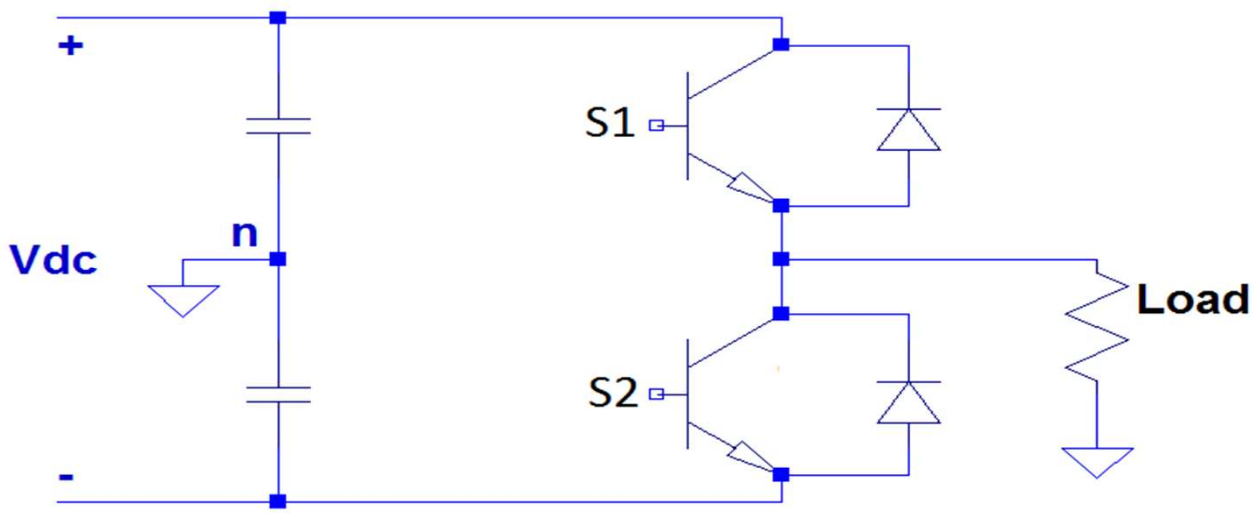
Resonance and Multilevel converters

Industrial Electrical Engineering and Automation
Lund University, Sweden

Conventional 2-level Converter

- **Topology reference**
- **Two level output: ±Vdc/2**
- **High dv/dt** $(=\frac{V_{dc}}{t_{sw}})$
- **Few components**
- **Easy to control**
- **EMC reducing implementation required**



One phase leg of a 2-level inverter

Multilevel Converters

Introduction:

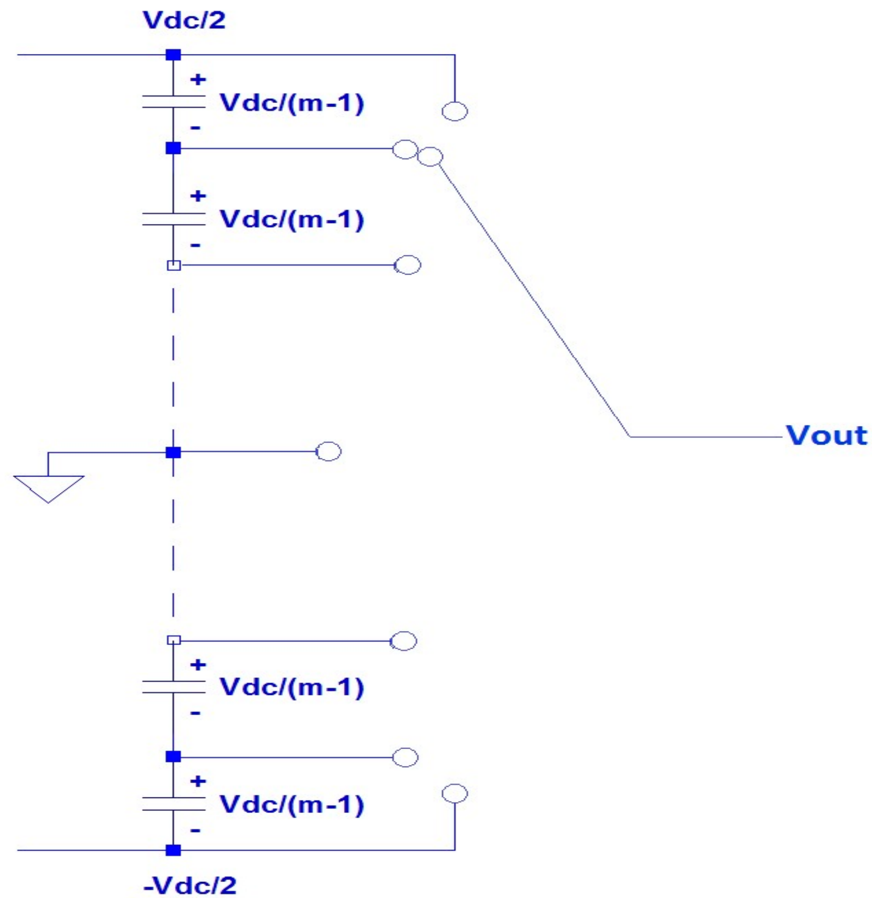
- Inverters with 3+ voltage levels are called multilevel inverters
- m-1 capacitors split the DC voltage into m levels (m-1 levels in the line voltage)
- The switches select the correct level
- The output only changes 1 level up/down at a time $(\frac{dv}{dt} = \frac{V_{dc}/(m-1)}{t_{sw}})$

Example in figure:

Assume m = 5 which means 4 capacitors are used to split up the DC voltage.

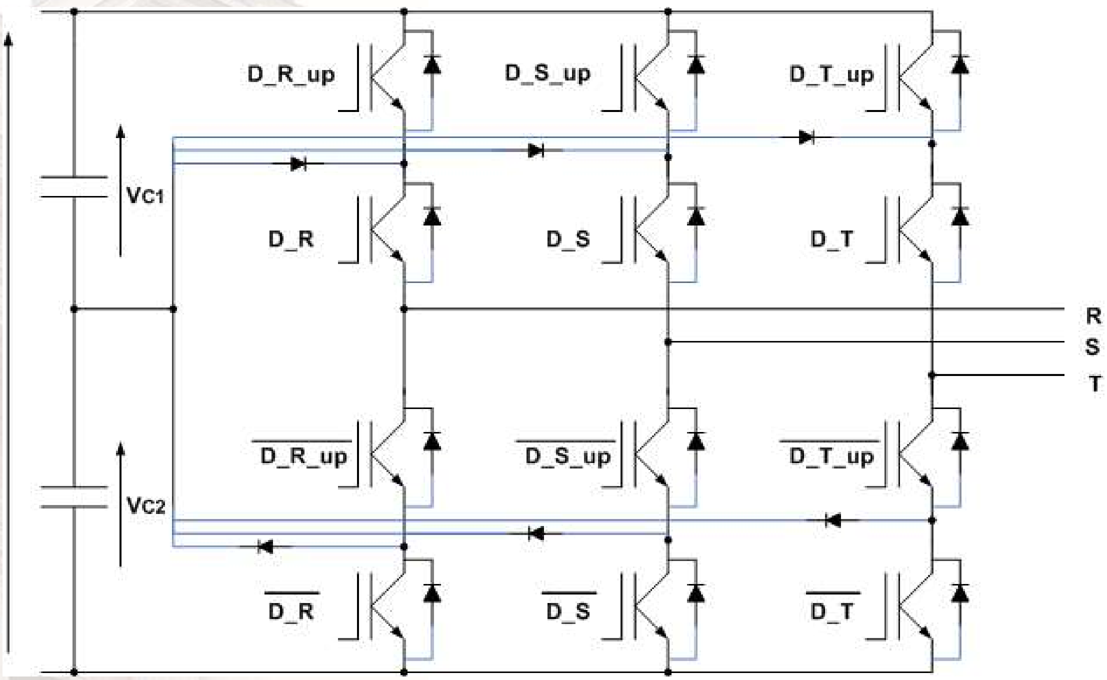
Then the output shown in the figure is:

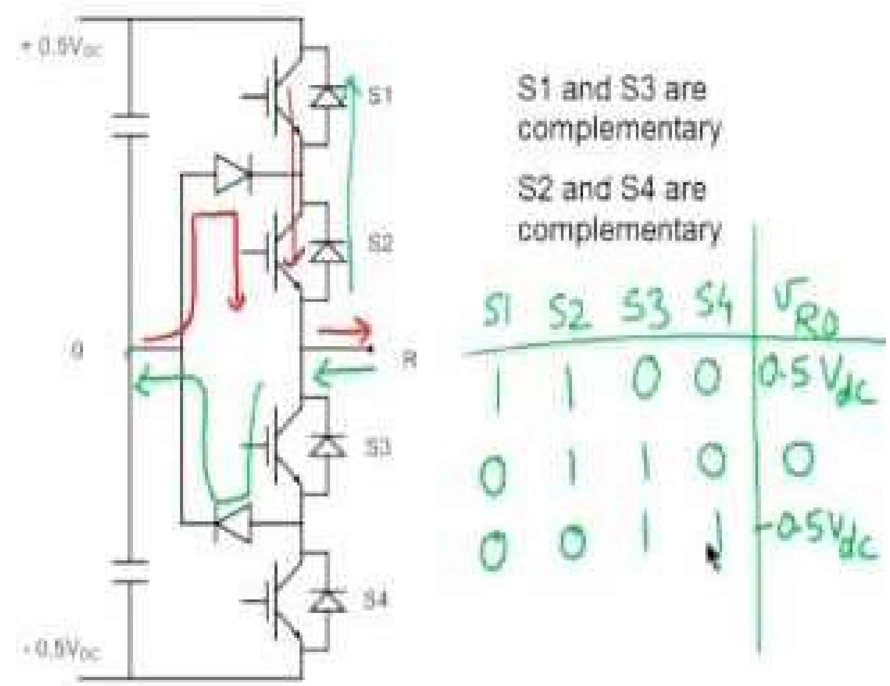
$$V_{out} = \frac{V_{dc}}{2} - \frac{V_{dc}}{4} = \frac{V_{dc}}{4}$$



Simplified m-level inverter

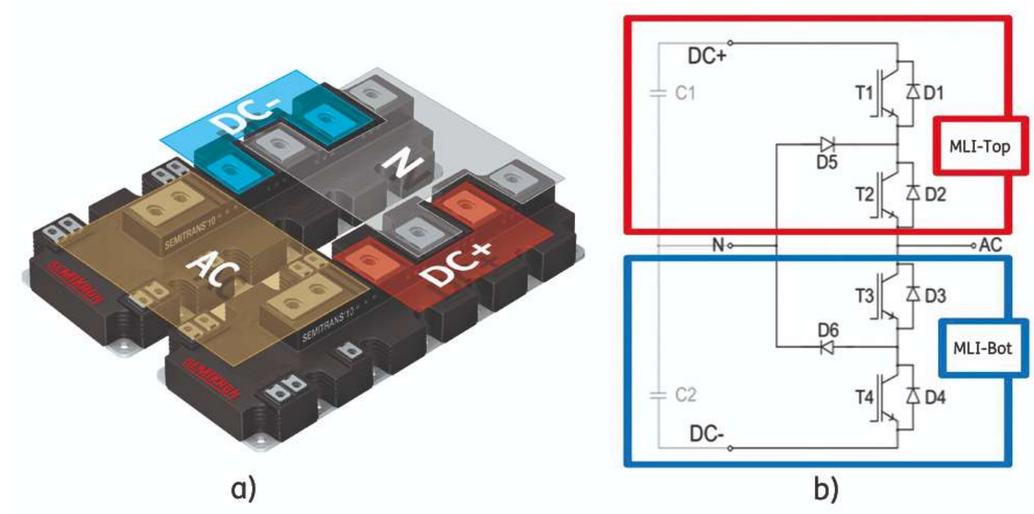
3-level in(con)verter





Example from Semikron

- SEMITRANS 10 MLI
- …for these type of inverters SEMIKRON introduced the SEMITRANS 10 MLI modules where the NPC topology is split to two halves. With current rating of 1200A and the use of 1200V medium power IGBT chips in combination with SEMIKRON CAL4F diodes SEMITRANS 10 MLI enables air cooled power blocks up to 750kW without paralleling of modules.

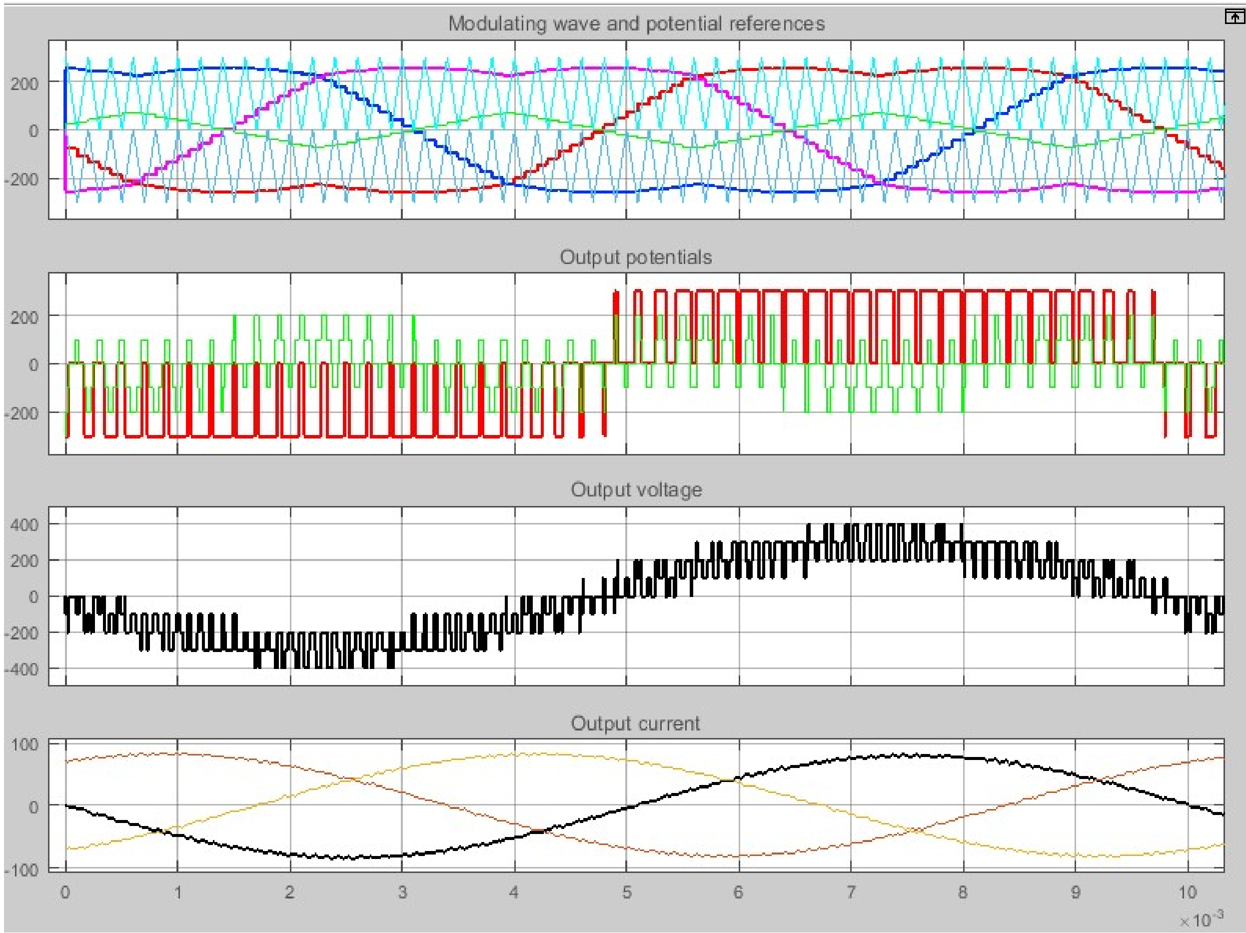


Product range
Half-bridges 1200V / 1400A and 1700V 1000A/1400A
MLI 1200V/1200A

https://www.semikron.com/zh/about-semikron/technical-articles/module-solutions-for-1500v-solar-inverters.html

