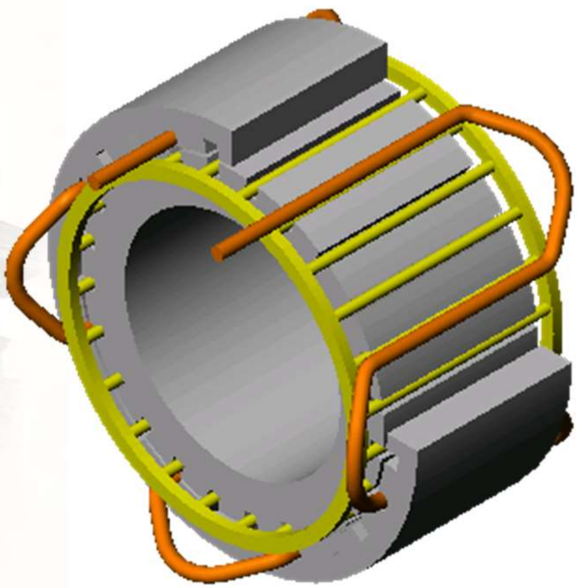
Torque Control of

Induction Machines ...

… as compared to PMSM

Power Electronics / Induction Machine

The Induction Machine



Three Phase Stator

No Magnets in the Rotor

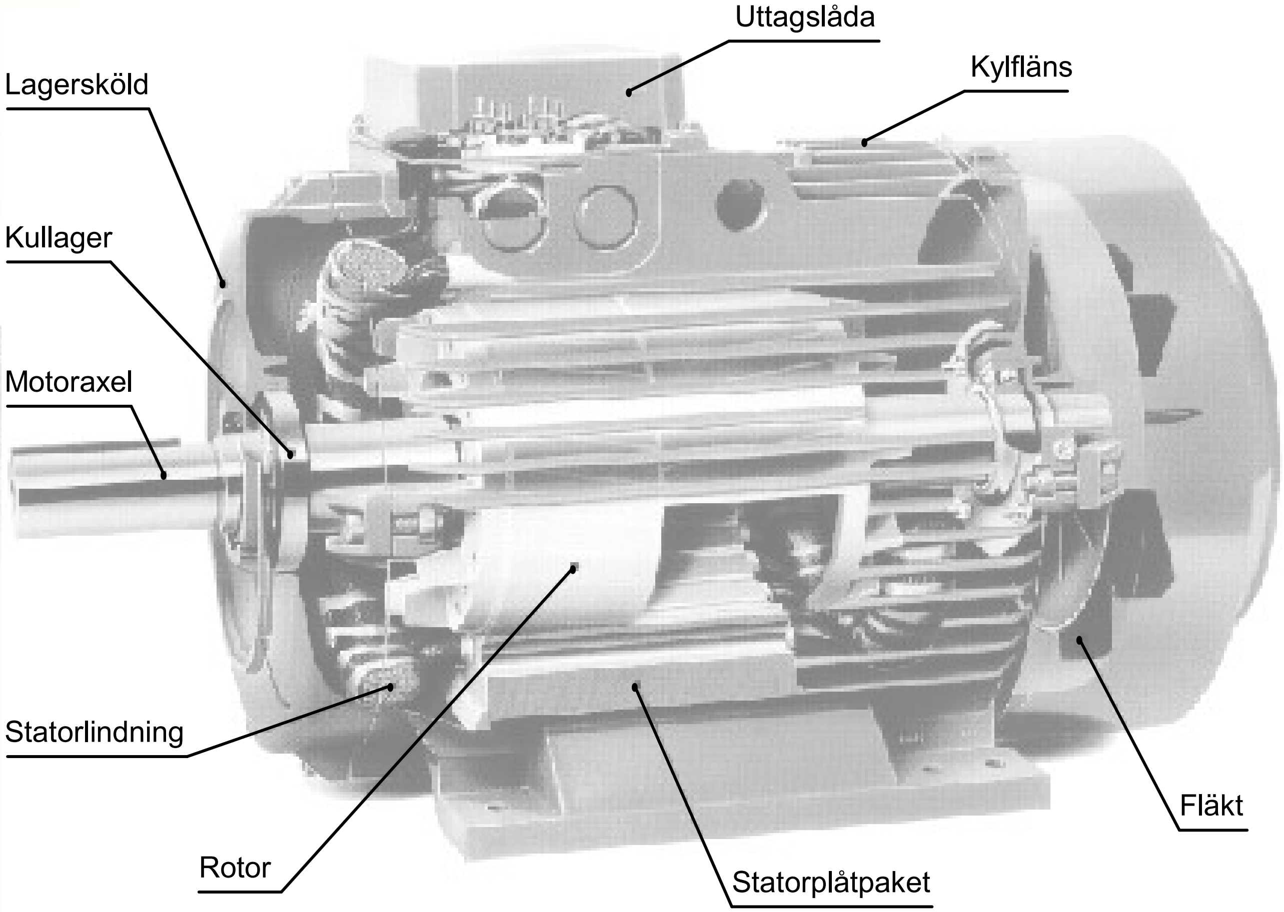
Rotor as Short Circuited ”Cage”

The rotor current must be induced

Three Phase Stator

Three phase power electronics

A look under the hood

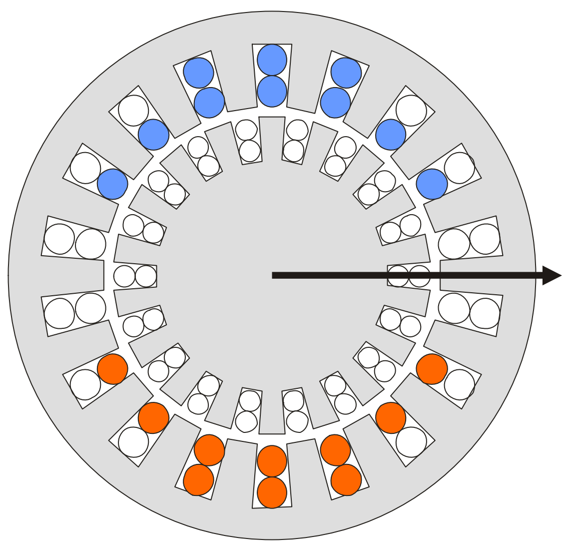


Advantages

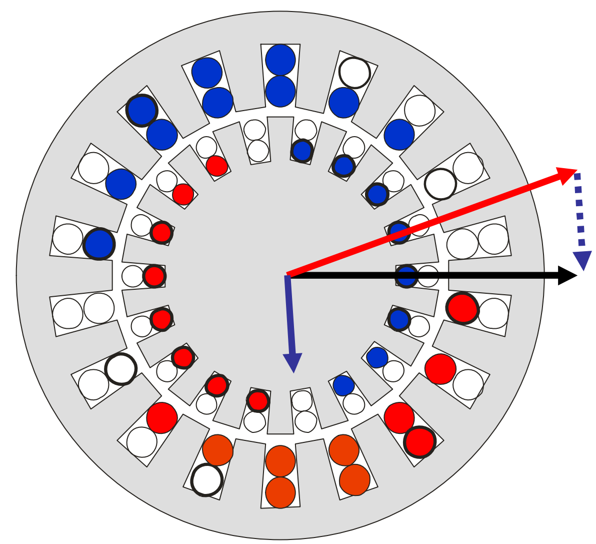
- Self starting when connected to the power grid
- Robust and reliable
- Cheap
- Low maintenance
- Standardized

