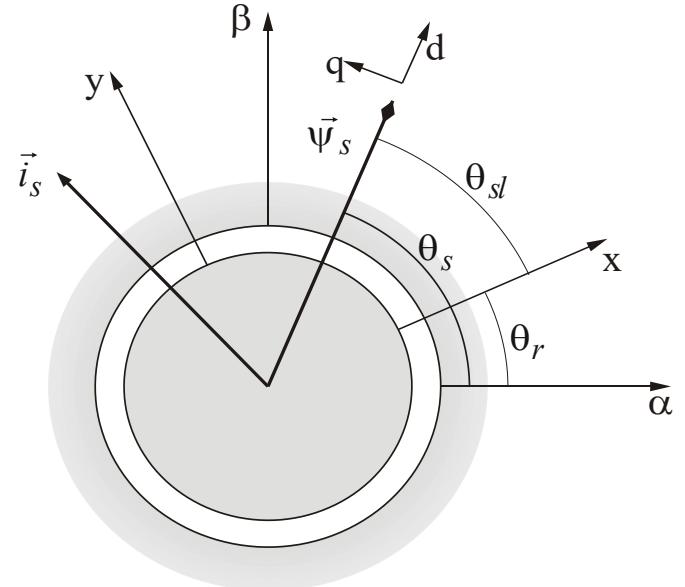
$$\begin{aligned} \frac{d \vec{\psi}_s}{dt} &= A_{obs,11} \cdot \hat{\vec{\psi}}_s + A_{obs,12} \cdot \hat{\vec{\psi}}_r + \vec{u}_s + R_s \cdot k_1 \cdot \vec{i}_s \\ &= A_{obs,11} \cdot \hat{\vec{\psi}}_s + A_{obs,12} \cdot \hat{\vec{\psi}}_r + R_s \cdot k_1 \cdot \vec{i}_s + \vec{u}_s \end{aligned}$$

Solved partly outside the microprocessor

$$\frac{d \vec{\psi}_r}{dt} = A_{obs,21} \cdot \hat{\vec{\psi}}_s + A_{obs,22} \cdot \hat{\vec{\psi}}_r + 0 + R_r \cdot k_2 \cdot \vec{i}_s$$

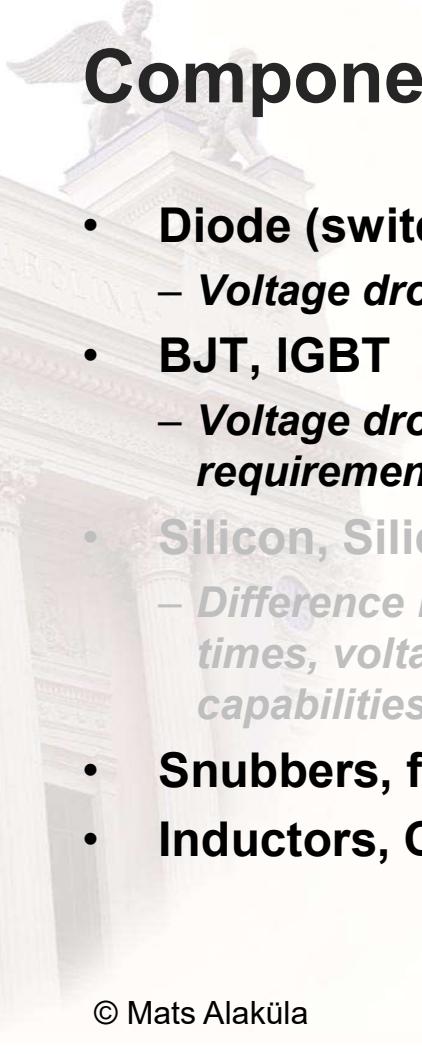
Solved entirely in the microprocessor

Simplified Stator Flux Model 1



$$\vec{u}_s = R \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt} = R \cdot (\vec{i}_s - \vec{i}_{s0}) + R \cdot \vec{i}_{s0} + \frac{d\vec{\psi}_s}{dt}$$

$$\frac{d\vec{\psi}_s}{dt} = -R \cdot \vec{i}_{s0} + [\vec{u}_s - R \cdot (\vec{i}_s - \vec{i}_{s0})] = \begin{cases} \text{no load running} \\ \vec{i}_s = \vec{i}_{s0} = \frac{\vec{\psi}_{s0}}{L_s} \end{cases} = -R \cdot \frac{\vec{\psi}_{s0}}{L_s} + \vec{u}_s = -\frac{\vec{\psi}_{s0}}{\tau_s} + \vec{u}_s$$