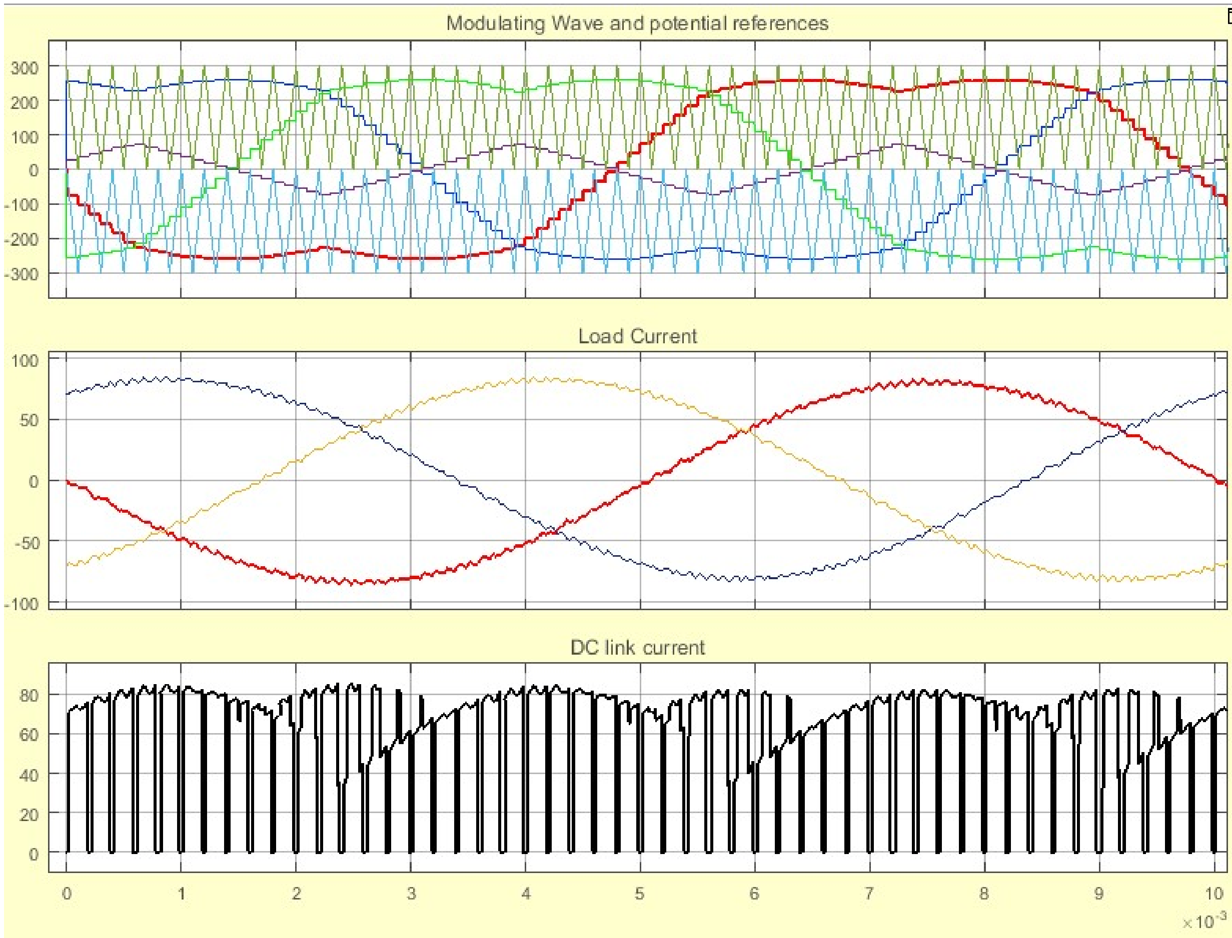
Example: 3-level converter

- **Two modulating waves:**
  - ***One between upper and mid***
  - ***One between mid and lower***
- **The rest is the same!**

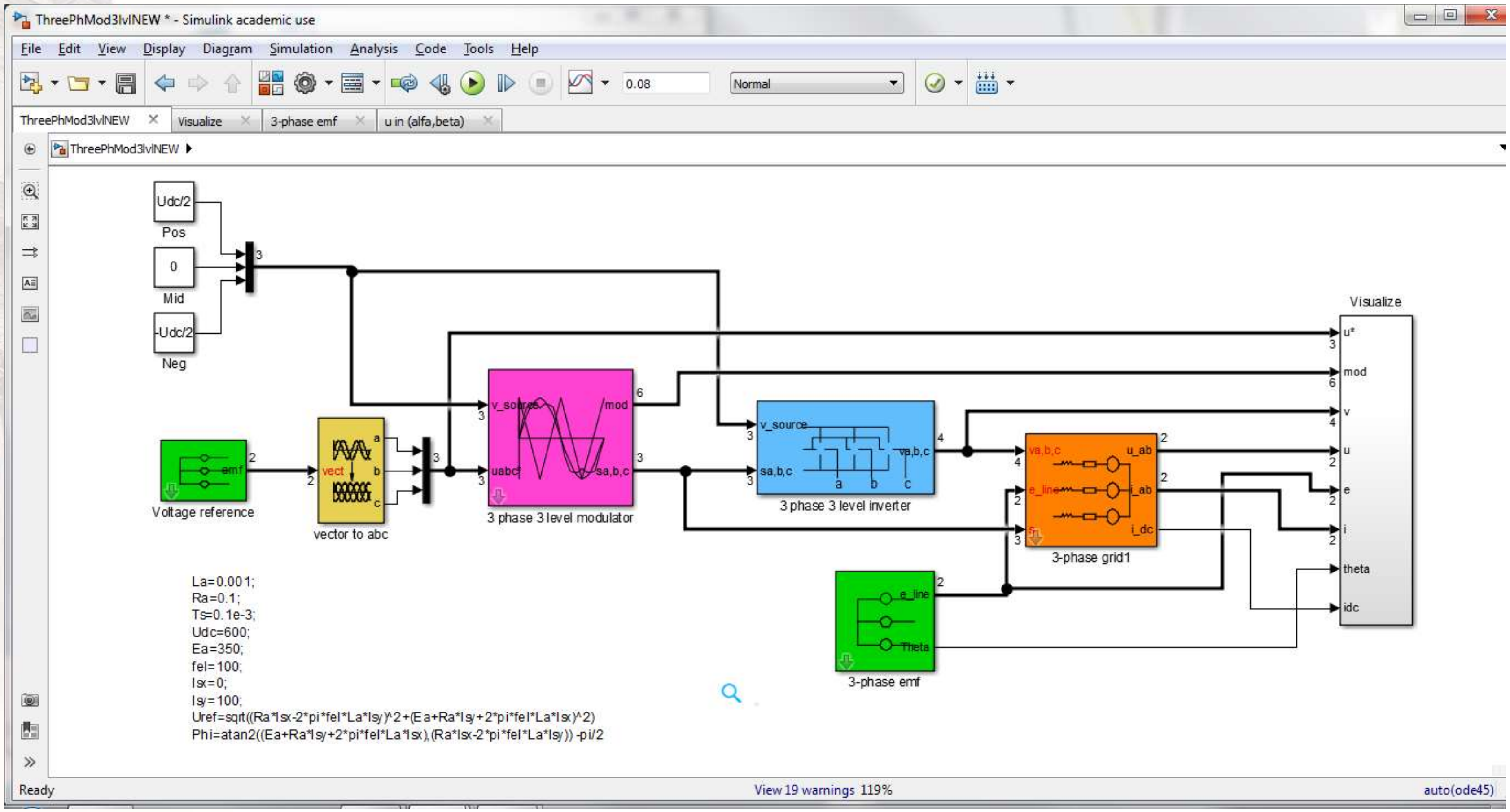


Example: 3-level converter

- **Two modulating waves:**
  - ***One between upper and mid***
  - ***One between mid and lower***
- **The rest is the same!**



To Simulink


ThreePhMod3lvlNEW * - Simulink academic use
File Edit View Display Diagram Simulation Analysis Code Tools Help
0.08
Normal
ThreePhMod3lvlNEW
Visualize
3-phase emf
u in (alfa,beta)
Udc/2
Pos
0
Mid
-Udc/2
Neg
Voltage reference
vector to abc
3 phase 3 level modulator
3 phase 3 level inverter
3-phase grid1
3-phase emf
Visualize
u*
mod
v
u
e
i
theta
idc
La=0.001;
Ra=0.1;
Ts=0.1e-3;
Udc=600;
Ea=350;
fel=100;
Isx=0;
Isy=100;
Uref=sqrt((Ra*Isx-2*pi*fel*La*Isy)^2+(Ea+Ra*Isy+2*pi*fel*La*Isx)^2)
Phi=atan2((Ea+Ra*Isy+2*pi*fel*La*Isx),(Ra*Isx-2*pi*fel*La*Isy)) -pi/2
Ready
View 19 warnings 119%
auto(ode45)

Multilevel Inverters pros/cons

Benefits:

- Less total harmonic distortion
- Can run on lower switching frequency
- Less component stress
- Lower component voltage rating

Drawbacks:

- Higher complexity
- Higher amount of components

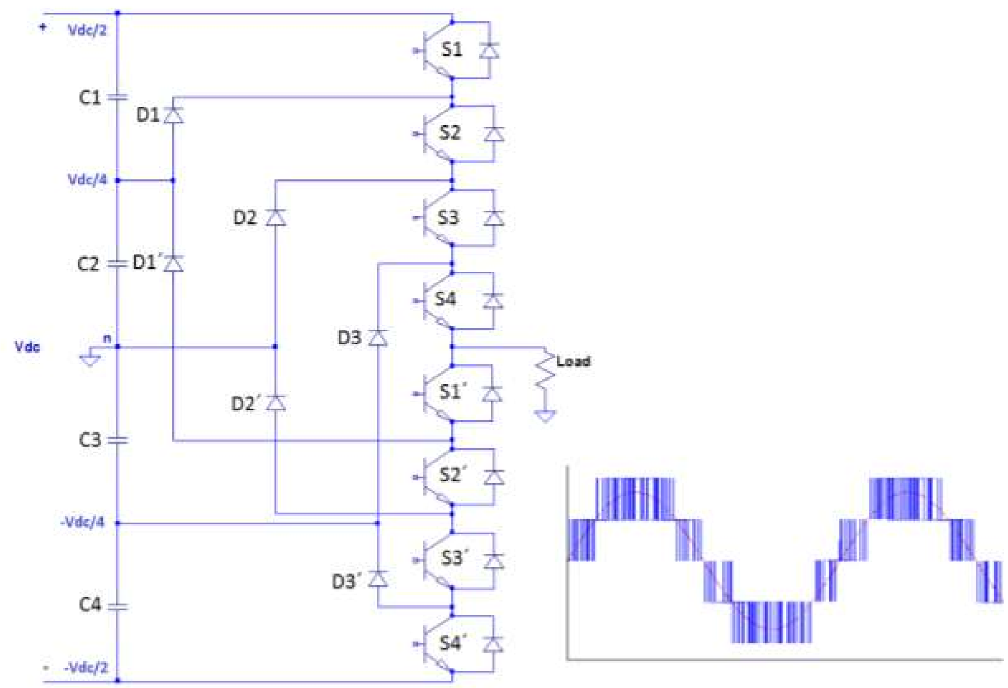
Neutral Point Clamped Multilevel Converter (NPCMLC)

Principle:

- The DC voltage is split into smaller levels by the capacitors.
- Diodes are used to clamp each switch to one capacitor voltage level.
- Switch state determines output voltage

Components:

- Number of capacitors: m-1
- Number of clamping diodes: (m-1)(m-2) per phase
- Number of switches: 2(m-1) per phase
- All components must have voltage rating higher than Vdc/(m-1)

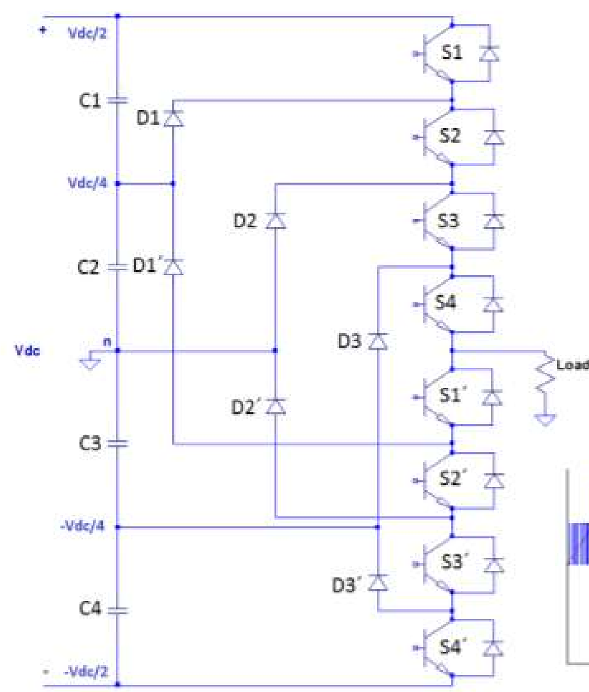


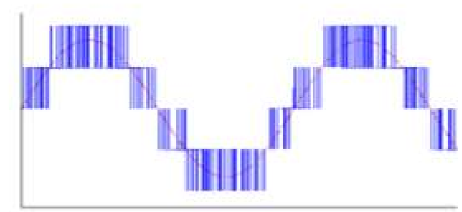
5-level Natural Point Clamped Converter

NPCMLC Control

- **Available switch states and corresponding output for the NPCMLC**

	Switch state							
Output	**S1**	**S2**	**S3**	**S4**	**S1′**	**S2′**	**S3′**	**S4′**
$\mathbf{V_{DC}/2}$	1	1	1	1	0	0	0	0
$\mathbf{V_{DC}/4}$	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0
$\mathbf{-V_{DC}/4}$	0	0	0	1	1	1	1	0
$\mathbf{-V_{DC}/2}$	0	0	0	0	1	1	1	1





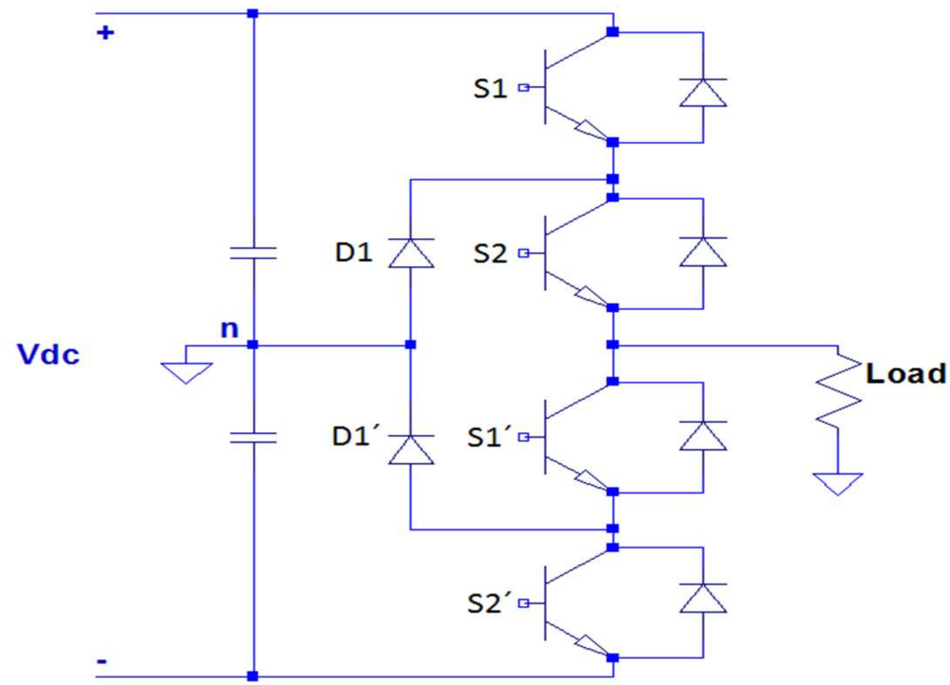
NPCMLC pros/cons

Benefits:

- Easy to control.

Drawbacks:

- High amount of clamping diodes when number of voltage levels is high.
- Capacitor unbalance occurs when transferring real power.

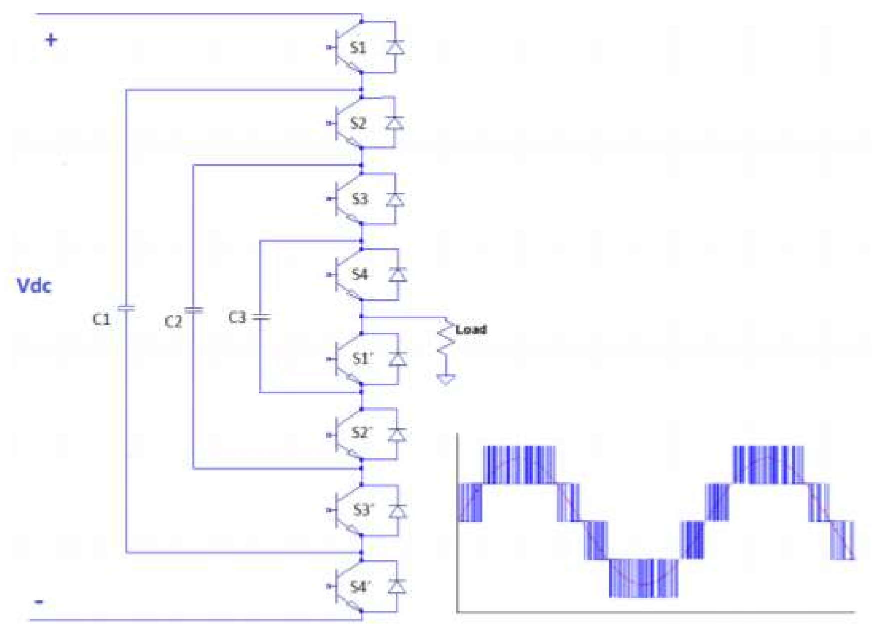


3-level Natural Point Clamped Converter

Capacitor Clamped Multilevel Converter (CCMLC)

Principle:

- Same basic principle as the NPCMLI
- Capacitors are used to clamp the device voltage to one voltage level
- Has redundant switching states, which makes capacitor balancing possible.