IM - dynamics

Start here …

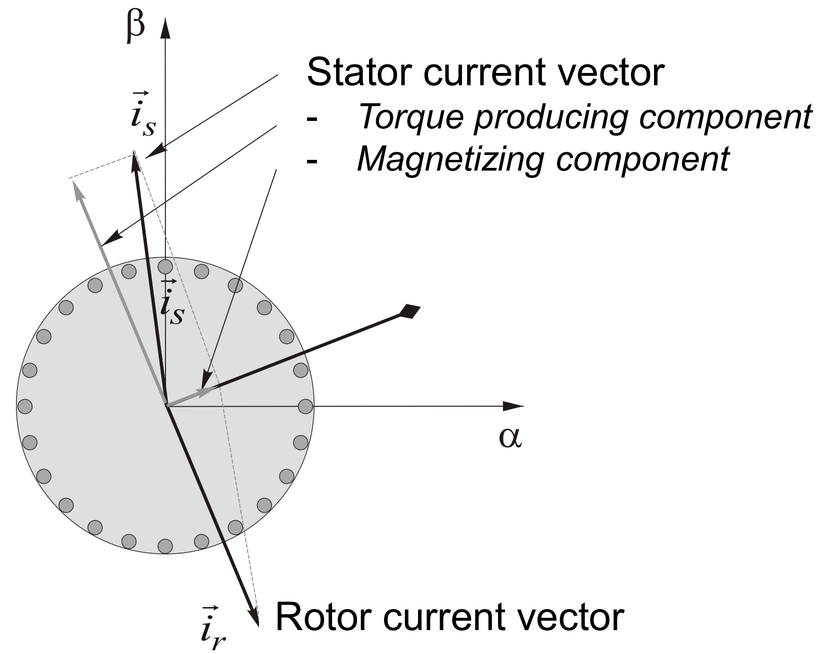


Move the stator current one step – what happens in the rotor?



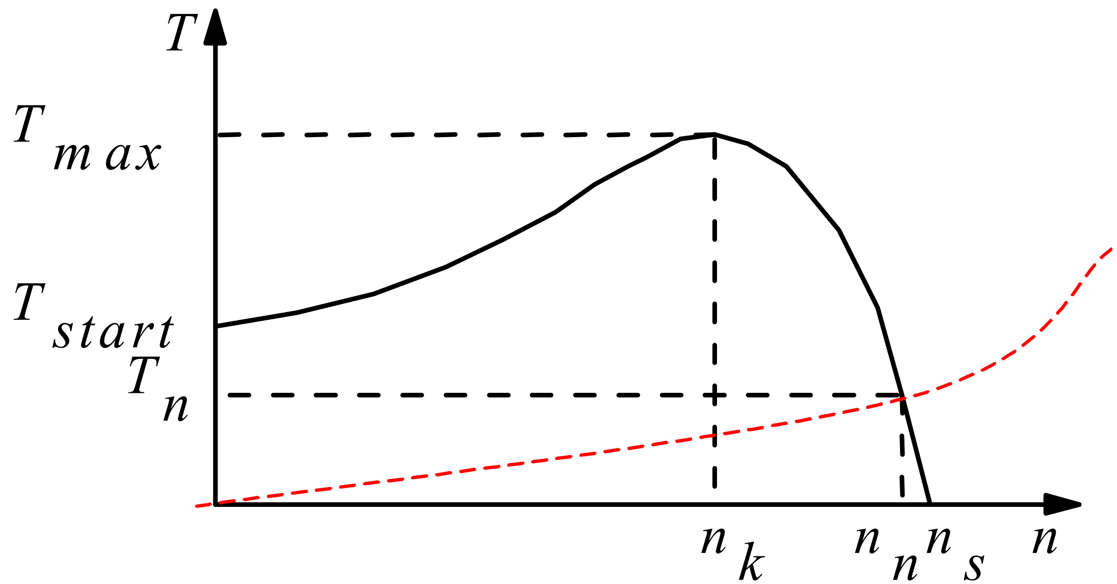
Consequences

- **The rotor current is**
  - ***Proportional to***
    The stator flux and
    The speed difference (both tangentially and radially)
  - ***Inversely proportional to the rotor resistance***
  - ***90 degrees lag versus the flux***



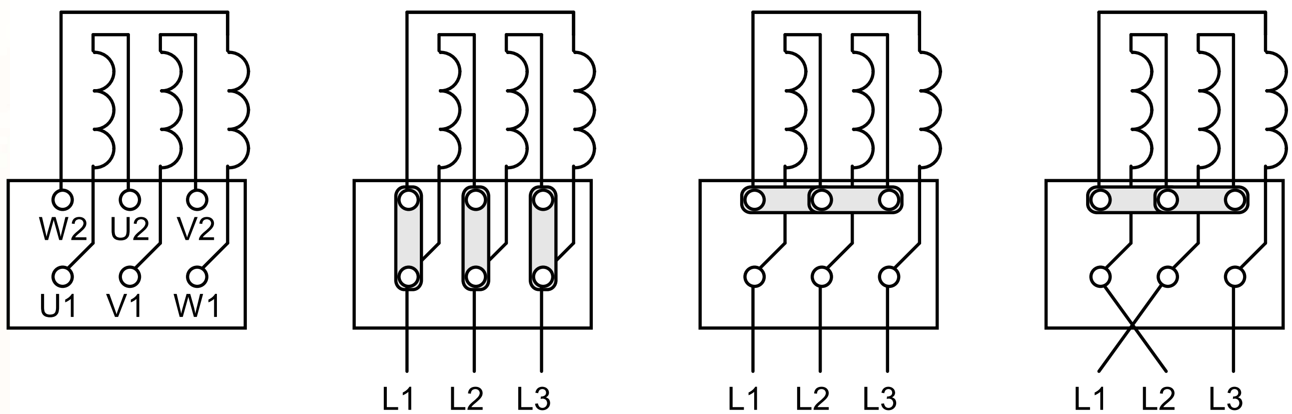
Torque properties

$$T = \psi_s \cdot i_r = (\omega - \omega_r) \cdot \frac{\hat{\psi}_s^2}{R_r}$$



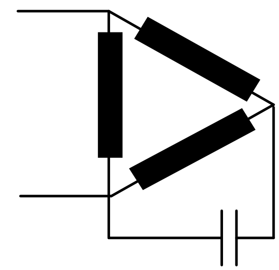
Connection

Motor 3~	50 Hz		IEC 34-1
			No.
2.2 kW	2820 r/min		
	cos φ 0,89		
Y 380 V 4.7 A	Δ 220 V 8.15 A		
	19.0 kg	IP 54	Cl. F

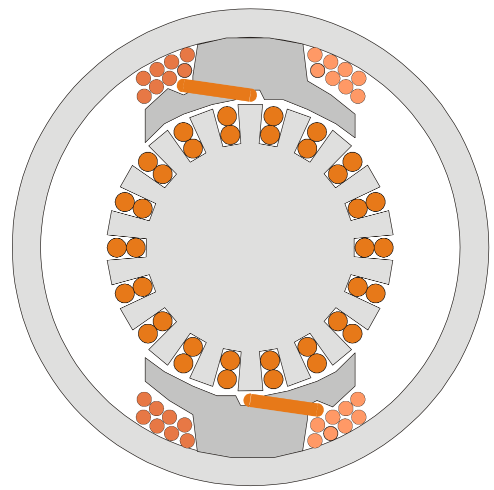


Single phase operation

- **Steinmetz connection**

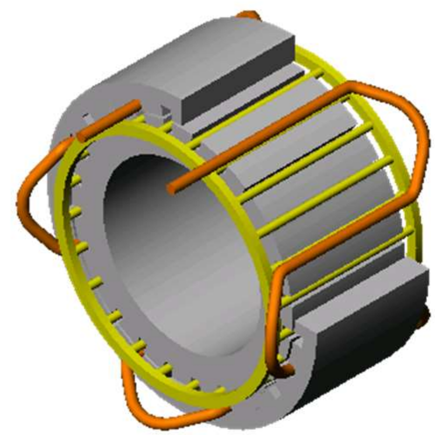


- **Shaded pole motor**



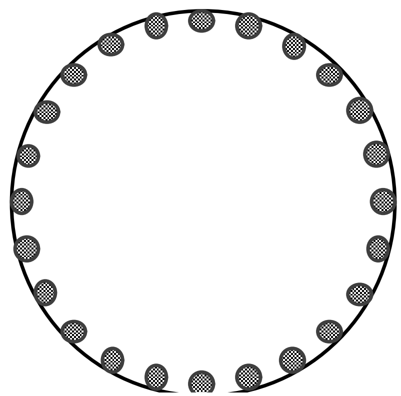
Summering AM

- **Rotation**
  - *With AC winding in the Stator*
- **Voltage**
  - *Proportional to frequency*
- **Speed**
  - *Also proportional to frequence, BUT WITH A SLIP @ Non-Zero Torque*
- **Torque**
  - *Proportional to y-axis current*