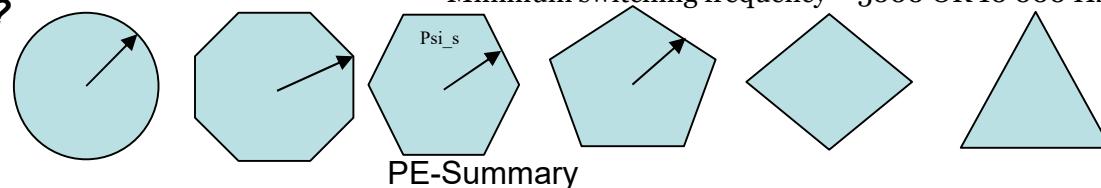
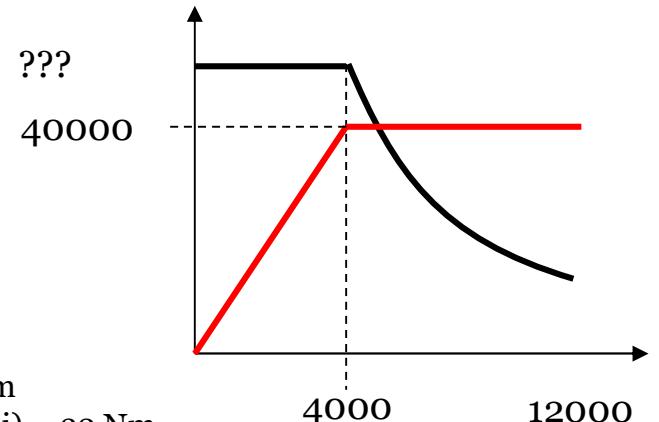


Components

- **Diode (switch, rectifier), Thyristor**
 - *Voltage drop, reverse recovery*
- **BJT, IGBT**
 - *Voltage drop, Gate drive requirements*
- **Silicon, Silicon Carbide**
 - *Difference in properties (switching times, voltage drops, temperature capabilities)*
- **Snubbers, function of**
- **Inductors, Capacitors**

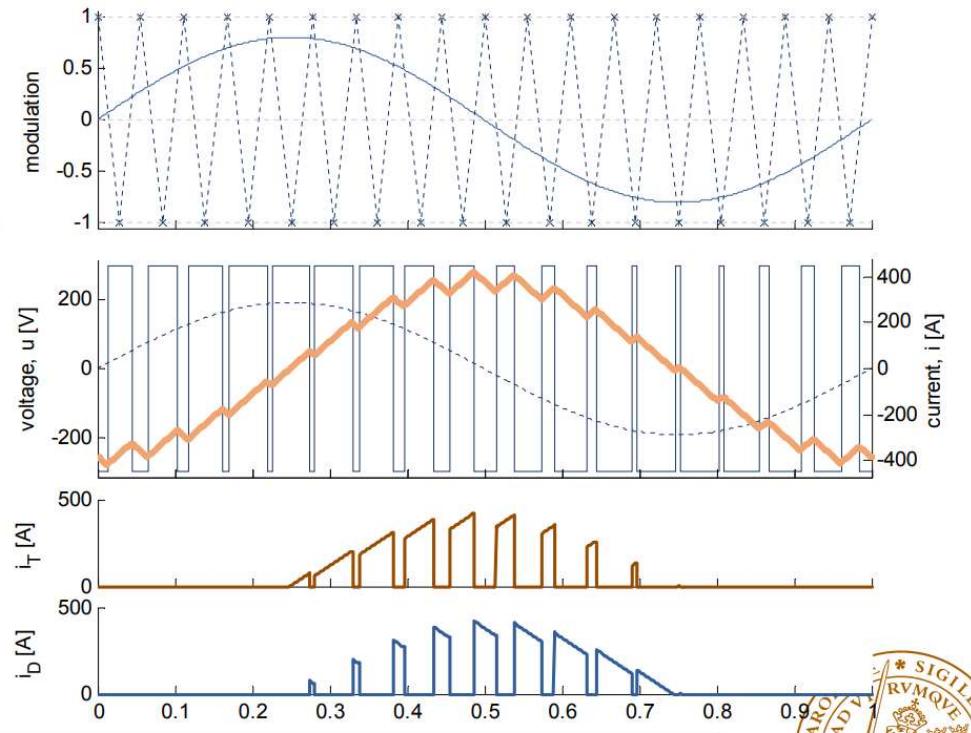
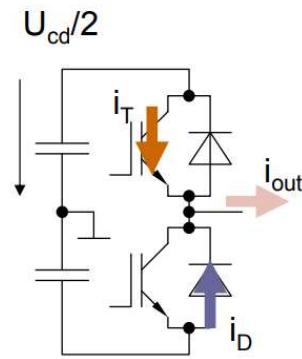
An example