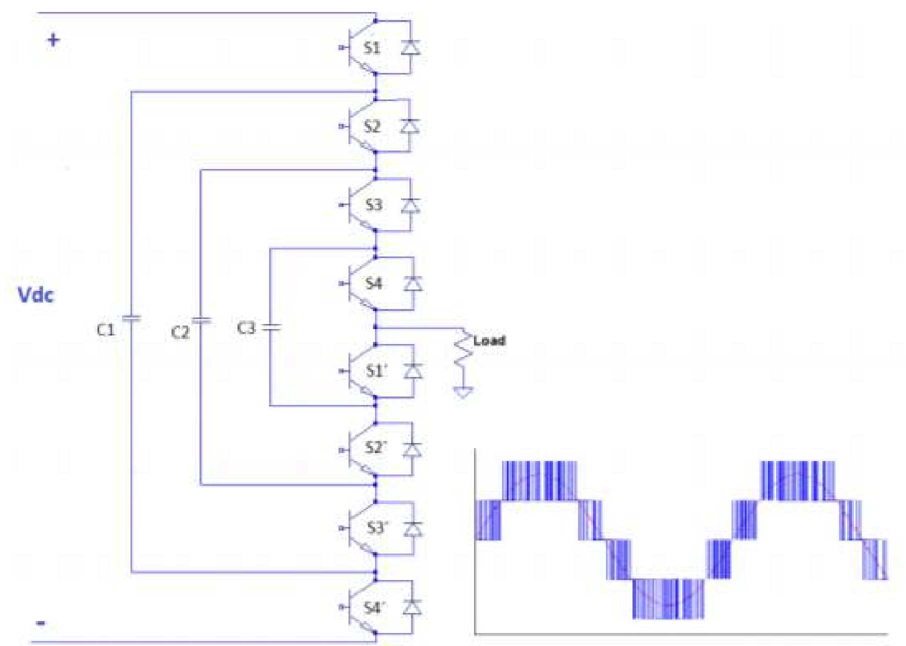
Components:

- Number of DC-bus capacitors: m-1
- Number of clamping capacitors: (m-1)(m-2)/2 per phase
- Number of switches: 2(m-1) per phase
- All components must have voltage rating higher than Vdc/(m-1)

CCMLC Control

	Switch state							
Output	S1	S2	S3	S4	S1′	S2′	S3′	S4′
$V_{DC}/2$	1	1	1	1	0	0	0	0
$V_{DC}/4$	1	1	1	0	1	0	0	0
	0	1	1	1	0	0	0	1
	1	0	1	1	0	0	1	0
0	1	1	0	0	1	1	0	0
	0	0	1	1	0	0	1	1
	1	0	1	0	1	0	1	0
	1	0	0	1	0	1	1	0
	0	1	0	1	0	1	0	1
	0	1	1	0	1	0	0	1
$V_{DC}/4$	1	0	0	0	1	1	1	0
	0	0	0	1	0	1	1	1
	0	0	1	0	1	0	1	1
$-V_{DC}/2$	0	0	0	0	1	1	1	1

- **Redundant switch states used for voltage balancing**



CCMLI pros/cons

Benefits:

- Redundant switch combinations makes voltage balancing possible

Drawbacks:

- High amount of clamping capacitors when number of voltage levels is high
- Capacitors are more expensive and bulky than diodes
- Balancing modulation is very complicated and requires high switching frequency resulting in high switching losses

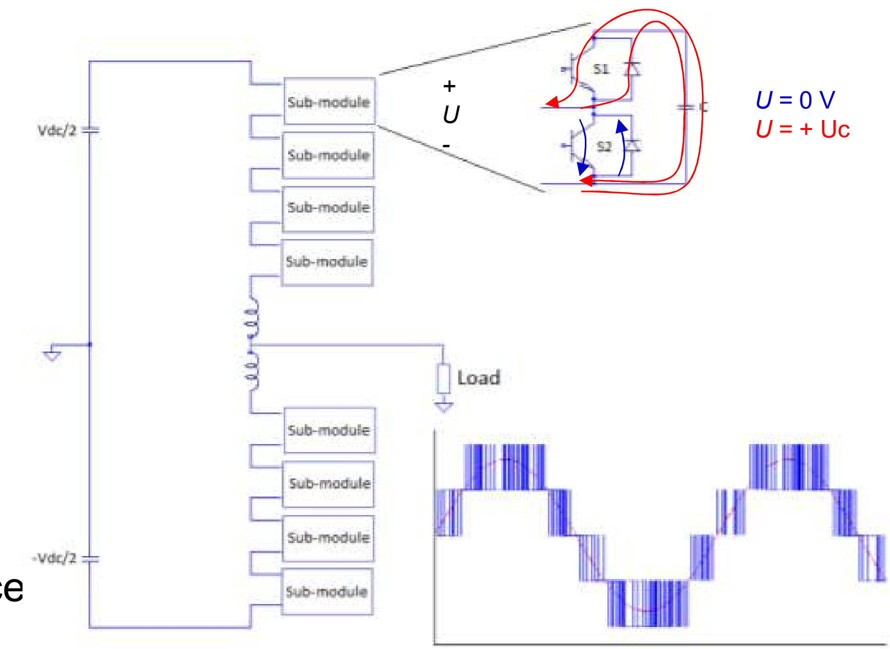
Modular Multilevel Inverter (MMI)

Principle:

- Modularized setup with submodules.
- Each submodule have a capacitor charged to Vdc/(m-1).
- The submodules can be inserted to make their capacitor contribute to the output.
- Has redundant switching states, which makes capacitor balancing possible.

Components:

- Number of capacitors: 2(m-1) per phase +2
- Number of switches: 4(m-1) per phase
- 2 inductors per phase (to take up voltage difference when switching occurs)



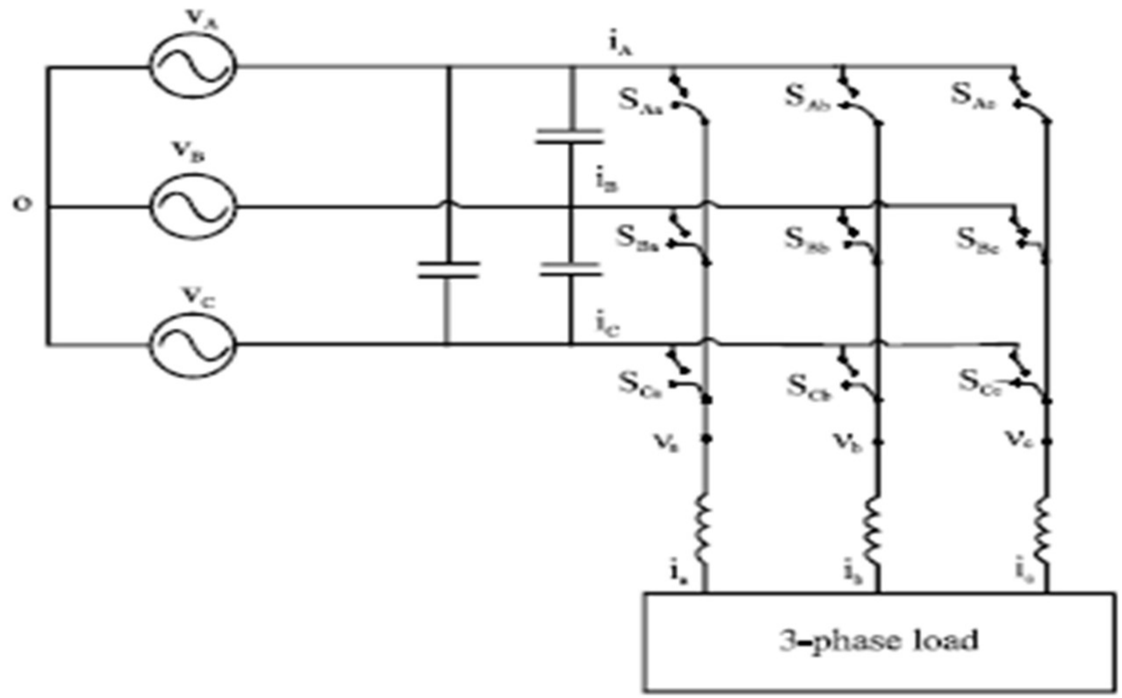
MMI pros/cons

Benefits:
- Modularized setup
- Redundant switch combinations makes voltage balancing possible
- The number of required components dose not grow quadratic with number of levels.

Drawbacks:
- Complicated balancing modulation

Matrix Converter

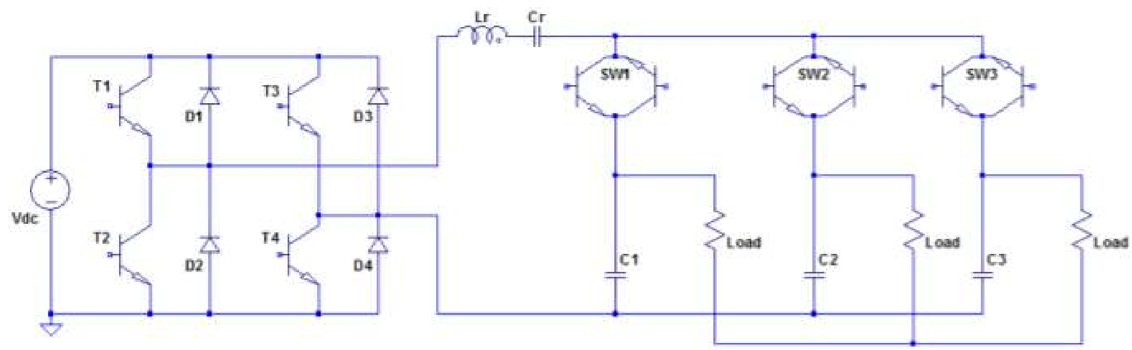
- Three VARYING levels IN.
- Modulation to any number of potentials OUT.
- No intermediate Energy Storage.
- Simple modulation
  - *Sort the three input levels in [min med max] and ”think 3-level”.*
- Difficult Switching.

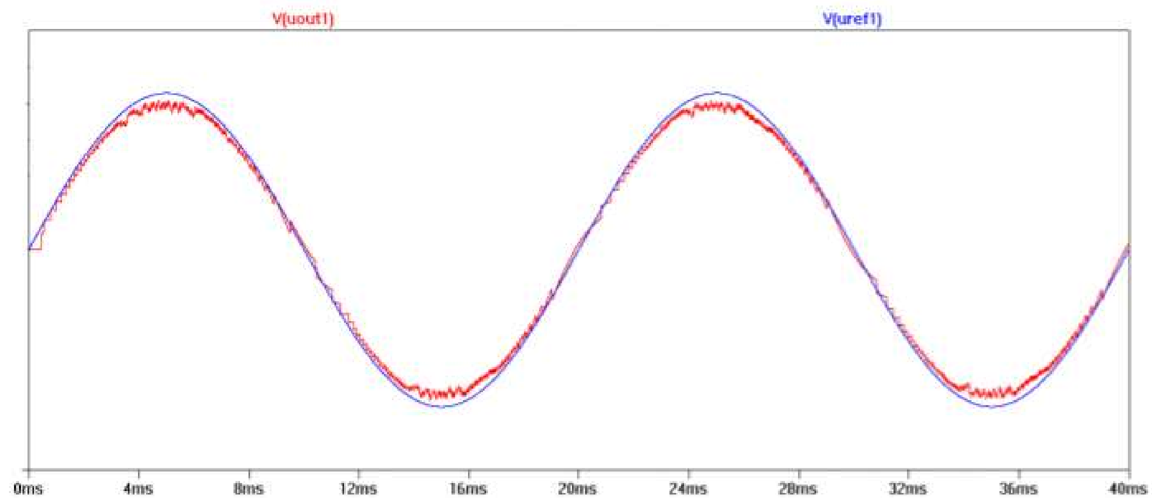


Star C

Principle:

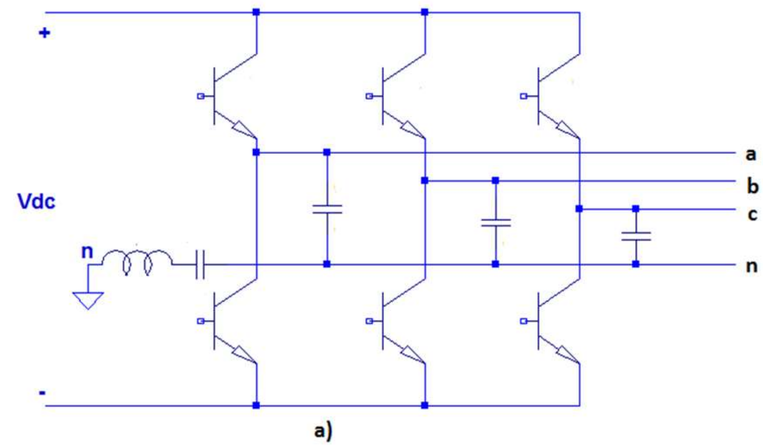
- A resonant voltage source (the 4Q converter with LC load) supplies three capacitors via three bidirectional switches.
- The bidirectional switches use positive OR negative half periods of the resonance to charge/dis-charge the capacitors.
- The load is connected to the capacitors

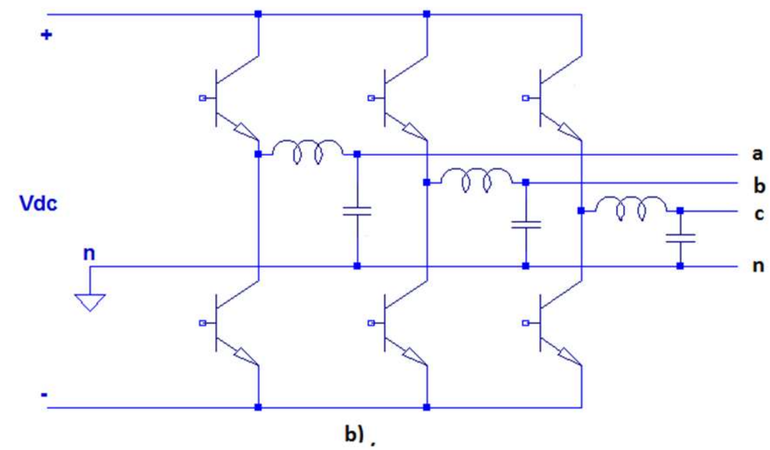




Typical output voltage waveform from the Star-C (red) and voltage reference (blue).