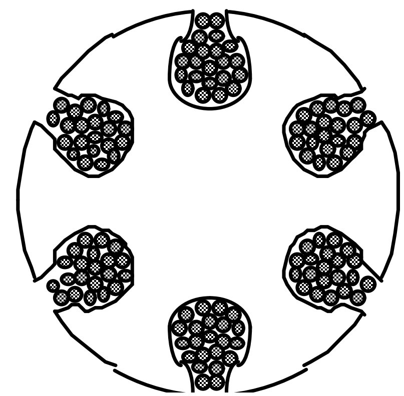


Mechanical design of IM

- **Stator same as PMSM**
- **Rotor:**

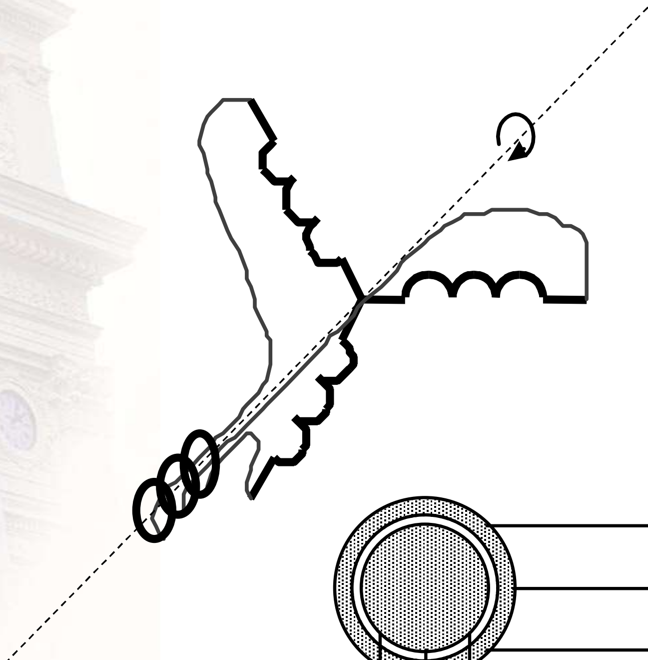


Cast aluminum



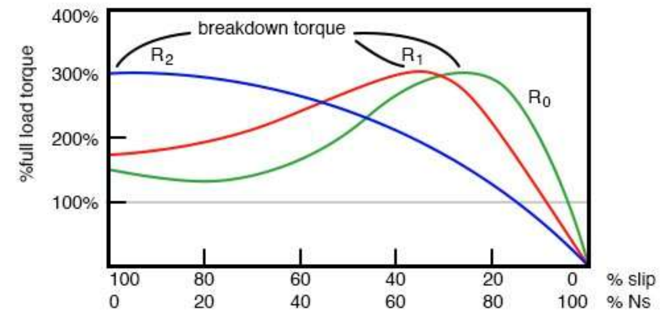
Wound copper /Slip ring

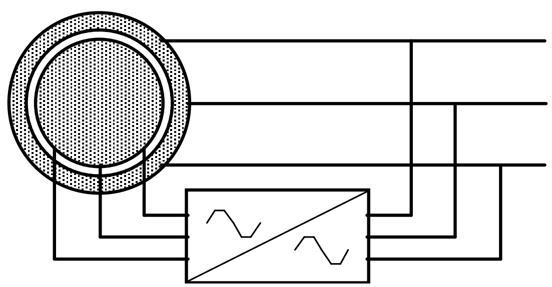
Slip ring rotor

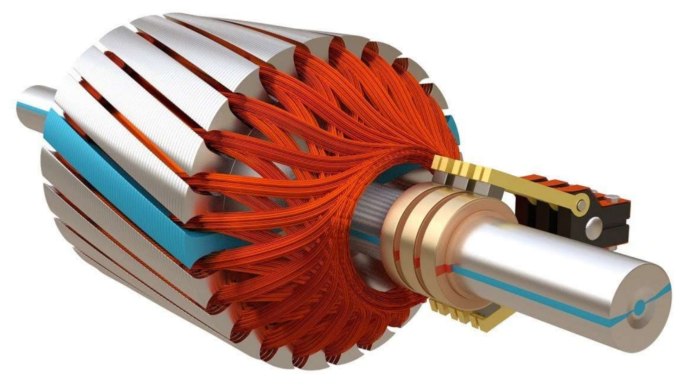


The rotor can be:

1. *Short circuited*
2. *Given an external rotor resistance*
3. *Fed by power electronics*