- **40 kW drive (Power El Conv + El Machine)**
- **Base speed 4000 rpm, max speed 12000 rpm**
 - **Rated torque ?** $T_{nom} = 40000/(4000/60*2*\pi) = 96 \text{ Nm}$
 - **Torque at max speed ?** $T@max_speed = 40000/(12000/60*2*\pi) = 32 \text{ Nm}$
- **400 V DC link**
 - **Max phase-phase voltage?** Assume symmetrized modulation $U_{pp_peak} = U_{dc} \rightarrow U_{pp_rms} = U_{dc}/\sqrt{2}$
 - **Max phase current?** $P = \sqrt{3} * U_{pp_rms} * I_{phase_rms} * \cos(\phi)$
Assume $\cos(\phi)=1!$
- **Pole number p=20**
 - **Minimum Sample&Control frequency?** Highest electric frequency? $f_{el_max} = f_{mech_max} * p/2 = 2000 \text{ Hz}$
 - **Minimum switching frequency** Highest mechanical frequency? $f_{mech_max} = 12000/60 = 200 \text{ Hz}$
Sample&Control frequency = 6...8...10 samples per period. IF 10 then the sample & control frequency is $10 \times 2000 = 20000 \text{ Hz}$.
- **PM machine**
 - **Sensors needed?** Minimum switching frequency = 5000 OR 10 000 Hz