Star C alternative version





a) Resonant version
b) Hard switched version

Star C pros/cons

Benefits:

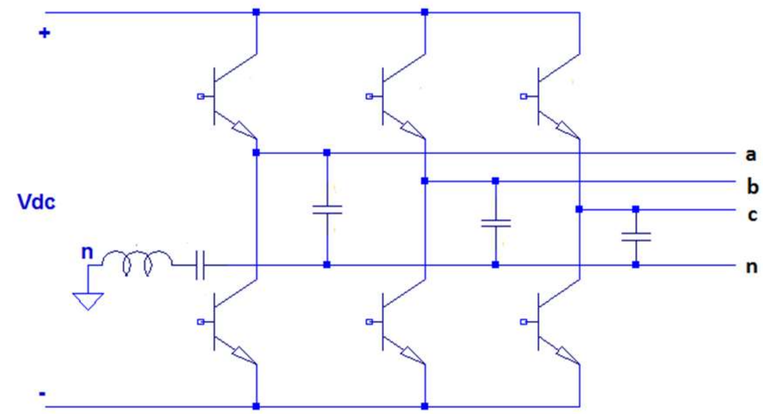
- Low number of semiconductor switches (compared to MLI:s)
- Very low output voltage derivatives

Drawbacks of series resonant Star C:

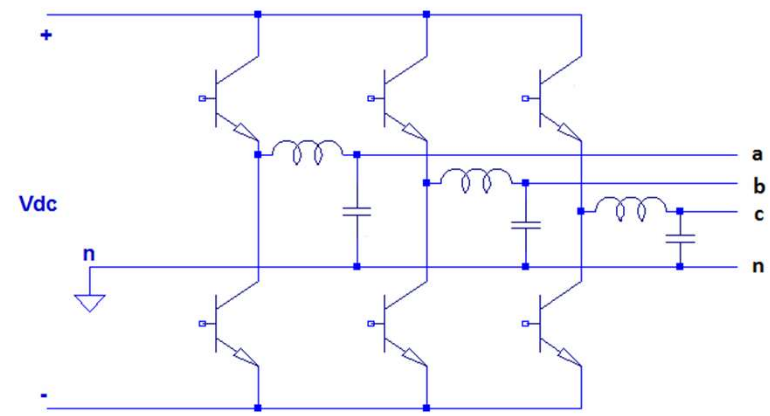
- May run in to problems if the load is unsymmetrical

Drawbacks of inductive Star C:

- Higher switching losses as a result of hard switching



a)

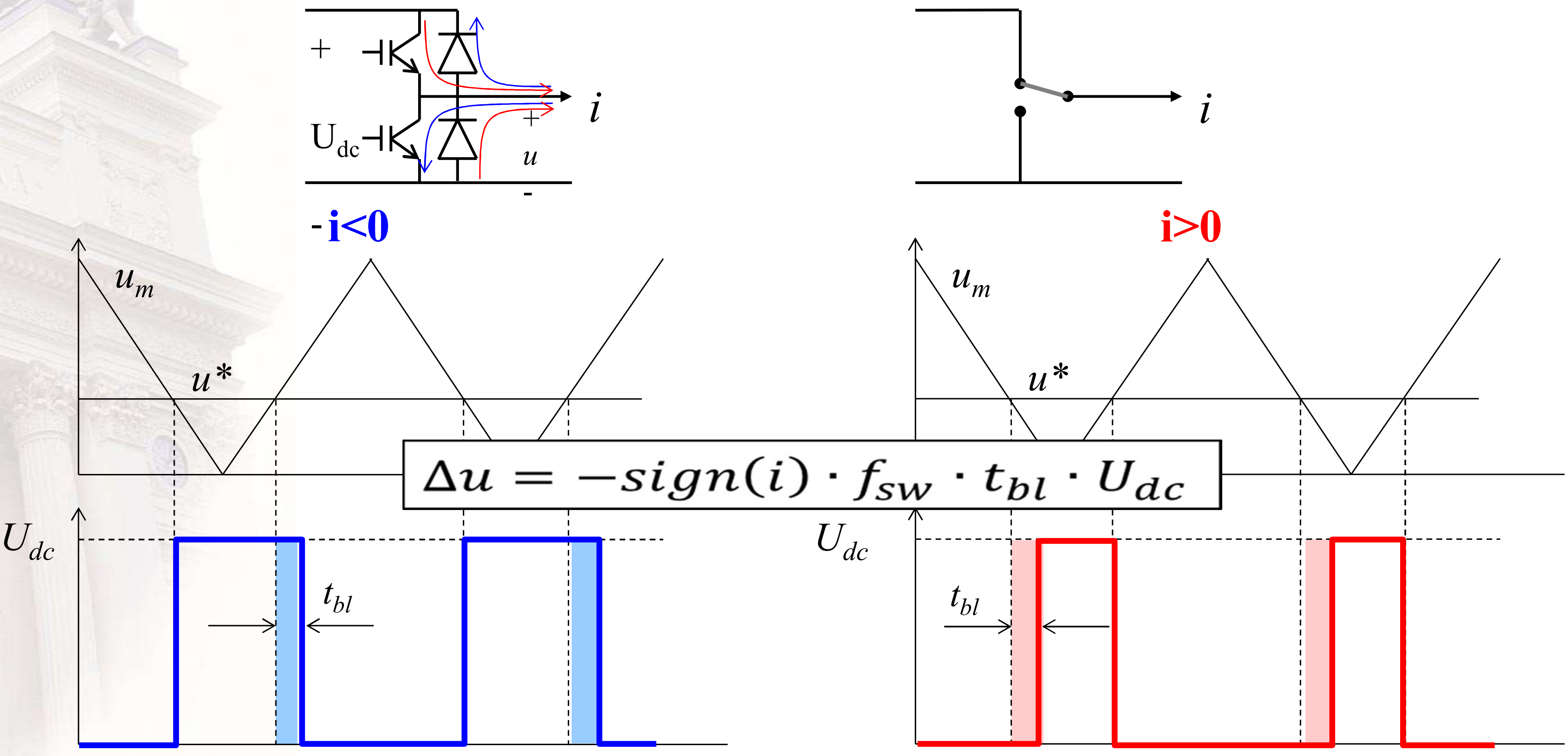


b)

The non ideal converter

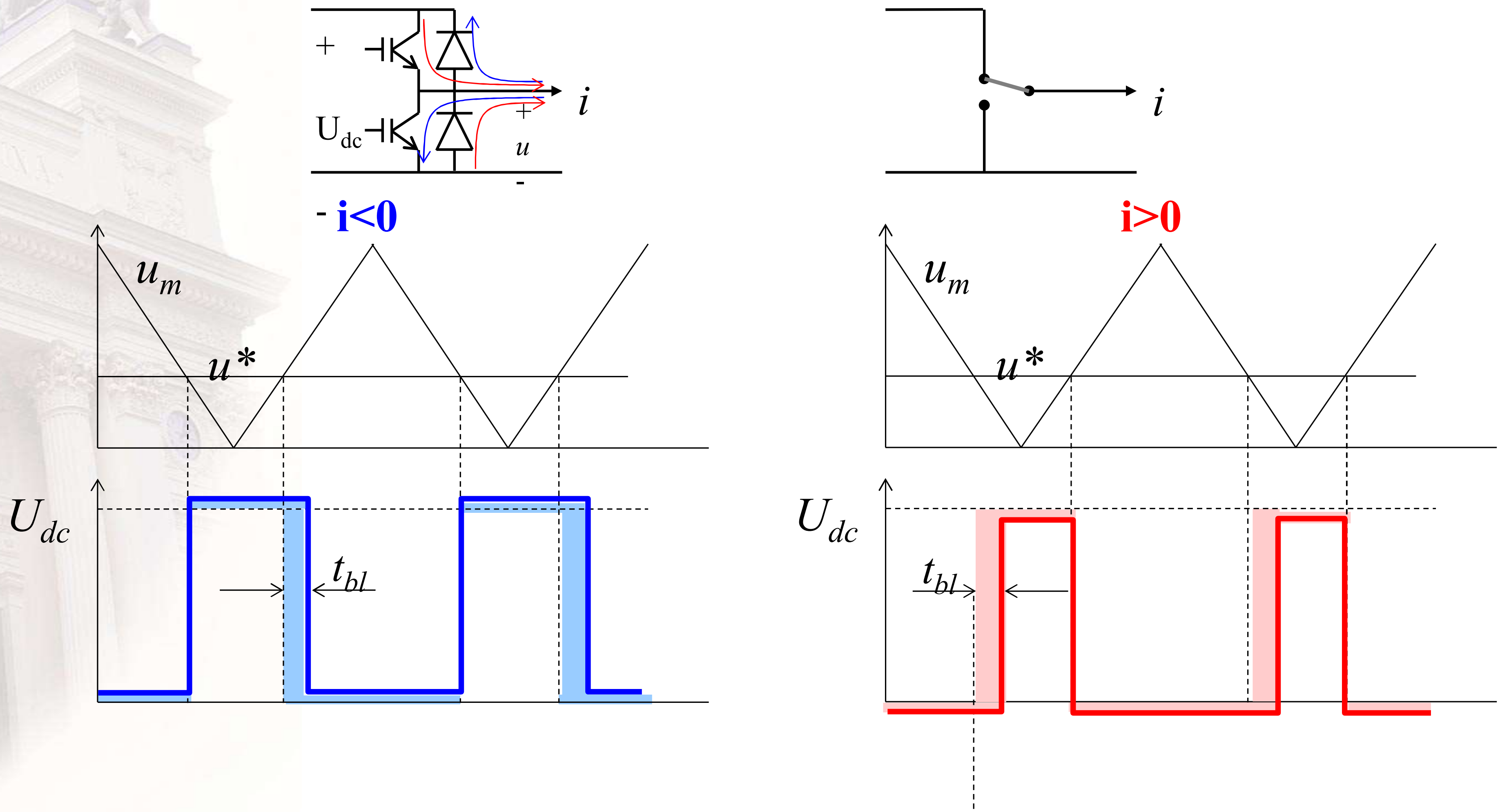
Of switched Power Electronics

Blanking Time

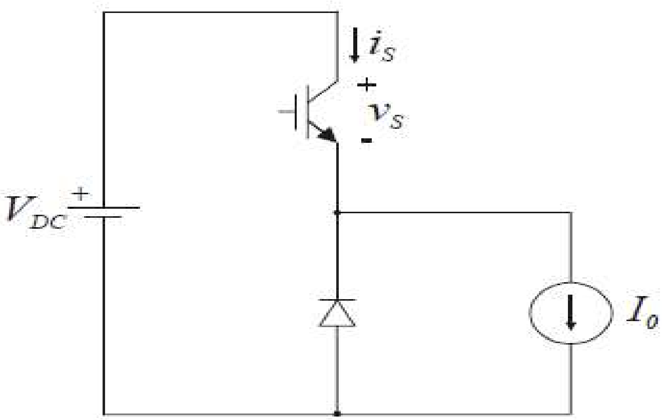


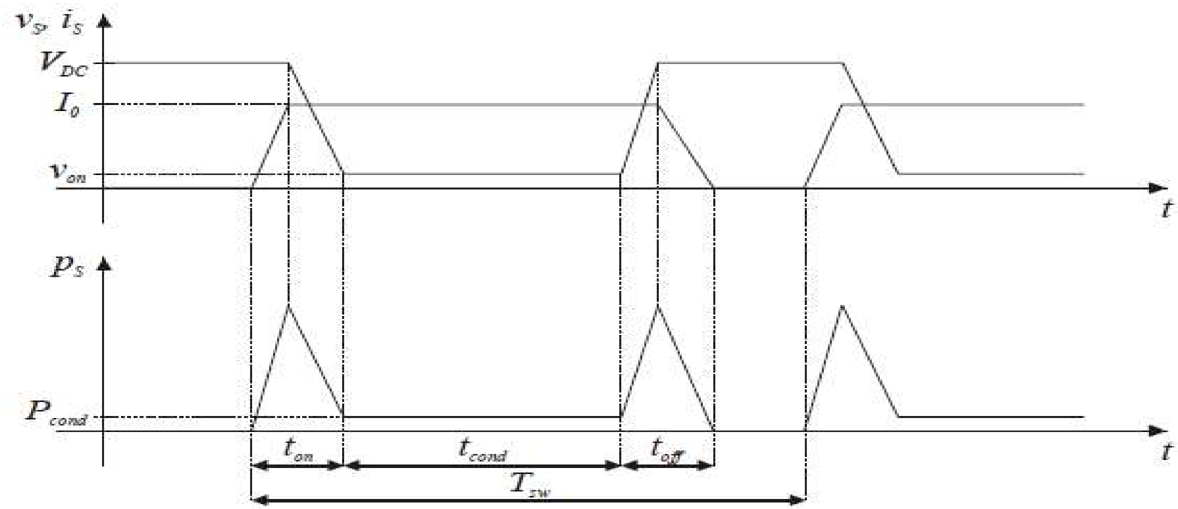
$$\Delta u = -sign(i) \cdot f_{sw} \cdot t_{bl} \cdot U_{dc}$$

Blanking Time + Voltage Drops



Simple Converter Loss Model





$$p_S(t) = v_S(t) \cdot i_S(t)$$

Switching and Conduction losses

Energy losses: $E_S(T_{sw}) = \int_{T_{sw}} p_S(\tau)d\tau = E_{S,on}(T_{sw}) + E_{S,cond}(T_{sw}) + E_{S,off}(T_{sw})$

$$E_{S,on}(T_{sw}) = \int_{t_{on}} p_S(\tau)d\tau = V_{DC} \cdot I_0 \cdot \frac{t_{on}}{2}$$

$$E_{S,cond}(T_{sw}) = \int_{t_{cond}} p_S(\tau)d\tau = V_{S(on)} \cdot I_0 \cdot t_{cond}$$

Note $V_{S(on)} = V_{S0} + R_S \cdot I_0$

$$E_{S,off}(T_{sw}) = \int_{t_{off}} p_S(\tau)d\tau = V_{DC} \cdot I_0 \cdot \frac{t_{off}}{2}$$

Power losses: $P_S(T_{sw}) = \frac{E_S(T_{sw})}{T_{sw}} = P_{S,on}(T_{sw}) + P_{S,cond}(T_{sw}) + P_{S,off}(T_{sw})$

$$P_{S,on}(T_{sw}) = \frac{E_{S,on}(T_{sw})}{T_{sw}} = E_{S,on}(T_{sw}) \cdot f_{sw} = \frac{V_{DC} \cdot I_0 \cdot t_{on}}{2} \cdot f_{sw}$$

$$P_{S,cond}(T_{sw}) = \frac{E_{S,cond}(T_{sw})}{T_{sw}} = V_{S(on)} \cdot I_0 \cdot \frac{t_{cond}}{T_{sw}} = V_{S(on)} \cdot I_0 \cdot D_S$$

$$P_{S,off}(T_{sw}) = \frac{E_{S,off}(T_{sw})}{T_{sw}} = E_{S,off}(T_{sw}) \cdot f_{sw} = \frac{V_{DC} \cdot I_0 \cdot t_{off}}{2} \cdot f_{sw}$$

$$P_{S,sw}(T_{sw}) = P_{S,on}(T_{sw}) + P_{S,off}(T_{sw})$$

Reverse recovery Losses

If specified, use:

$$E_{S,on}(T_{sw})=\frac{E_{on,n}}{V_{DC,n}\cdot I_{0,n}}\cdot V_{DC}\cdot I_0$$

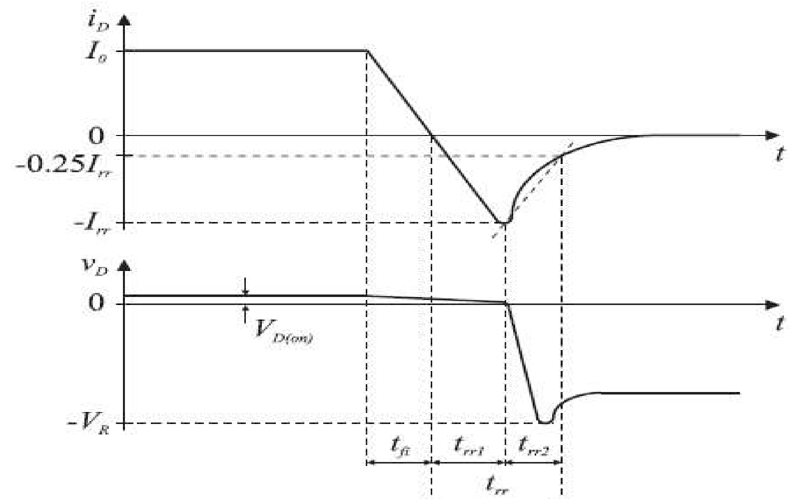
$$E_{S,off}(T_{sw})=\frac{E_{off,n}}{V_{DC,n}\cdot I_{0,n}}\cdot V_{DC}\cdot I_0$$

For the freewheeling diode:

$$P_{D,cond}(T_{sw})=V_{D(on)}\cdot I_0\cdot D_D \qquad V_{D(on)}=V_{D0}+R_D\cdot I_0$$

$$D_D\approx 1-D_S$$

$$P_{D,rr}=V_{DC}\cdot Q_f\cdot f_{sw} \qquad Q_f\approx\frac{1}{S+1}\cdot Q_{rr}\ \text{where}\ S=\frac{t_{rr1}}{t_{rr2}}$$

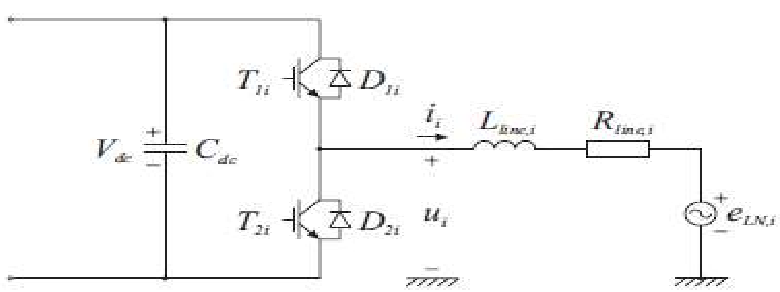


Fig

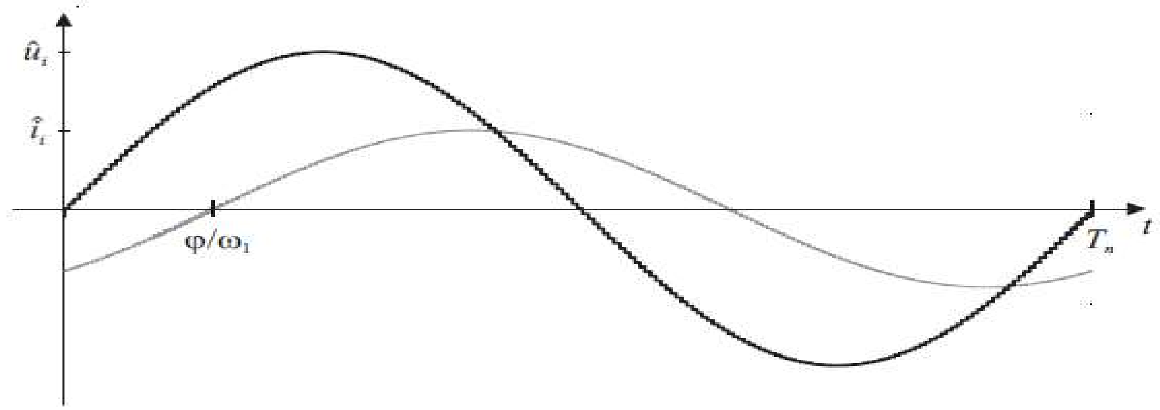
If specified, use:

$$P_{D,off}=E_{D,off}(T_{sw})\cdot f_{sw}\ ,\ E_{D,off}(T_{sw})=\frac{E_{off,n}}{V_{DC,n}\cdot I_{0,n}}\cdot V_{DC}\cdot I_0 \qquad Q_f=\frac{Q_{f,n}}{I_{0,n}}\cdot I_0$$

3-phase converter losses



One half-bridge of a three-phase voltage source converter.