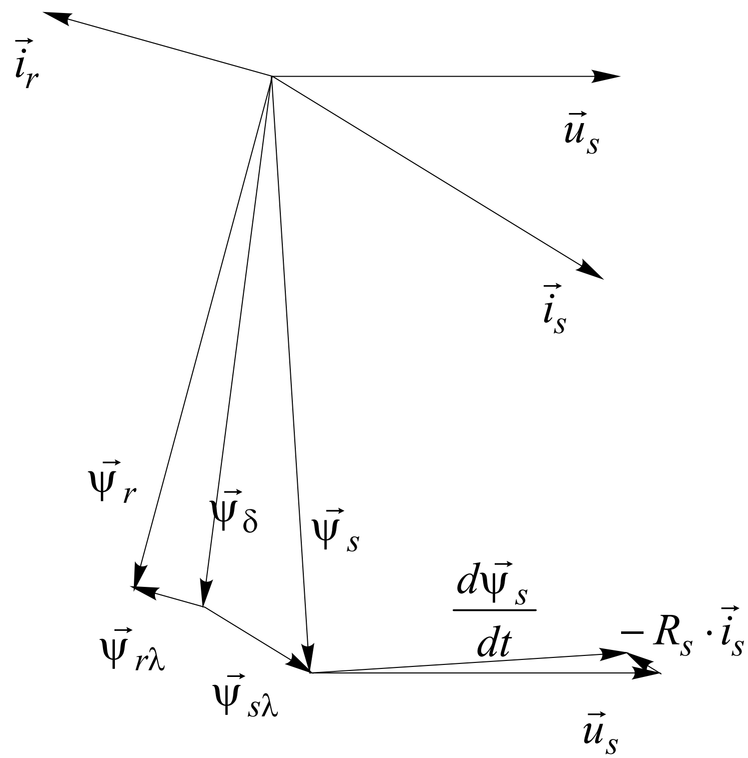
Mathematical model

- **Stator**

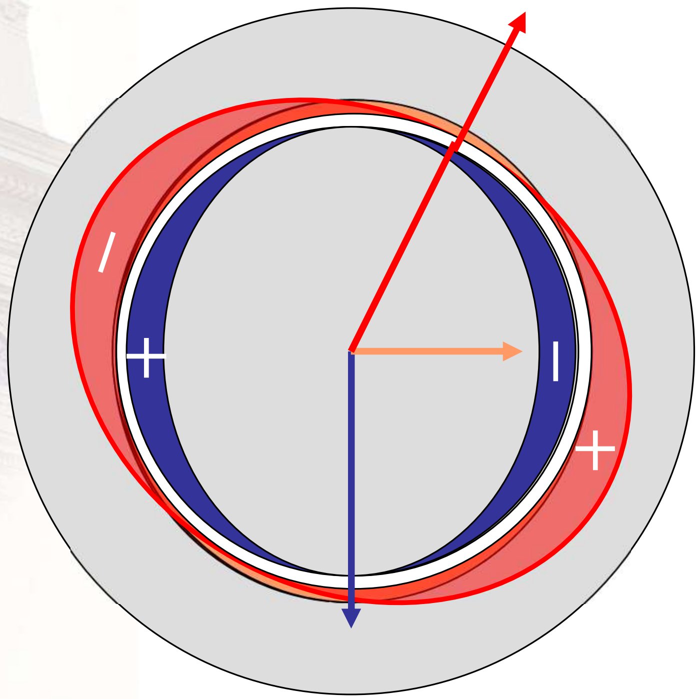
- **Rotor**

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}$$

$$\vec{u}_r = R_s \cdot \vec{i}_r + \frac{d\vec{\psi}_r}{dt}$$



How does it work?



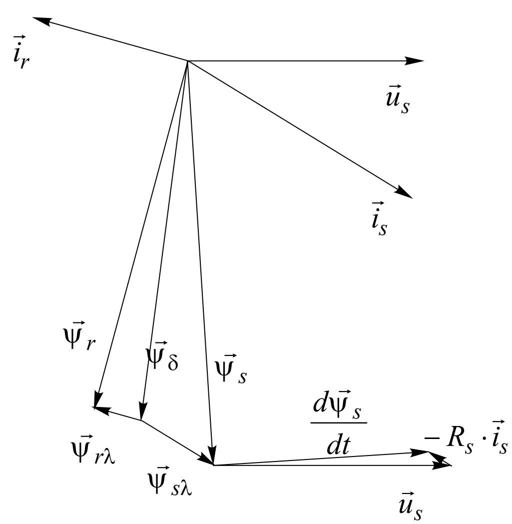
Imagine the following sequence :

1 A magnetizing stator current is stationary vs. the rotor.

2 The current is instantly moved and increased.

3 The rotor conserves the (rotor-)flux and thus the stator flux

Torque



Rotor flux orientation

$$T = \vec{\psi}_s \times \vec{i}_s = \vec{\psi}_r \times \vec{i}_r = -\frac{L_m}{L_s} \cdot \vec{\psi}_s \times \vec{i}_r = \frac{L_m}{L_s} \cdot \vec{\psi}_r \times \vec{i}_s = \vec{\psi}_\delta \times \vec{i}_s$$

Stator flux orientation

Vector control of Induction machines

- **Control usually made in EITHER:**
  - ***The stator flux oriented reference frame OR***
  - ***The rotor flux oriented reference frame***
- **Control, independen of frame, is usually the easiest**
- **Determining the reference frame is more difficult.**

Rotor flux estimation 1

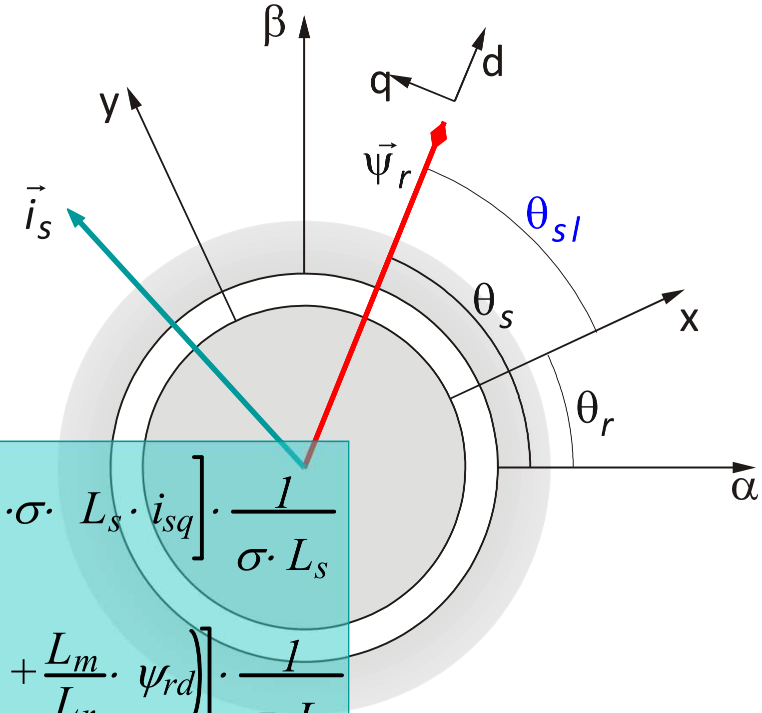
- **Rewriting the 2 (rotor and stator) <u>vector</u> equations -> 4 <u>scalar</u> equations**
- **That can be expressed like this (rotor equations first, stator equations last)**

$$\frac{d\psi_{rd}}{dt} = \frac{L_m}{\tau_r} \cdot i_{sd} - \frac{1}{\tau_r} \cdot \psi_{rd}$$

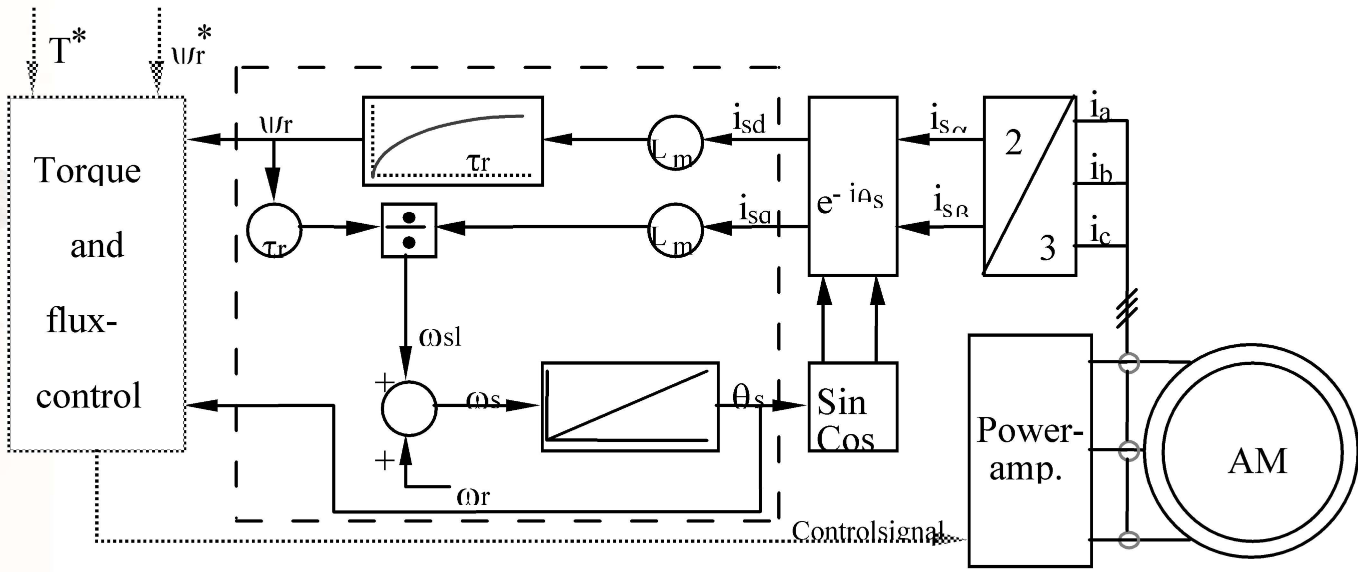
$$\frac{d\theta_{sl}}{dt} = \frac{L_m \cdot i_{sq}}{\tau_r \cdot \psi_{rd}}$$

$$\frac{di_{sd}}{dt} = \left[u_{sd} - R_s \cdot i_{sd} - \frac{L_m}{L_r} \cdot \frac{d\psi_{rd}}{dt} + \omega_s \cdot \sigma \cdot L_s \cdot i_{sq} \right] \cdot \frac{1}{\sigma \cdot L_s}$$

$$\frac{di_{sq}}{dt} = \left[u_{sq} - R_s \cdot i_{sq} - \omega_s \cdot \left(\sigma \cdot L_s \cdot i_{sd} + \frac{L_m}{L_r} \cdot \psi_{rd} \right) \right] \cdot \frac{1}{\sigma \cdot L_s}$$