Switching & component current

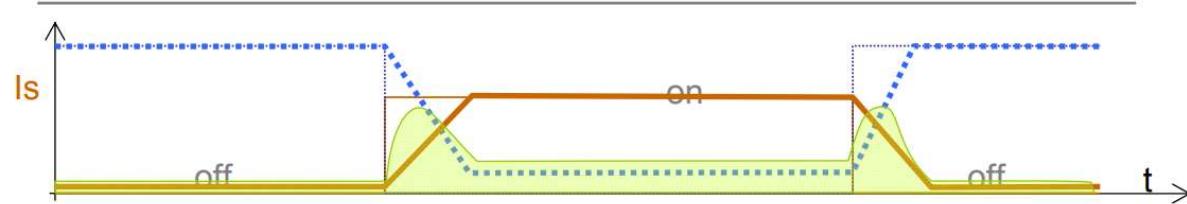


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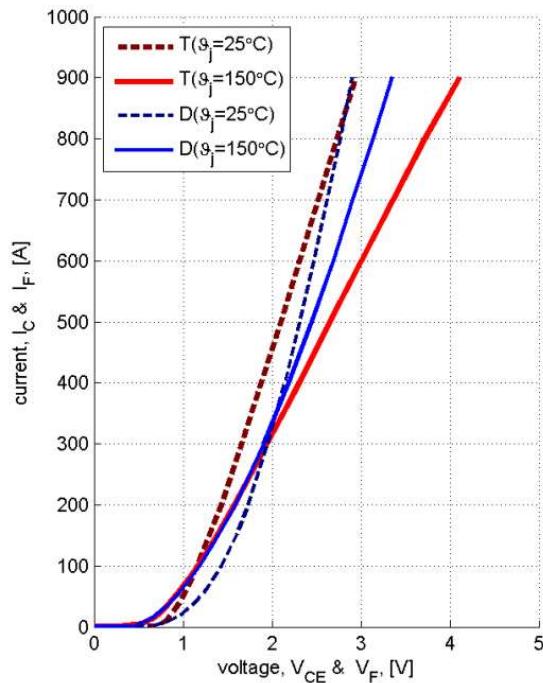
Inverter losses



- Switching cycle and power loss estimation conditions
 - Sinusoidal duty as a function of time
 - Voltage utilization or the degree of converter modulation $m = \hat{U}_{\text{out}} / (U_{\text{dc}}/2)$
 - Phase shift ϕ between the fundamental of AC voltage and current



Forward characteristics

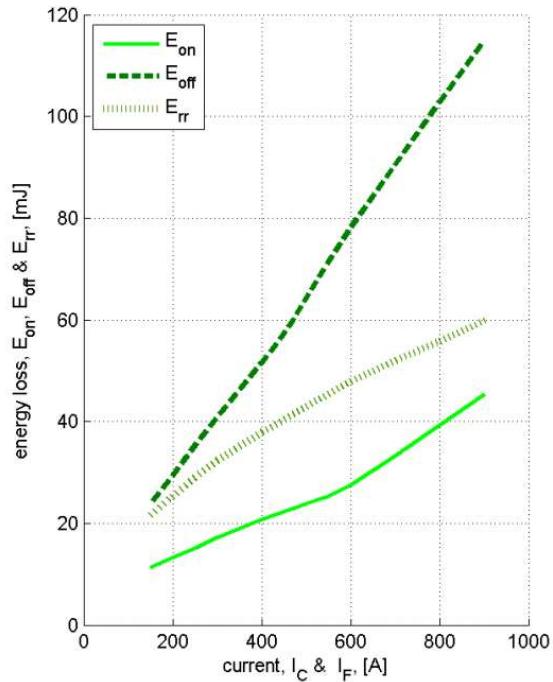


- $V_{ce0}(\theta_j)$ – temperature dependent threshold voltage of the transistor on-state characteristic
- $r_{ce}(\theta_j)$ – temperature dependent bulk resistance of the transistor on-state characteristic
- $V_{F0}(\theta_j)$ – temperature dependent threshold voltage of the diode on-state characteristic
- $r_F(\theta_j)$ – temperature dependent bulk resistance of the diode on-state characteristic





Switching energy losses



- E_{on} – turn on energy dissipation in the power transistor
- E_{off} – turn off energy dissipation in the power transistor
- E_{rr} – reverse recovery energy dissipation of the inverse diodes





Power losses in Transistor T

- On-state conducting losses

$$P_{cnd.T} = \left(\frac{1}{2\pi} + \frac{m \cdot \cos(\varphi)}{8} \right) \cdot V_{CE0}(g_j) \cdot \hat{I}_1 + \left(\frac{1}{8} + \frac{m \cdot \cos(\varphi)}{3\pi} \right) \cdot r_{CE}(g_j) \cdot \hat{I}_1^2$$

- Energy losses due to switching

$$P_{sw.T} = f_{sw} \cdot E_{on+off} \cdot \frac{1}{\pi} \frac{\hat{I}_1}{I_{ref}} \left(\frac{V_{dc}}{V_{ref}} \right)^{K_V} \cdot (1 + TC_{Esw}(g_j - g_{ref}))$$

\hat{I}_1	Amplitude of inverter output current
I_{ref} V_{ref} g_{ref}	Reference values of measured data
K_V	Voltage dependency of switching losses 1.3-1.4
TC_{Esw}	Temperature coefficient of switching losses 3e-3