Converter output voltage and current. The current is displaced by an angle φ relative to the voltage.

Loss estimation

Switching losses:

$$\overline{P}_{Ti,sw}=\frac{1}{T_n}\int_{T_n}\left(P_{on}+P_{off}\right)dt=\frac{f_{sw}}{T_n}\int_{T_n}\left(E_{on}+E_{off}\right)dt=\frac{E_{on,n}+E_{off,n}}{V_{dc,n}\cdot I_n}\cdot\frac{V_{dc}f_{sw}}{T_n}\int_{T_n}\left|\hat{i}_i\sin(\omega_1 t-\varphi)\right|dt=\frac{2\sqrt{2}}{\pi}\cdot\frac{E_{on,n}+E_{off,n}}{V_{dc,n}\cdot I_n}\cdot V_{dc}I_i f_{sw}$$

$$\overline{P}_{Di,sw}=\frac{1}{T_n}\int_{T_n}\left(P_{on}+P_{off}\right)dt=\frac{f_{sw}}{T_n}\int_{T_n}\left(E_{on}+E_{off}\right)dt=\frac{2\sqrt{2}}{\pi}\cdot\frac{E_{off,n}}{V_{dc,n}\cdot I_n}\cdot V_{dc}I_i f_{sw}=\frac{2\sqrt{2}}{\pi}\cdot\frac{E_{Drr,n}}{V_{dc,n}\cdot I_n}\cdot V_{dc}I_i f_{sw}$$

Conduction losses:

$$\overline{P}_{Ti,cond}=\left(\frac{\sqrt{2}}{\pi}\cdot V_{T0}I_i+\frac{1}{2}\cdot R_{T(on)}I_i^2\right)+\left(V_{T0}I_i+\frac{4\sqrt{2}}{3\pi}\cdot R_{T(on)}I_i^2\right)\cdot\frac{U_i\cos(\varphi)}{V_{dc}}$$

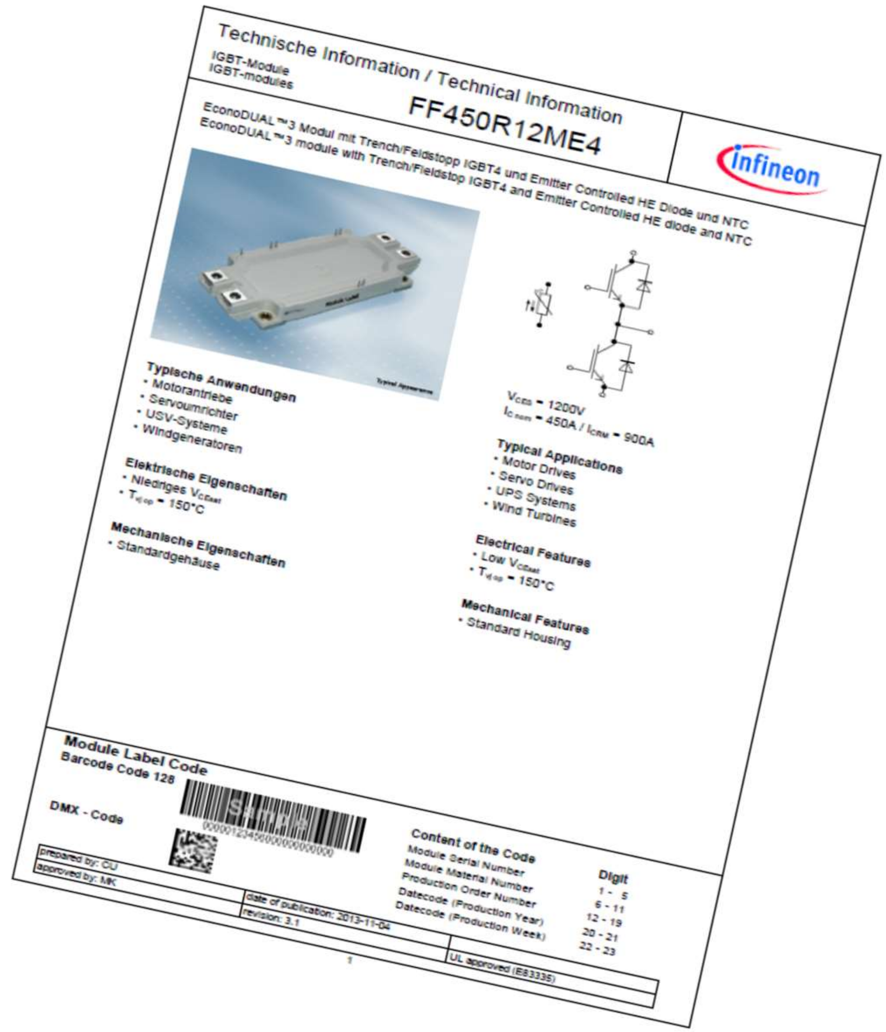
$$\overline{P}_{Di,cond}=\left(\frac{\sqrt{2}}{\pi}\cdot V_{D0}I_i+\frac{1}{2}\cdot R_{D(on)}I_i^2\right)-\left(V_{D0}I_i+\frac{4\sqrt{2}}{3\pi}\cdot R_{D(on)}I_i^2\right)\cdot\frac{U_i\cos(\varphi)}{V_{dc}}$$

Example :

V_to = 0.95; % [V]
V_do = 1.65; % [V]
R_t_on = 0.5/300;   % [Ohm]
R_d_on = 0; % [Ohm]
E_d_rr = 0.0485; % [J]
E_on = 26e-3; % [J]
E_off = 55.5e-3; % [J]

V_dc_n = 600; % [V]
I_n = 450; % [A]

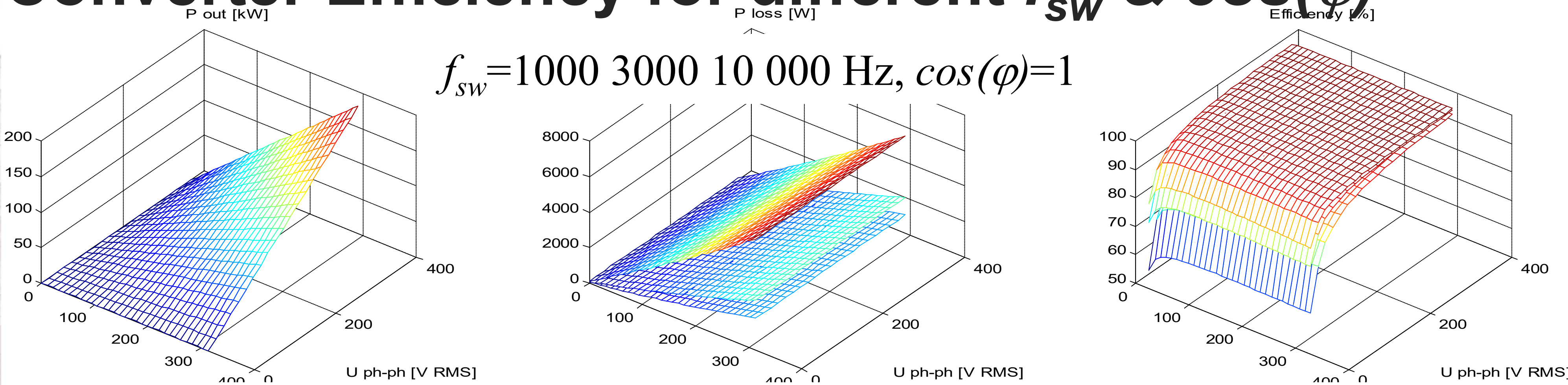
Udc = 600; % [V]
P_max = 200000; % [W]


Technische Information / Technical Information
IGBT-Module
IGBT-modules
FF450R12ME4
EconoDUAL™3 Modul mit Trench/Feldstopp IGBT4 und Emitter Controlled HE Diode und NTC
EconoDUAL™3 module with Trench/Fieldstop IGBT4 and Emitter Controlled HE diode and NTC

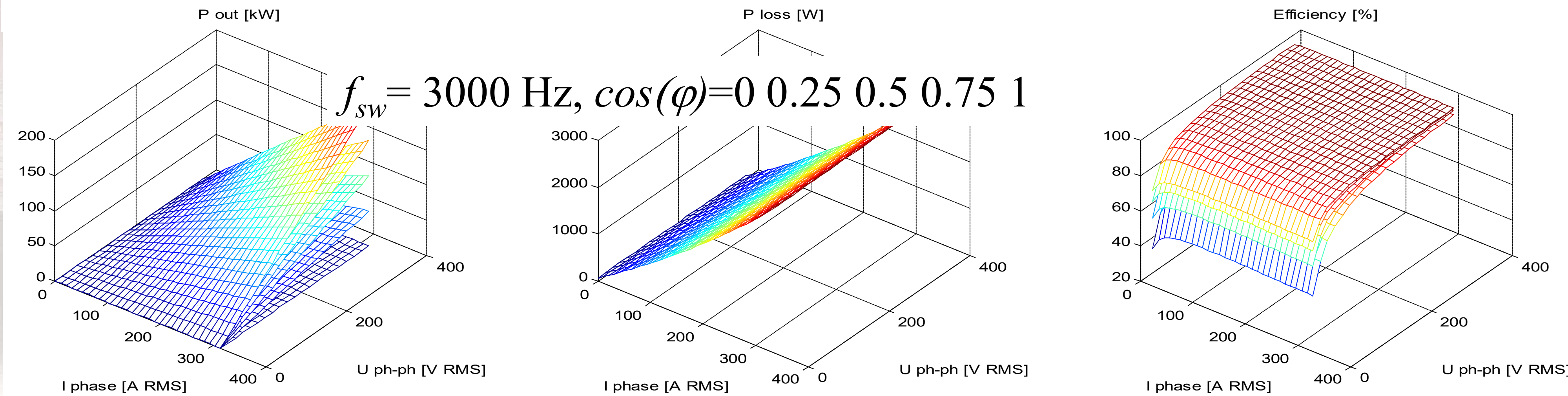
Typische Anwendungen
• Motorantriebe
• Servoumrichter
• USV-Systeme
• Windgeneratoren
Elektrische Eigenschaften
• Niedriges V_{CEsat}
• $T_{vj\,op}$ = 150°C
Mechanische Eigenschaften
• Standardgehäuse
V_{CES} = 1200V
$I_{C\,nom}$ = 450A / I_{CRM} = 900A
Typical Applications
• Motor Drives
• Servo Drives
• UPS Systems
• Wind Turbines
Electrical Features
• Low V_{CEsat}
• $T_{vj\,op}$ = 150°C
Mechanical Features
• Standard Housing
Module Label Code
Barcode Code 128
DMX - Code
Content of the Code
Module Serial Number
Module Material Number
Production Order Number
Datecode (Production Year)
Datecode (Production Week)
Digit
1 - 5
6 - 11
12 - 19
20 - 21
22 - 23
prepared by: CU
approved by: MK
date of publication: 2013-11-04
revision: 3.1
UL approved (E83335)
1

Converter Efficiency for different f_{sw} & $cos(\varphi)$

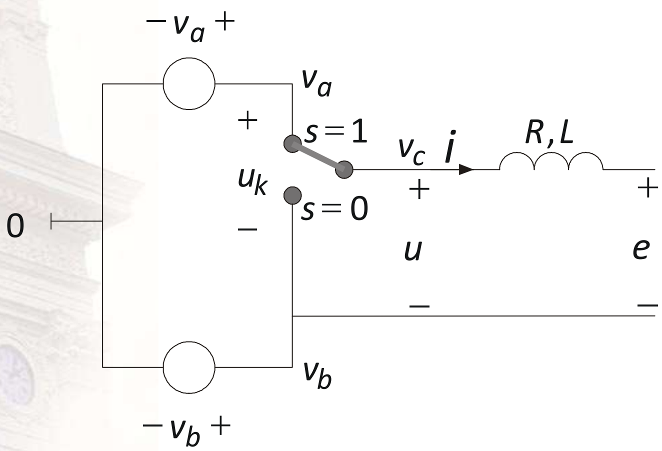
f_{sw}=1000 3000 10 000 Hz, $cos(\varphi)$=1



f_{sw}= 3000 Hz, $cos(\varphi)$=0 0.25 0.5 0.75 1



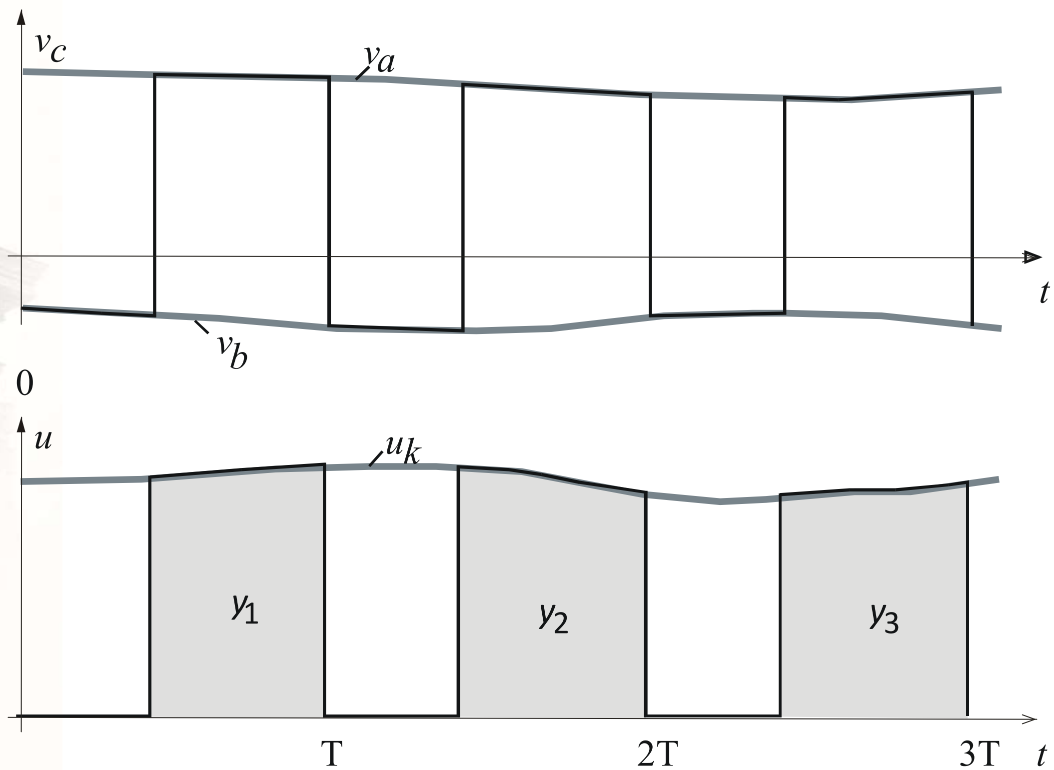
Modulation - Control of voltage time area



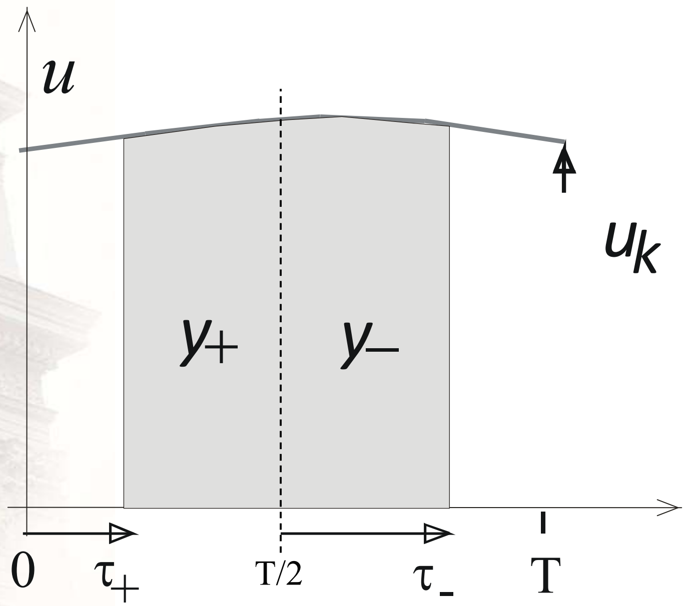
$$v_c = \begin{cases} v_a \text{ when } s = 1 \\ v_b \text{ when } s = 0 \end{cases}$$