Rotor flux estimation 2



Full Flux Observer 1 (to estimate both rotor and stator flux in the same time)

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}$$

$$0 = R_r \cdot \vec{i}_r + \frac{d\vec{\psi}_r}{dt} - j \cdot \omega_r \cdot \vec{\psi}_r$$

$$\Downarrow$$

$$\underbrace{\begin{bmatrix} \vec{u}_s \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_s \\ \vec{i}_r \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -j \cdot \omega_r \end{bmatrix} \cdot \begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix}}_{u = R \cdot i + \frac{d\psi}{dt} + \Omega \cdot \psi}$$

Full Flux Observer 2

$$\frac{d\psi}{dt} = u - R \cdot i - \Omega \cdot \psi = \begin{Bmatrix} \psi = L \cdot i \\ i = L^{-1} \cdot \psi \end{Bmatrix} = -\left(R \cdot L^{-1} + \Omega\right) \cdot \psi + u$$

$$\frac{d\hat{\psi}}{dt} = -\left(R \cdot L^{-1} + \Omega\right) \cdot \hat{\psi} + u + R \cdot k \cdot \left(i_s - C \cdot \hat{\psi}\right) =$$

$$= -\left(R \cdot L^{-1} + \Omega - R \cdot k \cdot C\right) \cdot \hat{\psi} + u + R \cdot k \cdot i_s =$$

$$= A_{obs} \cdot \hat{\psi} + B \cdot u + R \cdot k \cdot i_s$$

where

$$R = \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix}; \; L = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix}; \; \psi = \begin{bmatrix} \vec{\psi}_s \\ \vec{\psi}_r \end{bmatrix}; \; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$u = \begin{bmatrix} \vec{u}_s \\ 0 \end{bmatrix}; \; C = L^{-1}(1,:) \; ; \; \Omega = \begin{bmatrix} 0 & 0 \\ 0 & -j \cdot \omega_r \end{bmatrix}$$

Full Flux Observer 3

$$\hat{\psi}(k+1) = \boldsymbol{F}_{obs} \cdot \hat{\psi}(k) + \boldsymbol{G}_{obs} \cdot \overline{\boldsymbol{u}}(k,k+1) + \boldsymbol{K}_{obs} \cdot \overline{\underline{i}}_s(k,k+1)$$

$$\boldsymbol{F}_{obs} = e^{\boldsymbol{A}_{obs} \cdot T_s} = e^{-(\boldsymbol{R} \cdot \boldsymbol{L}^{-1} + \Omega + \boldsymbol{R} \cdot \boldsymbol{k} \cdot \boldsymbol{C}) \cdot T_s} \approx$$

$$\approx e^{-\Omega \cdot T_s/2} \cdot e^{-(\boldsymbol{R} \cdot \boldsymbol{L}^{-1} + \boldsymbol{R} \cdot \boldsymbol{k} \cdot \boldsymbol{C}) \cdot T_s} \cdot e^{-\Omega \cdot T_s/2} =$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & e^{j \cdot \omega_r \cdot T_s/2} \end{bmatrix} \cdot \boldsymbol{F}_{obs,\omega_r=0} \cdot \begin{bmatrix} 0 & 0 \\ 0 & e^{j \cdot \omega_r \cdot T_s/2} \end{bmatrix}$$

$$\boldsymbol{F}_{obs} = \left[I - \boldsymbol{A}_{obs} \cdot \frac{T_s}{2} \right]^{-1} \cdot \left[I + \boldsymbol{A}_{obs} \cdot \frac{T_s}{2} \right] \approx \left[I + \boldsymbol{A}_{obs} \cdot \frac{T_s}{2} \right]^2$$

Voltage measurement

- **Estimation by software**
  - *Error prone at low speeds*
- **Analogue filtering and sampling**
  - *slow*
- **Direct integration**
  - *Needs galvanic isolation*

Split equation solving

$$\begin{bmatrix} \frac{d\vec{\psi}_s}{dt} \\ \frac{d\vec{\psi}_r}{dt} \end{bmatrix} = \begin{bmatrix} A_{obs,11} & A_{obs,12} \\ A_{obs,21} & A_{obs,22} \end{bmatrix} \cdot \begin{bmatrix} \hat{\vec{\psi}}_s \\ \hat{\vec{\psi}}_r \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \vec{u}_s + \begin{bmatrix} R_s \cdot k_1 \cdot \vec{i}_s \\ R_r \cdot k_2 \cdot \vec{i}_s \end{bmatrix}$$

$$\frac{d\vec{\psi}_s}{dt} = A_{obs,11} \cdot \hat{\vec{\psi}}_s + A_{obs,12} \cdot \hat{\vec{\psi}}_r + \vec{u}_s + R_s \cdot k_1 \cdot \vec{i}_s$$

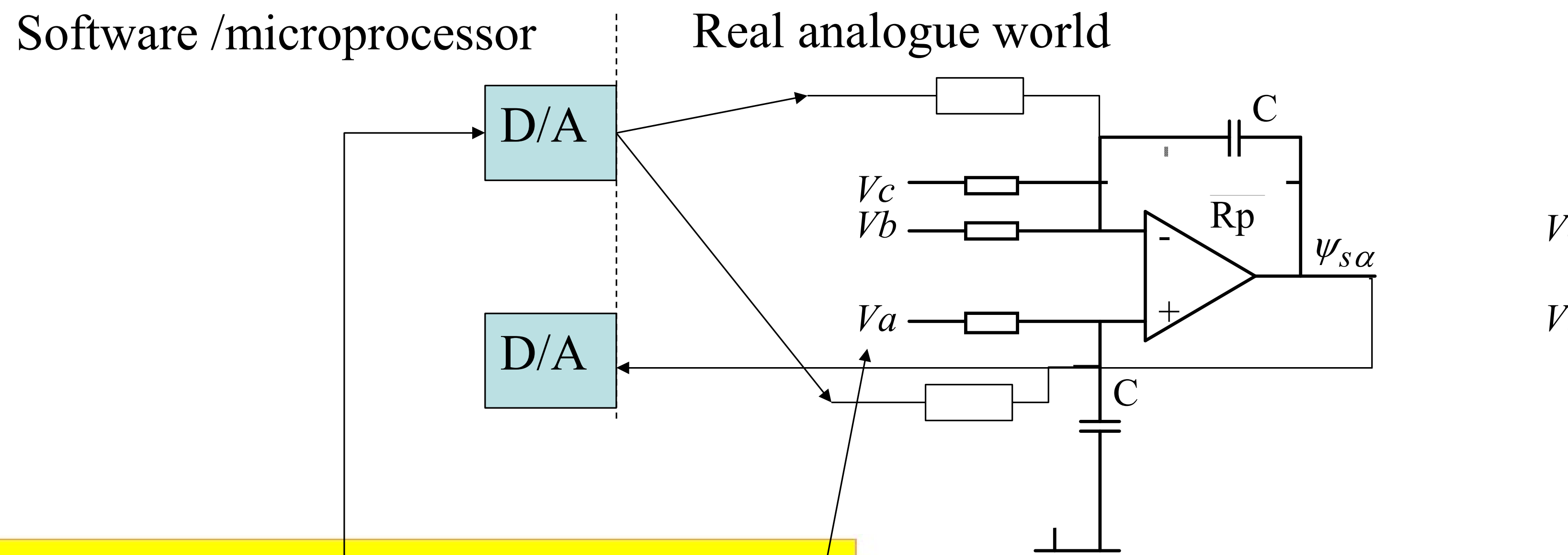
$$= A_{obs,11} \cdot \hat{\vec{\psi}}_s + A_{obs,12} \cdot \hat{\vec{\psi}}_r + R_s \cdot k_1 \cdot \vec{i}_s \quad + \quad \vec{u}_s$$