Power losses in Diode D

- On-state conducting losses

$$P_{cnd.D} = \left(\frac{1}{2\pi} - \frac{m \cdot \cos(\varphi)}{8} \right) \cdot V_{F0}(\vartheta_j) \cdot \hat{I}_1 + \left(\frac{1}{8} - \frac{m \cdot \cos(\varphi)}{3\pi} \right) \cdot r_F(\vartheta_j) \cdot \hat{I}_1^2$$

- Energy losses due to switching

$$P_{sw.T} = f_{sw} \cdot E_{rr} \cdot \left(\frac{1}{\pi} \frac{\hat{I}_1}{I_{ref}} \right)^{K_I} \left(\frac{V_{dc}}{V_{ref}} \right)^{K_V} \cdot (1 + TC_{Esw}(\vartheta_j - \vartheta_{ref}))$$

K_V	Voltage dependency of switching losses ~0.6
K_I	Current dependency of switching losses ~0.6
TC_{Esw}	Temperature coefficient of switching losses $6e^{-3}$

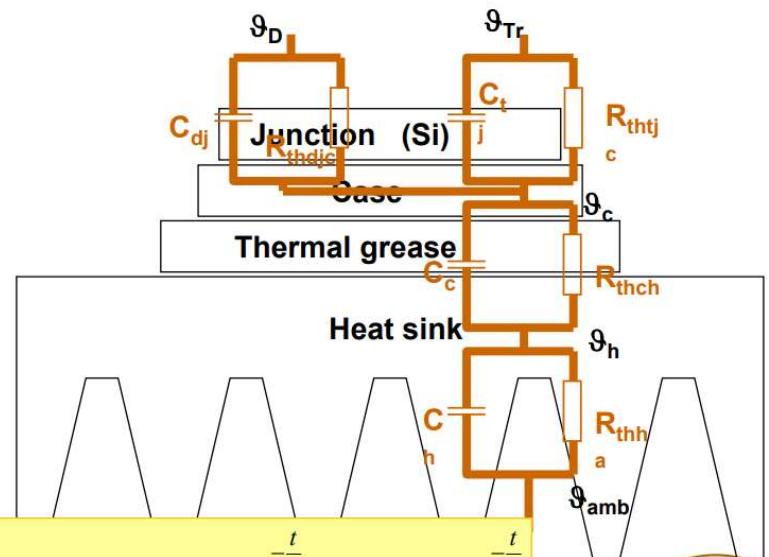




Thermal circuit

- Heat flow
 - Heat sources: losses in diodes and transistors
 - Heat sink: natural but preferably forced convection
 - Thermal resistance: components and thermal connections between

$$\vartheta_{l,end} = (R_{th} \cdot P + \vartheta_{amb}) \cdot (1 - e^{-\frac{t}{\tau}}) + \vartheta_{l,start} \cdot e^{-\frac{t}{\tau}}$$



Another Example

- **Battery Charge
24 V -> 12 V**
- **500 W**
- **10 % ripple, 5 kHz switching frequency**
- **Type of converter?**
- **Inductance?**
- **Current step 0-20 A?**

Use a buck converter (1Q DC/DC step down converter)

$$I = 500/12 = 42 \text{ A}$$

$$\text{Ripple} = 10\% = 4.2 \text{ A}$$

Inductance?

$$di/dt = (24-12)/L \text{ OR } (-12/L)$$

$$Di = 4.2 \text{ A} = di/dt * Dt$$

$$Dt = 0.5 * 1/5000.$$

$$Di/Dt = 4.2/(0.5 * 1/5000) = 12/L \rightarrow L$$

$$L=0.3 \text{ mH}$$

$$Ts = 1/5000/2$$

$$U^* = (L/Ts+R/2)*((i^*-i) + \text{Int})+ e =$$

$$L/Ts*20 + 12 = 0.3e-3/1e-4*20+12 = 72 =$$

$$24 + 24 + 24$$