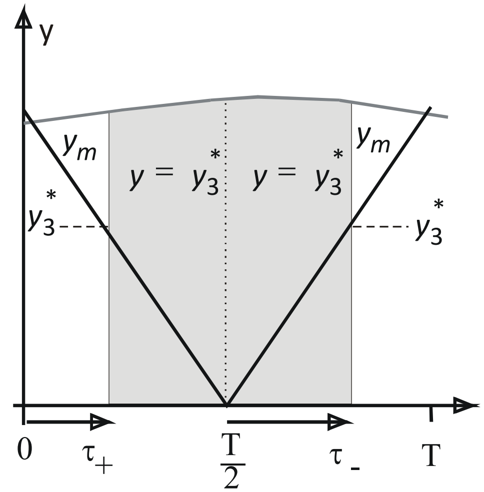
$$u = s \cdot (v_a - v_b) = s \cdot u_k = \begin{cases} u_k \text{ when } s = 1 \\ 0 \text{ when } s = 0 \end{cases}$$

Output voltage



Control with both flanks





$$Y_0 = \int_0^{T/2} u_k \cdot dt \qquad y_+ = \int_{\tau_+}^{T/2} u_k \cdot dt = Y_0 - \int_0^{\tau_+} u_k \cdot dt$$

$$y(\tau_+, \tau_-) = y_+ + y_- \qquad y_- = \int_{T/2}^{T/2+\tau_-} u_k \cdot dt$$

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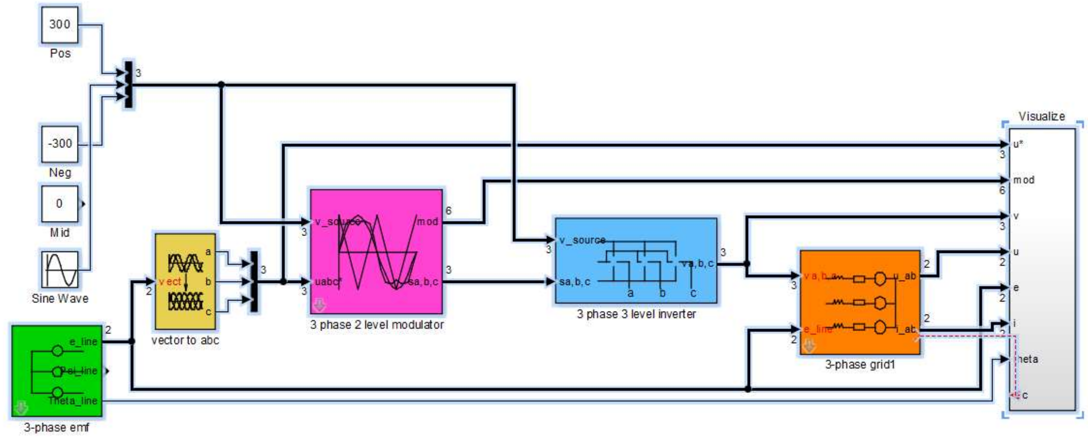
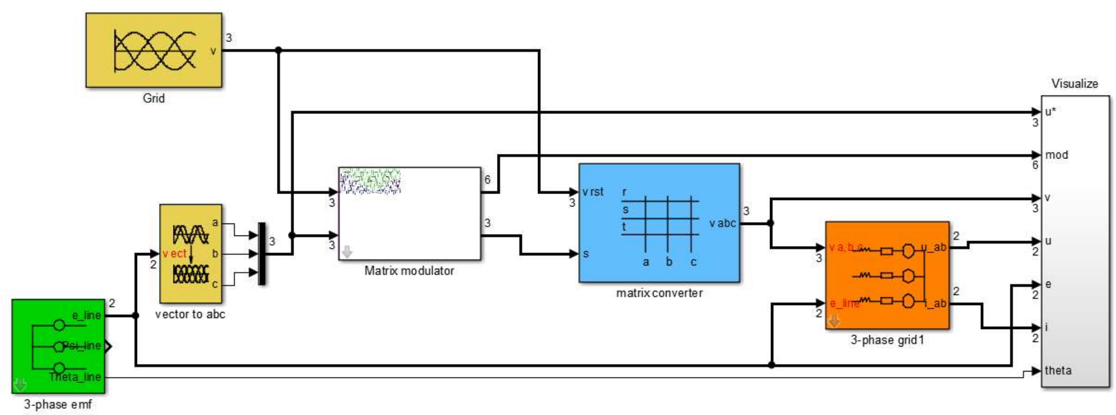
To Simulink !


300
Pos
-300
Neg
0
Mid
Sine Wave
3-phase emf
vector to abc
3 phase 2 level modulator
3 phase 3 level inverter
3-phase grid1
Visualize
u*
mod
v
u
e
i
theta


Grid
3-phase emf
vector to abc
Matrix modulator
matrix converter
3-phase grid1
Visualize
u*
mod
v
u
e
i
theta

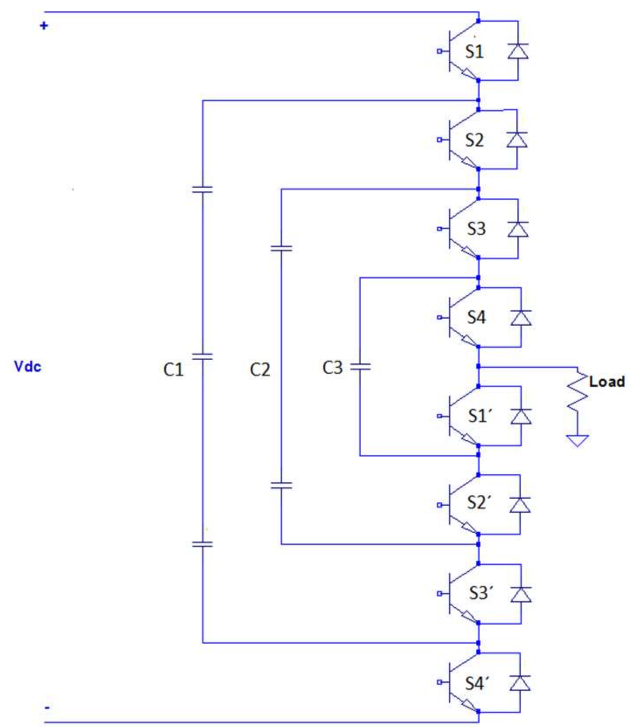
Balancing the capacitor clamped inverter

TABLE I
SWITCHING STATES OF A FIVE-LEVEL FLYING-CAPACITOR INVERTER LEG

State	sc_4	sc_3	sc_2	sc_1	Output	ΔVc_3	ΔVc_2	ΔVc_1	Level
0	0	0	0	0	0	0	0	0	0
01	0	0	0	1	¼ E	0	0	-	1
02	0	0	1	0	¼ E	0	-	+	1
03	0	0	1	1	½ E	0	-	0	2
04	0	1	0	0	¼ E	-	+	0	1
05	0	1	0	1	½ E	-	+	-	2
06	0	1	1	0	½ E	-	0	+	2
07	0	1	1	1	¾ E	-	0	0	3
08	1	0	0	0	¼ E	+	0	0	1
09	1	0	0	1	½ E	+	0	-	2
0A	1	0	1	0	½ E	+	-	+	2
0B	1	0	1	1	¾ E	+	-	0	3
0C	1	1	0	0	½ E	0	+	0	2
0D	1	1	0	1	¾ E	0	+	-	3
0E	1	1	1	0	¾ E	0	0	+	3
0F	1	1	1	1	E	0	0	0	4
Switching states						ΔV_{ck} = Flying capacitors voltage evolutions			

ΔV_{ck} : + = Positive evolution - = Negative evolution 0 = Without evolution

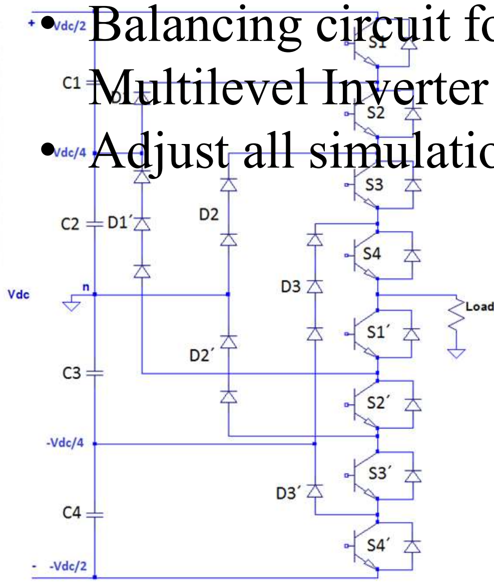
Available switch states and corresponding voltage evolution
(from "Flying Capacitor Multilevel Inverters and DTC Motor Drive Applications" by M. Escalante, J. Vannier, A. Arzandé)



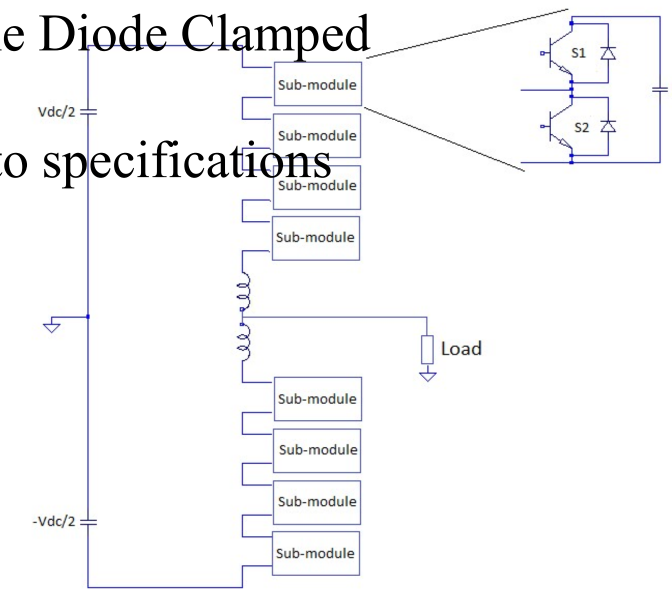
One phase leg of a 5-lvl capacitor clamped inverter

Simulation work to do

- Add the motor model
- Balancing modulation for the Modular Multilevel Inverter
- Balancing circuit for the Diode Clamped Multilevel Inverter
- Adjust all simulations to specifications



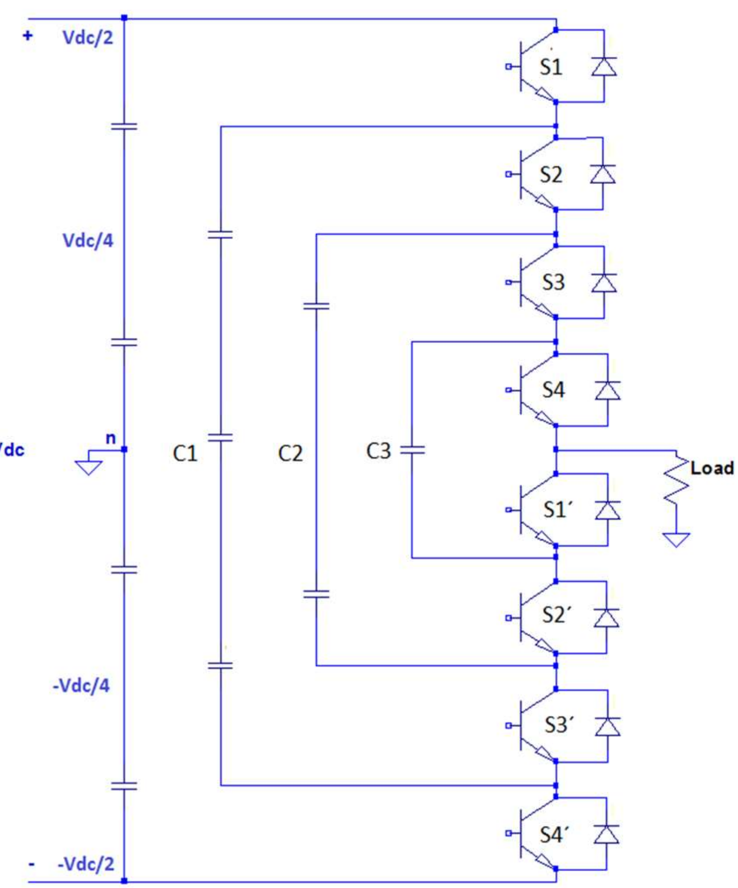
One phase leg of the 5-lvl diode clamped inverter



One phase leg of the 5-lvl modular inverter

Capacitor Clamped MLI

- Kapacitanser i serie
- Mäta alla kapacitans värden
- Look-up Table i Spice



CCMLI - Balansering

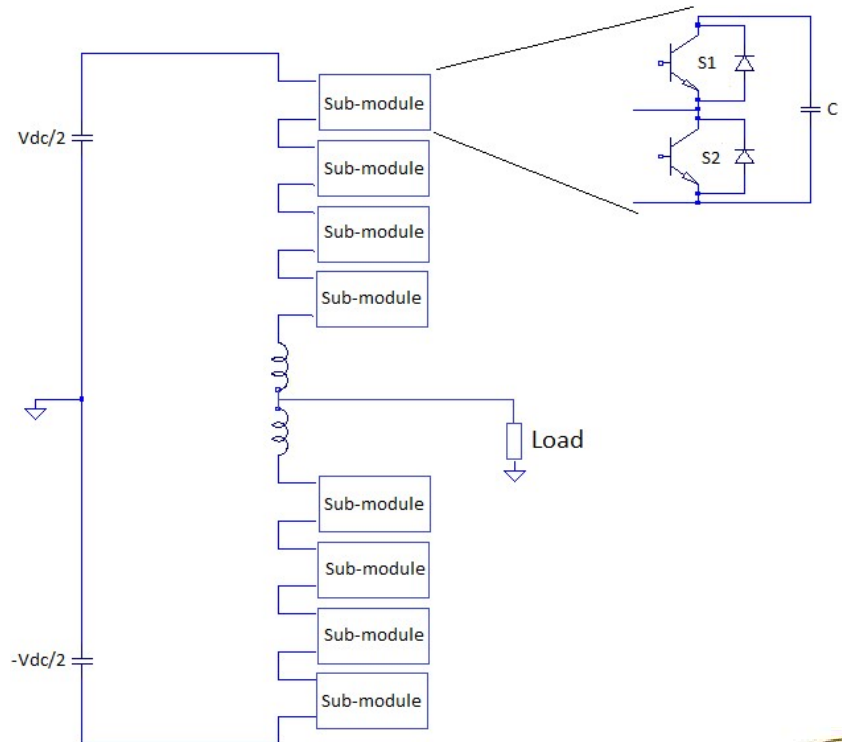
State	sc_4	sc_3	sc_2	sc_1	Output	ΔVc_3	ΔVc_2	ΔVc_1	Level
0	*0*	*0*	*0*	*0*	0	0	0	0	0
01	*0*	*0*	*0*	*1*	¼ E	0	0	-	1
02	*0*	*0*	*1*	*0*	¼ E	0	-	+	1
03	*0*	*0*	*1*	*1*	½ E	0	-	0	2
04	*0*	*1*	*0*	*0*	¼ E	-	+	0	1
05	*0*	*1*	*0*	*1*	½ E	-	+	-	2
06	*0*	*1*	*1*	*0*	½ E	-	0	+	2
07	*0*	*1*	*1*	*1*	¾ E	-	0	0	3
08	*1*	*0*	*0*	*0*	¼ E	+	0	0	1
09	*1*	*0*	*0*	*1*	½ E	+	0	-	2
0A	*1*	*0*	*1*	*0*	½ E	+	-	+	2
0B	*1*	*0*	*1*	*1*	¾ E	+	-	0	3
0C	*1*	*1*	*0*	*0*	½ E	0	+	0	2
0D	*1*	*1*	*0*	*1*	¾ E	0	+	-	3
0E	*1*	*1*	*1*	*0*	¾ E	0	0	+	3
0F	*1*	*1*	*1*	*1*	E	0	0	0	4
Switching states						ΔV_{ck} = Flying capacitors voltage evolutions			

ΔV_{ck} : + = Positive evolution - = Negative evolution 0 = Without evolution

Capacitors voltage states						Switching state		
LS3	LI3	LS2	LI2	LS1	LI1	Level 1	Leve 2	Level 3
0	0	0	0	0	0	(q+1)	(q+1)	(q+1)
0	0	0	0	0	1	02	06	0E
0	0	0	0	1	0	01	09	0D
0	0	0	1	0	0	04	0C	0D
0	0	0	1	0	1	04	0C	0E
0	0	0	1	1	0	01	0C	0D
0	0	1	0	0	0	02	03	0B
0	0	1	0	0	1	02	0A	0E
0	0	1	0	1	0	01	03	0B
0	1	0	0	0	0	08	09	0B
0	1	0	0	0	1	08	0A	0E
0	1	0	0	1	0	08	09	0B
0	1	0	1	0	0	08	0C	0D
0	1	0	1	0	1	08	0C	0E
0	1	0	1	1	0	08	09	0D
0	1	1	0	0	0	08	03	0B
0	1	1	0	0	1	02	0A	0B
0	1	1	0	1	0	08	09	0B
1	0	0	0	0	0	04	06	07
1	0	0	0	0	1	04	06	07
1	0	0	0	1	0	01	05	07
1	0	0	1	0	0	04	0C	07
1	0	0	1	0	1	04	06	07
1	0	0	1	1	0	04	05	0D
1	0	1	0	0	0	02	03	07
1	0	1	0	0	1	02	06	07
1	0	1	0	1	0	01	03	07