Solved partly outside the microprocessor

$$\frac{d\vec{\psi}_r}{dt} = A_{obs,21} \cdot \hat{\vec{\psi}}_s + A_{obs,22} \cdot \hat{\vec{\psi}}_r + 0 + R_r \cdot k_2 \cdot \vec{i}_s$$

Solved entirely in the microprocessor

Direct integration implemented



$$\frac{d\vec{\psi}_s}{dt} = A_{obs,11}\cdot\hat{\vec{\psi}}_s + A_{obs,12}\cdot\hat{\vec{\psi}}_r + \vec{u}_s + R_s\cdot k_1\cdot\vec{i}_s$$

$$= \boxed{A_{obs,11}\cdot\hat{\vec{\psi}}_s + A_{obs,12}\cdot\hat{\vec{\psi}}_r + R_s\cdot k_1\cdot\vec{i}_s} + \vec{u}_s$$

Solved partly outside the microprocessor

Stator Flux Oriented Vector Control of Torque and Flux

3 advantages:

- In a synchronously rotating reference frame, there are no stationary errors since all reference values are constant in stationarity
- The stator flux vector can in most cases be observed with high accuracy and low parameter sensitivity
- Direct flux control can be applied

Equations, but ony the stator equations this time

- **2 vector equation, i.e. 4 scalar equations, rewritten gives :**

$$\frac{d\psi_{sd}}{dt} = \frac{L_s}{\tau_r} \cdot i_{sd} - \frac{1}{\tau_r} \cdot \psi_{sd} + \left[\sigma \cdot L_s \cdot \frac{di_{sd}}{dt} - \omega_{sl} \cdot \sigma \cdot L_s \cdot i_{sq} \right] = u_{sd} - R_s \cdot i_{sd}$$

$$\frac{d\theta_{sl}}{dt} = \frac{L_s \cdot i_{sq} + \sigma \cdot L_s \cdot \tau_r \cdot \frac{di_{sq}}{dt}}{\tau_r \cdot \psi_{sd} - \sigma \cdot L_s \cdot \tau_r \cdot i_{sd}} = \frac{u_{sq} - R_s \cdot i_{sq}}{\psi_{sd}} - \omega_r$$

Direct Flux Vector Control

- The stator flux can in this particular case be controlled directly, without using a current control loop:
  - *Use the d-axis stator voltage equation, and integrated over one sampling period.*
  - *Approximate the stator current to the stationary one.*
  - *This leads to a p-controller for the stator flux modulus.*
- This is a possible, and somewhat beautiful control concept, but suffers form the lack of an inner current control loop to limit current in case of failure (e.g. short circuit in a winding)

$$u_{sd}(t) = R_s \cdot i_{sd}(t) + \frac{d\psi_{sd}(t)}{dt}$$

$$u_{sd}^*(k) = R_s \cdot \bar{i}_{sd}(k,k+1) + \frac{\psi_{sd}^*(k) - \psi_{sd}(k)}{T_s} =$$

$$= \{\psi_{sd} = L_s \cdot i_{sd} \quad in\ stationarity\ \} =$$

$$\approx R_s \cdot \frac{\psi_{sd}(k)}{L_s} + \frac{\psi_{sd}^*(k) - \psi_{sd}(k)}{T_s}$$

Control of the Torque producing current component : 1

- The remaining equations can be combined into a version of the *q*-axis equation:

- This whos that the q-axis current controller is based on exactly the same type of equation as a current controller in the PMSM machine, just with different parameters:
  - *Stator+Rotor resistance*
  - *Stator leakage+Rotor leakages inductances*

$$u_{sq} = R_s \cdot i_{sq} + \omega_r \cdot \psi_{sd} + \left[\frac{L_s \cdot i_{sq} + \sigma \cdot L_s \cdot \tau_r \cdot \frac{di_{sq}}{dt}}{\tau_r \cdot \psi_{sd} - \sigma \cdot L_s \cdot \tau_r \cdot i_{sd}} \right] \cdot \psi_{sd}$$

$$u_{sq} = (R_s + R_r) \cdot i_{sq} + \sigma \cdot L_s \cdot \frac{di_{sq}}{dt} + \omega_r \cdot \psi_{sd} \approx$$

$$\approx (R_s + R_r) \cdot i_{sq} + (L_{s\lambda} + L_{r\lambda}) \cdot \frac{di_{sq}}{dt} + \omega_r \cdot \psi_{sd}$$

Control of the Torque producing current component : 2

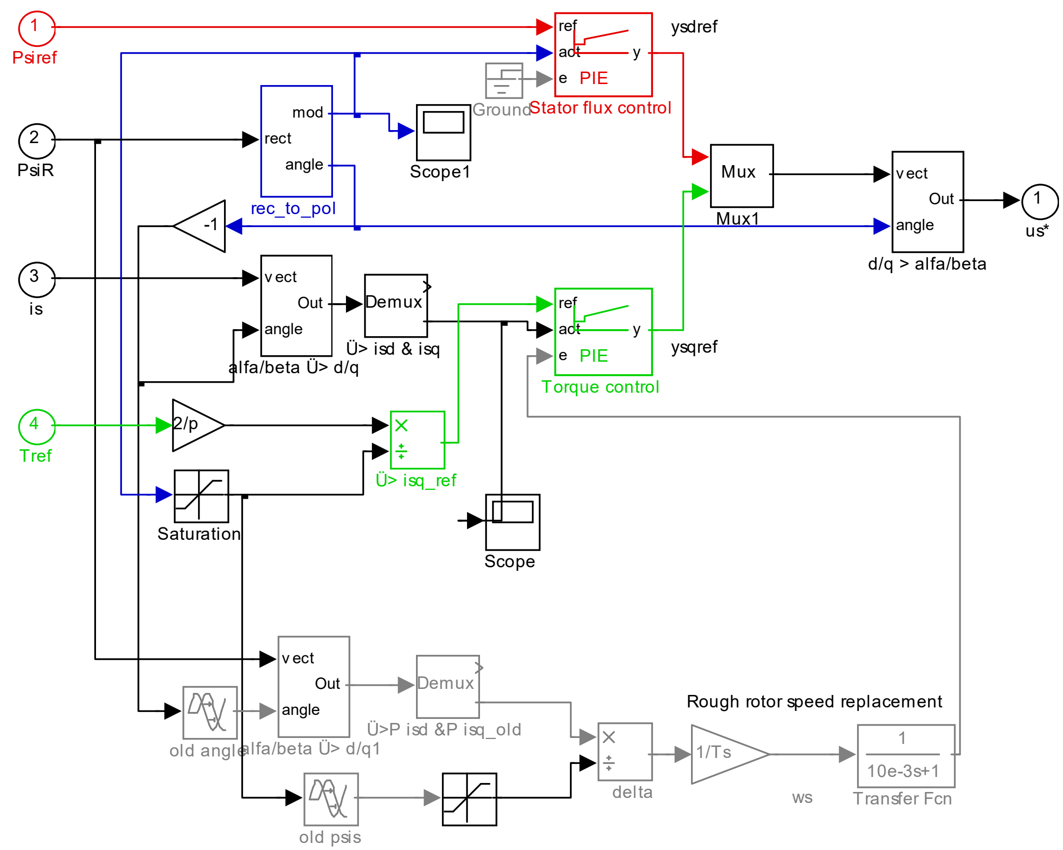
- Use generic 1-phase load ...

$$u_{sq}*(k)=$$

$$=\left(\frac{L_{s\lambda}+L_{r\lambda}}{T_s}+\frac{R_s+R_r}{2}\right)\cdot\left(\left(i_{sq}^*(k)-\hat{i}_{sq}(k)\right)+\frac{T_s}{\left(\frac{L_{s\lambda}+L_{r\lambda}}{R_s+R_r}+\frac{T_s}{2}\right)}\cdot\sum_{n=0}^{n=k-1}\left(i_{sq}^*(n)-\hat{i}_{sq}(n)\right)\right)+$$

$$+\,\omega_r\cdot\psi_{sd}(k)$$

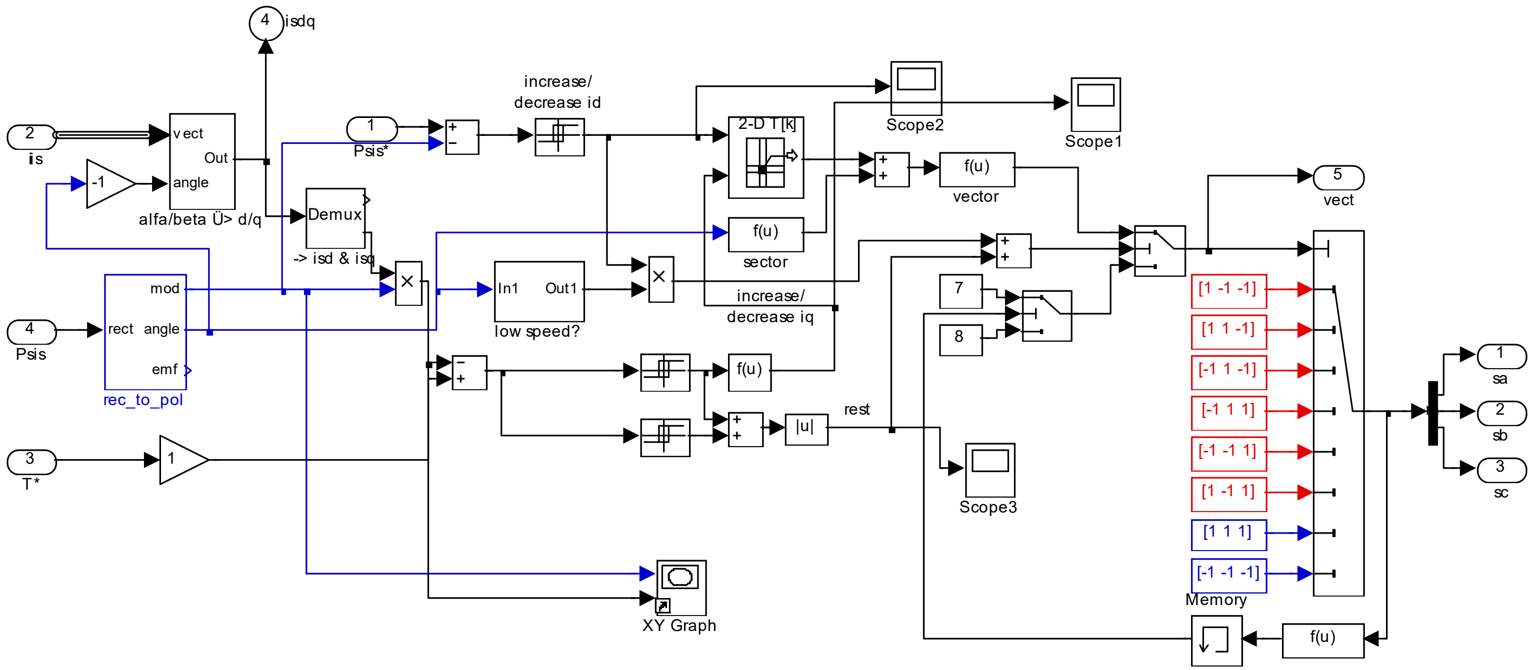
Vector Control System



DTC

- **Compare to DCC.**
  - *Replace* i_{sd} *with* ψ_{sd}
  - *Replace* i_{sq} *with T*

DTC system


isdq
increase/
decrease id
Scope2
Scope1
is
vect
Out
angle
Psis*
2-D T[k]
f(u)
vector
-1
alfa/beta Ü> d/q
Demux
-> isd & isq
sector
vect
In1
Out1
low speed?
increase/
decrease iq
mod
rect
emf
Psis
rec_to_pol
[1 -1 -1]
[1 1 -1]
[-1 1 -1]
[-1 1 1]
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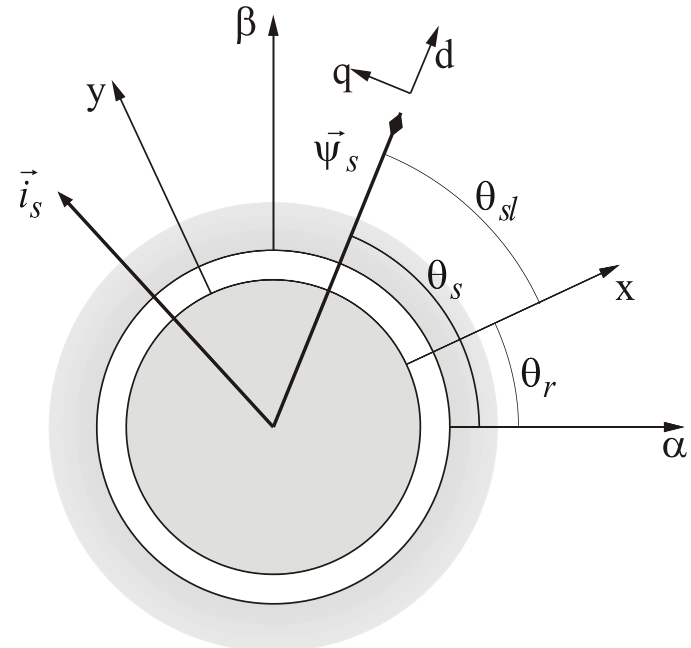
|u|
rest
Scope3
T*
sa
sb
sc
Memory
XY Graph

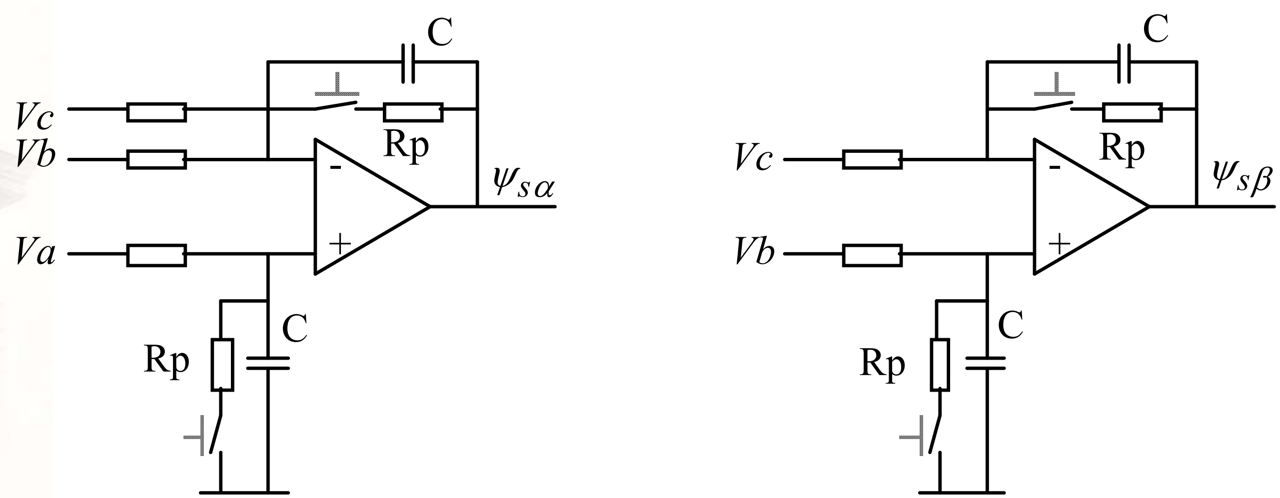
Simplified Stator Flux Model 1



$$\vec{u}_s = R\cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt} = R\cdot \left(\vec{i}_s - \vec{i}_{s0}\right) + R\cdot \vec{i}_{s0} + \frac{d\vec{\psi}_s}{dt}$$