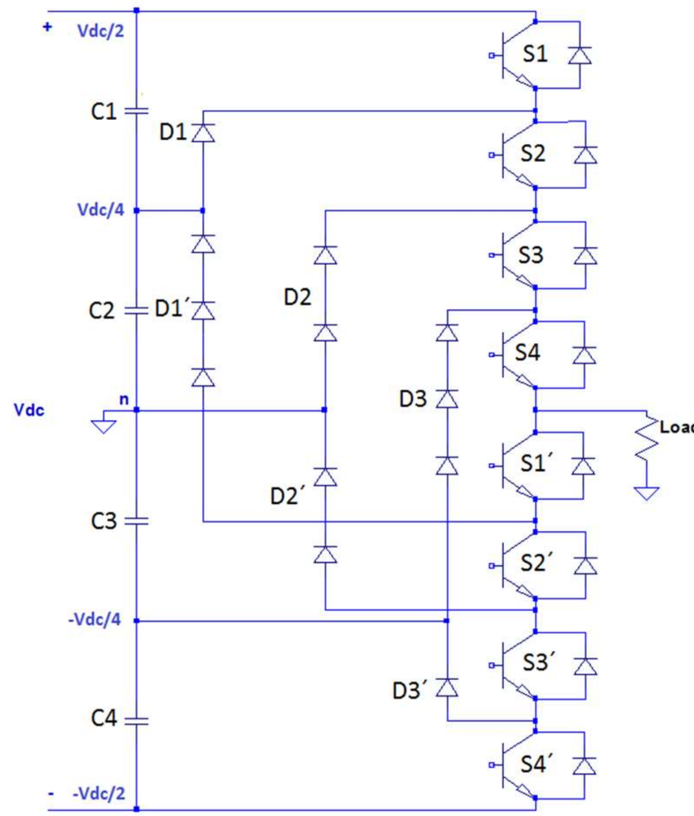
Modular MLI

- Balansera en fas som i CCMLI
- Balansera tre faser – Se modular.pdf

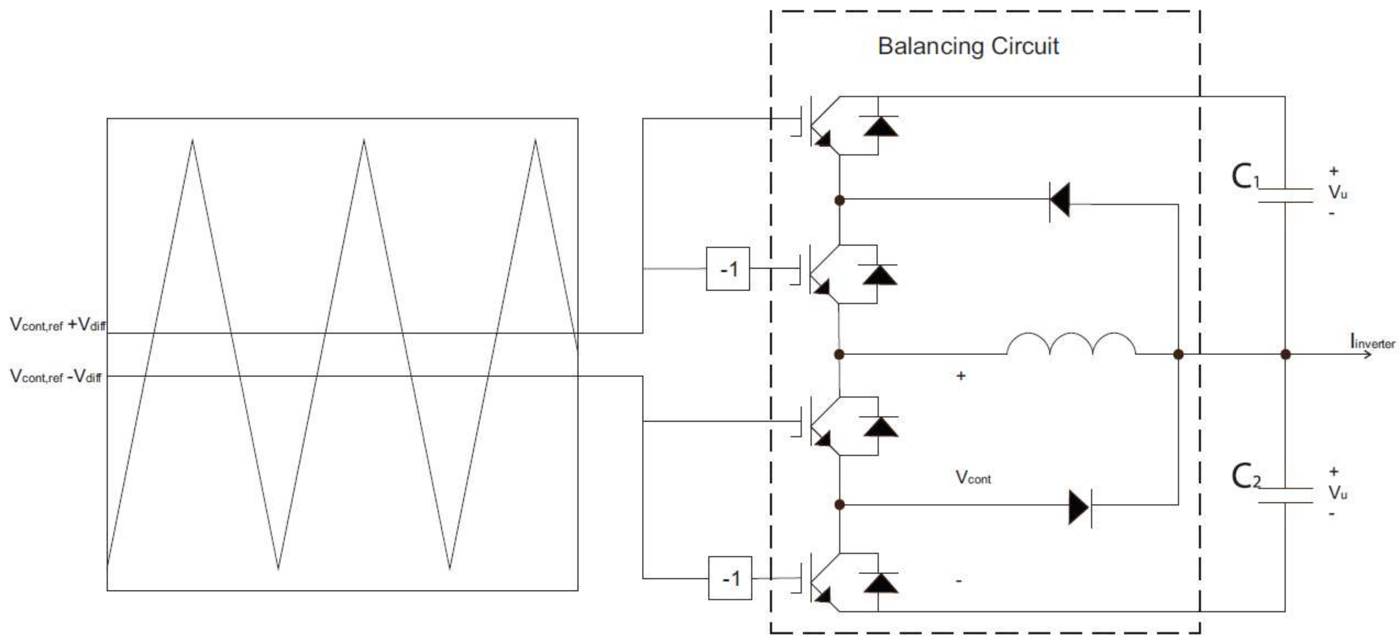


Natural Point Clamped MLI

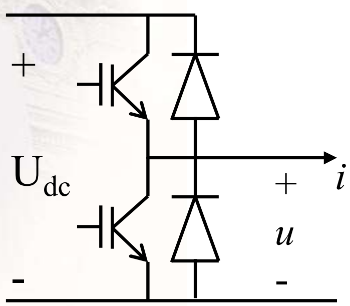
- 3-level version (simulering)
- Fler nivåer => Balanserings problem
- Balansera med extra krets

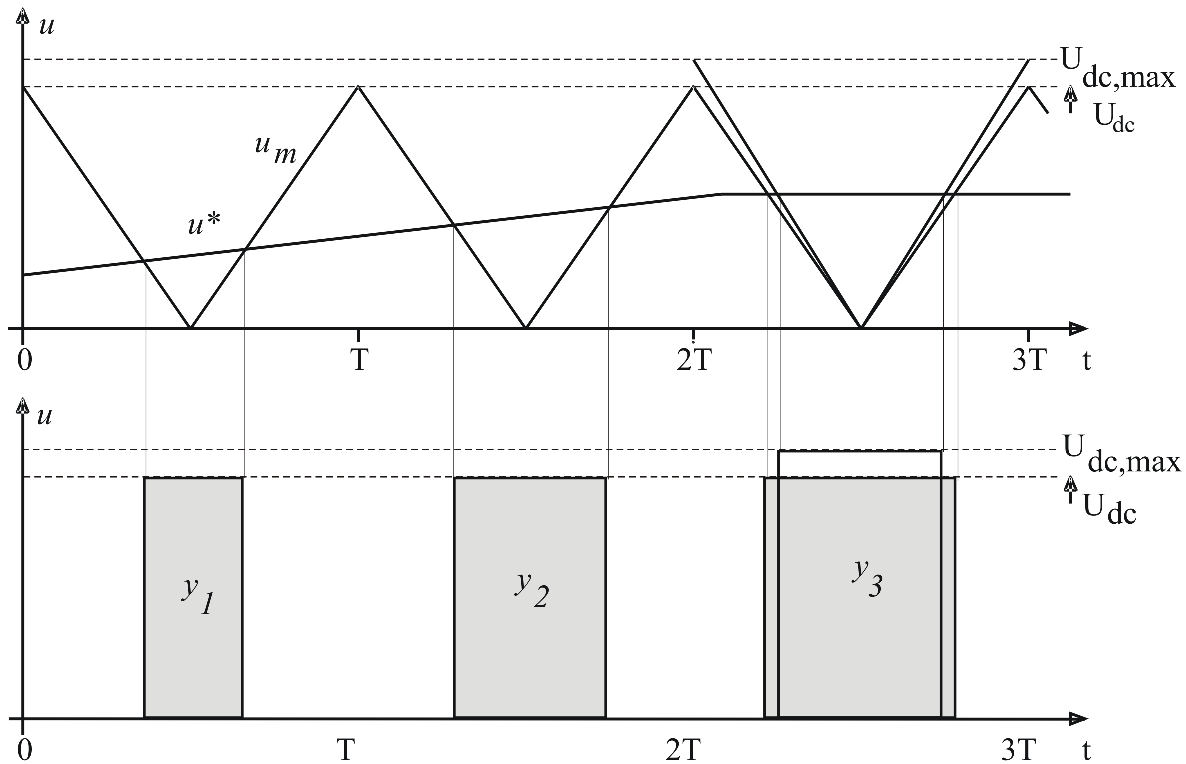


NPCMLI - Balansering

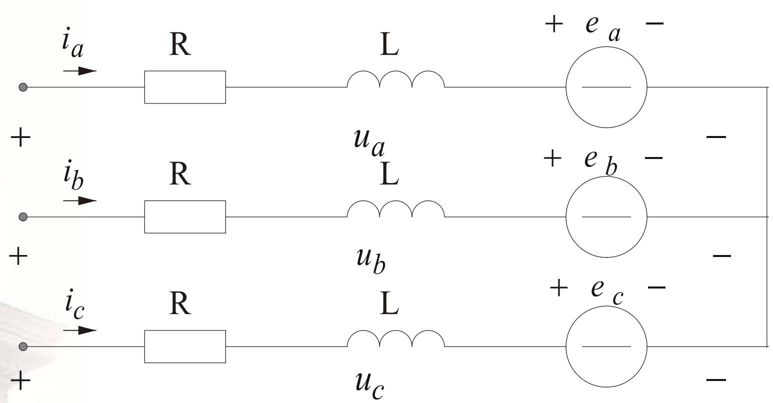


Two quadrant DC converters : II





The generic 3-phase load



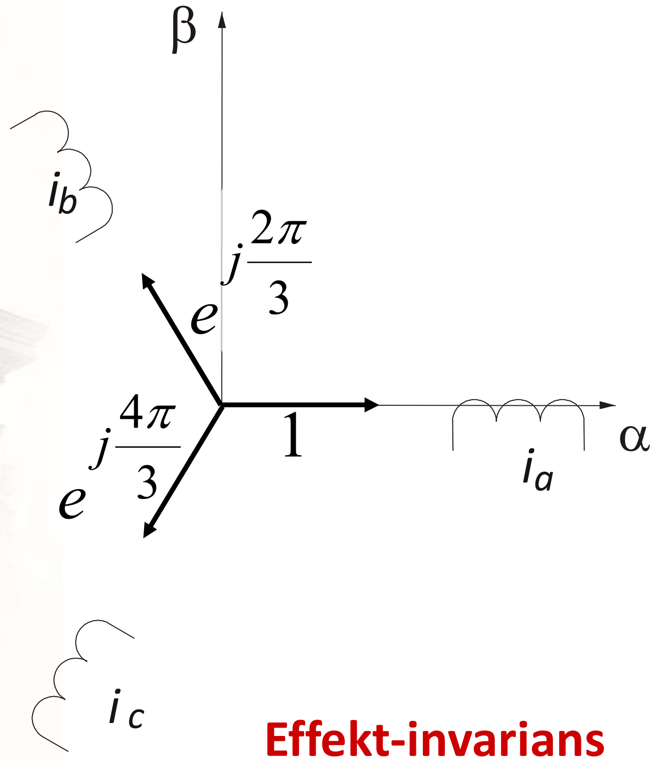
$$\sqrt{\frac{2}{3}}\left(u_a = R\cdot i_a + L\cdot\frac{di_a}{dt} + e_a\right)$$

$$\sqrt{\frac{2}{3}}\cdot e^{j\frac{2\pi}{3}}\cdot\left(u_b = R\cdot i_b + L\cdot\frac{di_a}{dt} + e_b\right)$$

$$+\ \sqrt{\frac{2}{3}}\cdot e^{j\frac{4\pi}{3}}\cdot\left(u_c = R\cdot i_c + L\cdot\frac{di_a}{dt} + e_c\right)$$

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{di}{dt} + \vec{e}$$

Vectors in 3-phase systems



$$\sqrt{\frac{2}{3}}\left(u_a = R\cdot i_a + L\cdot\frac{di_a}{dt} + e_a\right)$$

$$\sqrt{\frac{2}{3}}\cdot e^{j\frac{2\pi}{3}}\cdot\left(u_b = R\cdot i_b + L\cdot\frac{di_a}{dt} + e_b\right)$$

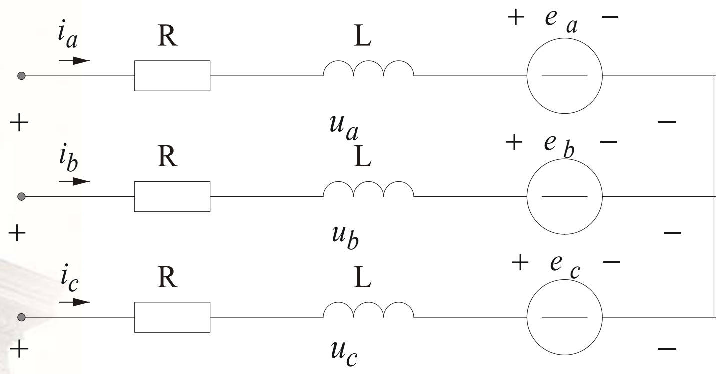
$$+\ \sqrt{\frac{2}{3}}\cdot e^{j\frac{4\pi}{3}}\cdot\left(u_c = R\cdot i_c + L\cdot\frac{di_a}{dt} + e_c\right)$$

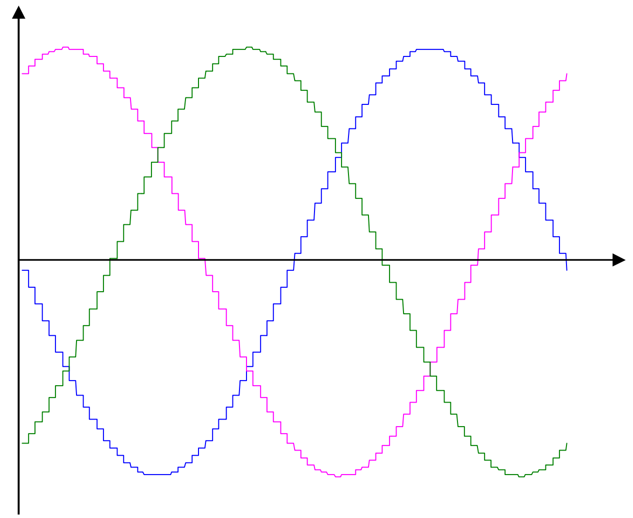
$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{di}{dt} + \vec{e}$$

Effekt-invarians

$$p(t) = u_a\cdot i_a + u_b\cdot i_b + u_c\cdot i_c = u_\alpha\cdot i_\alpha + u_\beta\cdot i_\beta$$

Symmetric emf





$$\begin{cases} e_a = \hat{e} \cdot \cos(\omega \cdot t) \\ e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \\ e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \end{cases}$$

Example, grid voltage vector

$$\vec{e} = \sqrt{\frac{2}{3}} \cdot \left(e_a + e_b \cdot e^{j\frac{2\pi}{3}} + e_c \cdot e^{j\frac{4\pi}{3}} \right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\cos(\omega \cdot t) + \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\begin{array}{l} \cos(\omega \cdot t) + \left(\cos(\omega \cdot t) \cdot \cos\left(\frac{2\pi}{3}\right) + \sin(\omega \cdot t) \cdot \sin\left(\frac{2\pi}{3}\right) \right) \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) \\ + \left(\cos(\omega \cdot t) \cdot \cos\left(\frac{4\pi}{3}\right) + \sin(\omega \cdot t) \cdot \sin\left(\frac{4\pi}{3}\right) \right) \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \end{array} \right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\cos(\omega \cdot t) \cdot \left(1 + \frac{1}{4} + \frac{1}{4}\right) + j \cdot \sin(\omega t \cdot t) \cdot \left(\frac{3}{4} + \frac{3}{4}\right) \right) =$$

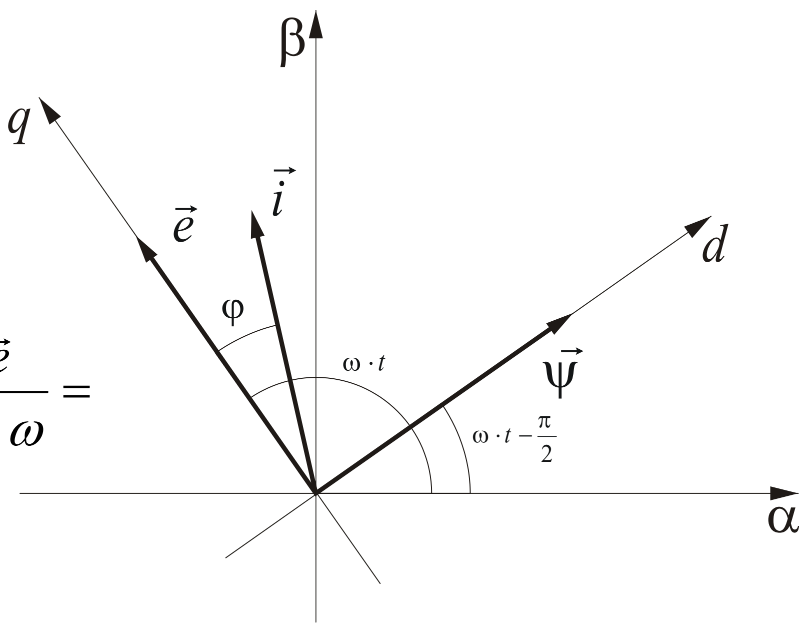
$$= \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot (\cos(\omega \cdot t) + j \cdot \sin(\omega t \cdot t)) = E \cdot e^{j\omega t}$$

Rotating reference frame

Use the integral of
The grid back emf
vector:

$$\vec{\psi} = \int_0^t \vec{e} \cdot dt = \int_0^t E \cdot e^{j\omega \cdot t} dt = \frac{\vec{e}}{j \cdot \omega} =$$

$$= \frac{E}{\omega} e^{j\left(\omega \cdot t - \frac{\pi}{2}\right)}$$



Voltage equation in the (d,q)-frame

$$\vec{u} = R \cdot \vec{i} + L \cdot \frac{d\vec{i}}{dt} + j \cdot \omega \cdot L \cdot \vec{i} + \vec{e}$$

Active power ...

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{d\vec{i}}{dt} + j\cdot\omega\cdot L\cdot\vec{i} + \vec{e}$$

$$p(t) = \mathrm{Re}\left\{\vec{u}\cdot\vec{i}^{\,*}\right\} = \mathrm{Re}\left\{R\cdot\vec{i}\cdot\vec{i}^{\,*} + L\cdot\frac{d\vec{i}}{dt}\cdot\vec{i}^{\,*} + j\cdot\omega\cdot L\cdot\vec{i}\cdot\vec{i}^{\,*} + \vec{e}\cdot\vec{i}^{\,*}\right\} =$$

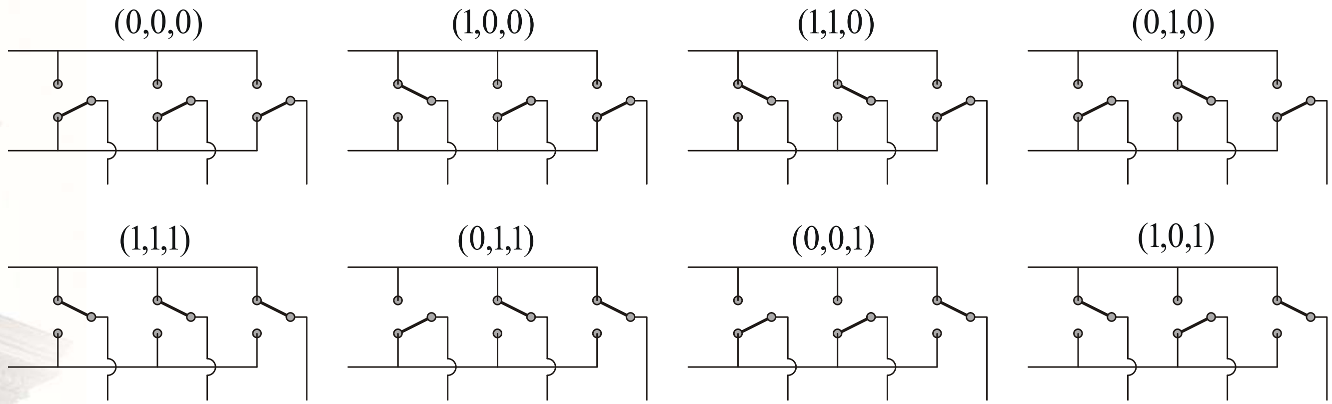
$$= \underbrace{Ri_d^2 + Ri_q^2}_{1} + \underbrace{L\frac{di_d}{dt}i_d + L\frac{di_q}{dt}i_q}_{2} + \underbrace{e_q i_q}_{3}$$

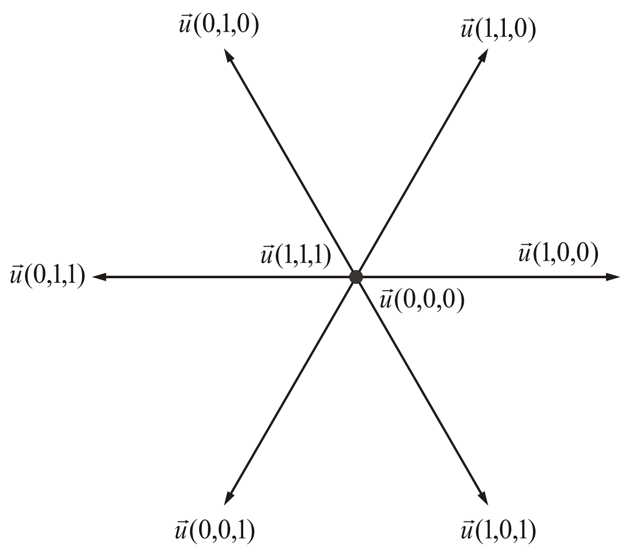
1. Resistive losses
2. Energizing inductances
3. Power absorbed by the grid back emf

Stationarity:

$$p(t) = E\cdot\left|\vec{i}\right|\cdot\cos(\varphi) = E\cdot\sqrt{\frac{3}{2}}\cdot\left|\hat{i}_{phase}\right|\cdot\cos(\varphi) = \sqrt{3}\cdot E\cdot I_{rms,\,phase}\cdot\cos(\varphi)$$

3-phase converters – 8 switch states





$$\vec{u}(1,0,0) = \sqrt{\frac{2}{3}} U_{dc} = -\vec{u}(0,1,1)$$

$$\vec{u}(0,1,0) = \sqrt{\frac{2}{3}} U_{dc} \cdot e^{j\frac{2\cdot\pi}{3}} = -\vec{u}(1,0,1)$$

$$\vec{u}(0,0,1) = \sqrt{\frac{2}{3}} U_{dc} \cdot e^{j\frac{4\cdot\pi}{3}} = -\vec{u}(1,1,0)$$

$$\vec{u}(0,0,0) = 0 = \vec{u}(1,1,1)$$

3-phase converters - sinusoidal references

$$\vec{u}^* = u^* \cdot e^{j\omega t} = u^* \cdot \left(\cos(\omega t) + j \cdot \sin(\omega t)\right) = u_\alpha^* + j \cdot u_\beta^*$$

$$u_a^* = \sqrt{\frac{2}{3}} u_\alpha^*$$

$$u_b^* = \frac{1}{\sqrt{2}} u_\beta^* - \frac{1}{\sqrt{6}} u_\alpha^*$$

$$u_c^* = -\frac{1}{\sqrt{2}} u_\beta^* - \frac{1}{\sqrt{6}} u_\alpha^*$$

$\Rightarrow$

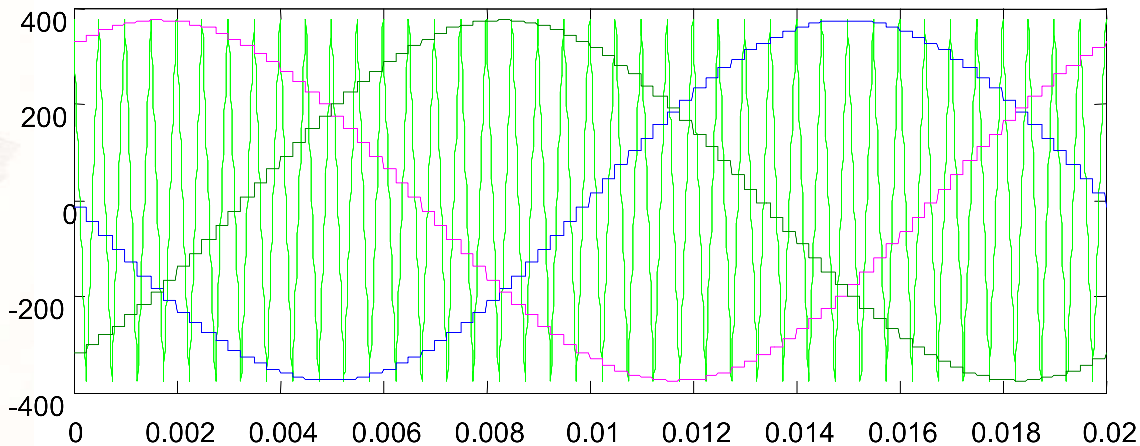
$$u_a^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t)$$

$$u_b^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t - \frac{2\pi}{3})$$

$$u_c^* = \sqrt{\frac{2}{3}} \cdot u^* \cos(\omega t - \frac{4\pi}{3})$$

3-phase converters modulation

Simplest with sinusoidal references...



... but the DC link voltage is badly utilized.

3-phase converters – symmetrization

3 phase potentials, only 2 vector components. One degree of freedom to be used for other purposes.

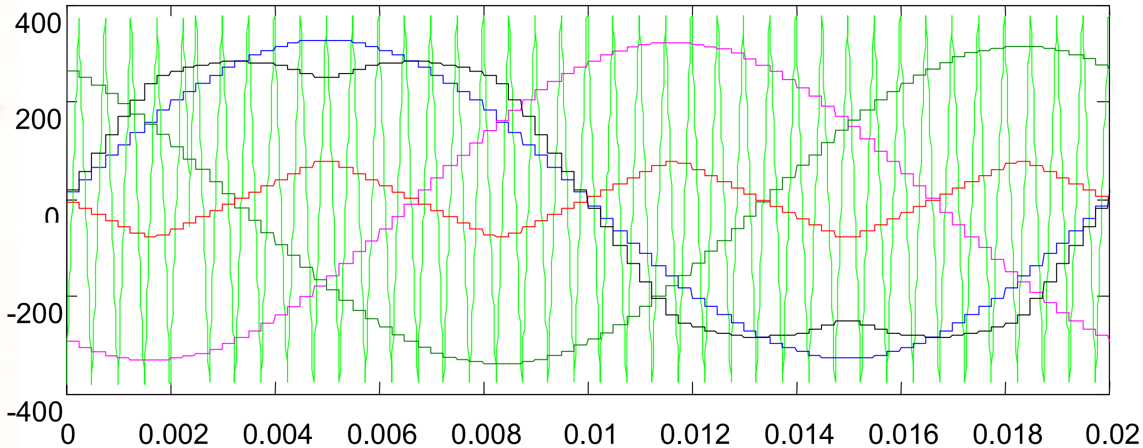
$$v_{az}^{*} = u_{a}^{*} - v_{z}^{*}$$

$$v_{bz}^{*} = u_{b}^{*} - v_{z}^{*}$$

$$v_{cz}^{*} = u_{c}^{*} - v_{z}^{*}$$

3-phase symmetrized modulation

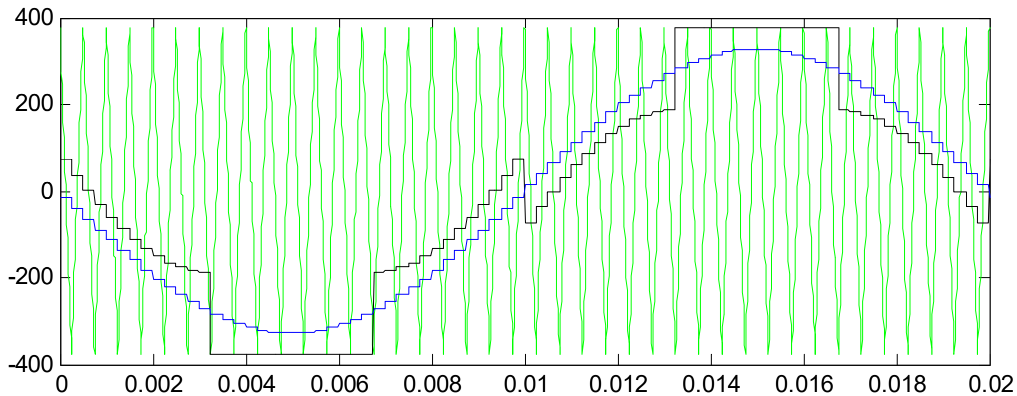
$$v_z^* = \frac{\max(u_a^*, u_b^*, u_c^*) + \min(u_a^*, u_b^*, u_c^*)}{2}$$



Maximum phase voltage with sinusoidal modulation : Udc/2
Maximum phase-to phase voltage with symmetrized modulation : Udc -> Phase voltage Udc/sqrt(3), i.e. 2/sqrt(3)=1.15 times larger than with sinusoidal modulation.

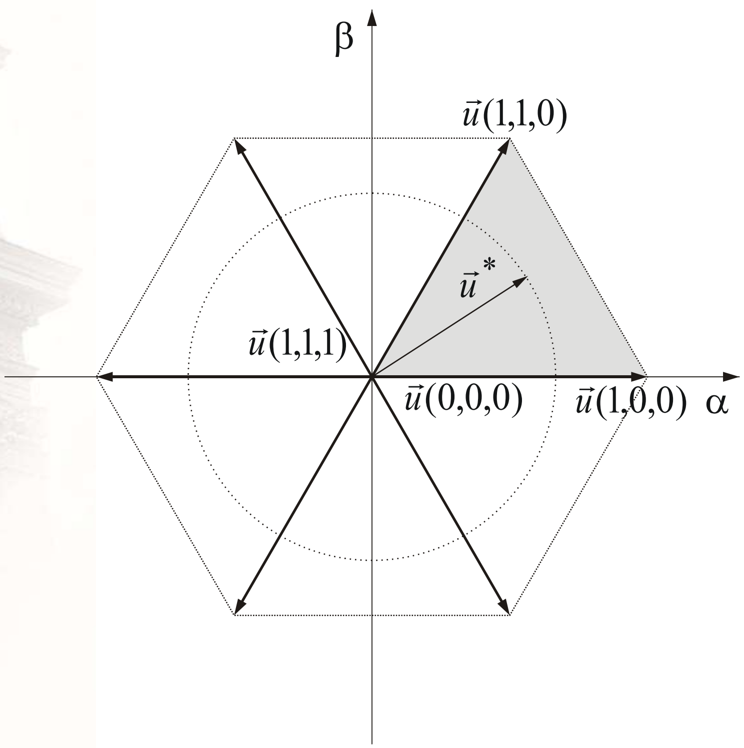
3-phase minimum switching modulation

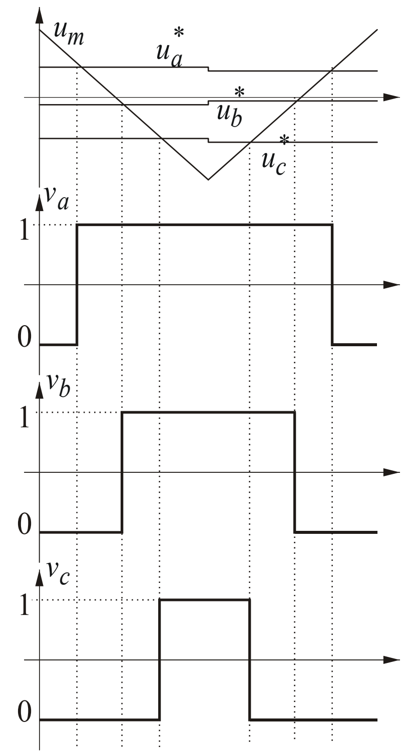
$$v_z{}^* = -\min\left(\frac{U_{dc}}{2} - \max\left(u_a^*, u_b^*, u_c^*\right), -\frac{U_{dc}}{2} - \min\left(u_a^*, u_b^*, u_c^*\right)\right)$$



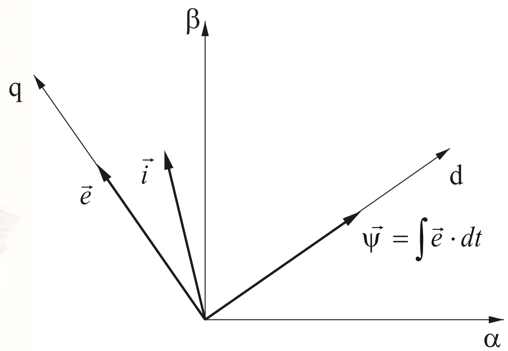
One phase is not switching for 2 60 degree intervals ...

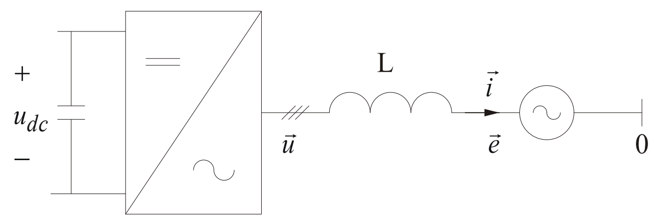
Modulation sequence vs. ripple





Modulation sequence vs. ripple





$$\frac{d\vec{i}}{dt} = \frac{\vec{u} - \vec{e}}{L}$$

Current ripple in the (d,q)-frame