$$\frac{d\vec{\psi}_s}{dt} = -R\cdot \vec{i}_{s0} + \left[\vec{u}_s - R\cdot \left(\vec{i}_s - \vec{i}_{s0}\right)\right] = \left\{\begin{matrix} no\ load\ running \\ \vec{i}_s = \vec{i}_{s0} = \frac{\vec{\psi}_{s0}}{L_s} \end{matrix}\right\} = -R\cdot \frac{\vec{\psi}_{s0}}{L_s} + \vec{u}_s = -\frac{\vec{\psi}_{s0}}{\tau_s} + \vec{u}_s$$

Simplified Stator flux model 2

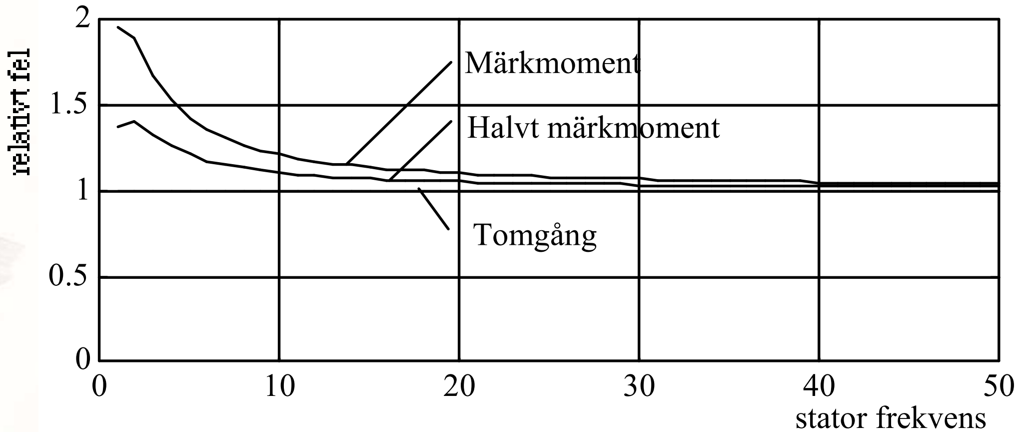


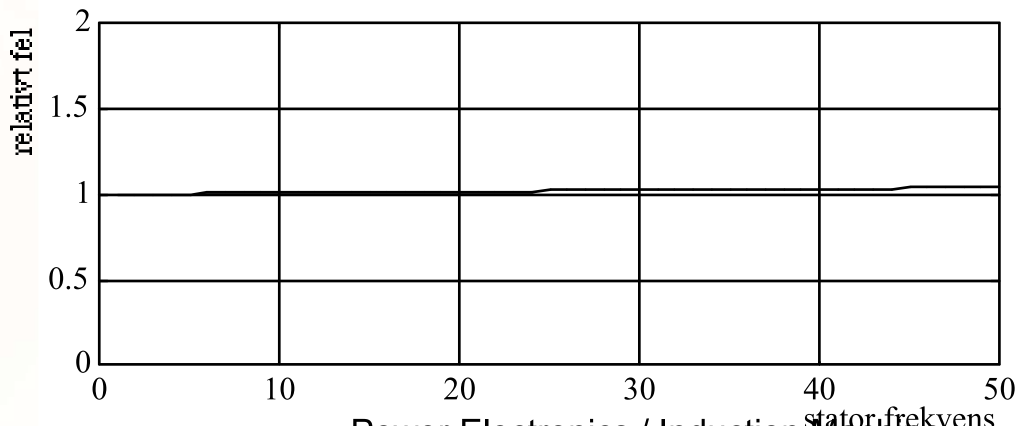
Simplified Stator Flux Model 3

$$\frac{d\psi_{sd}}{dt} = \frac{L_s}{\tau_r} \cdot i_{sd} - \frac{1}{\tau_r} \cdot \psi_{sd} + \left[\sigma \cdot L_s \cdot \frac{di_{sd}}{dt} - \omega_{sl} \cdot \sigma \cdot L_s \cdot i_{sq} \right] = u_{sd} - R_s \cdot i_{sd}$$

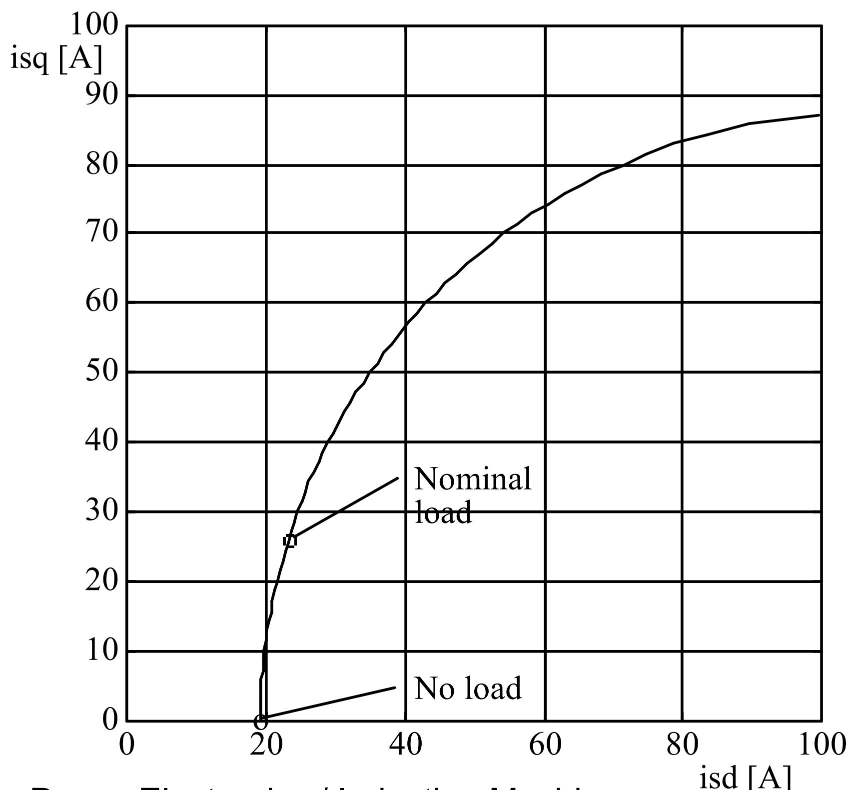
$$\frac{d\theta_{sl}}{dt} = \frac{L_s \cdot i_{sq} + \sigma \cdot L_s \cdot \tau_r \cdot \dfrac{di_{sq}}{dt}}{\tau_r \cdot \psi_{sd} - \sigma \cdot L_s \cdot \tau_r \cdot i_{sd}} = \frac{u_{sq} - R_s \cdot i_{sq}}{\psi_{sd}} - \omega_r$$

Simplified estimation error





Relation iq/id in stator flux frame

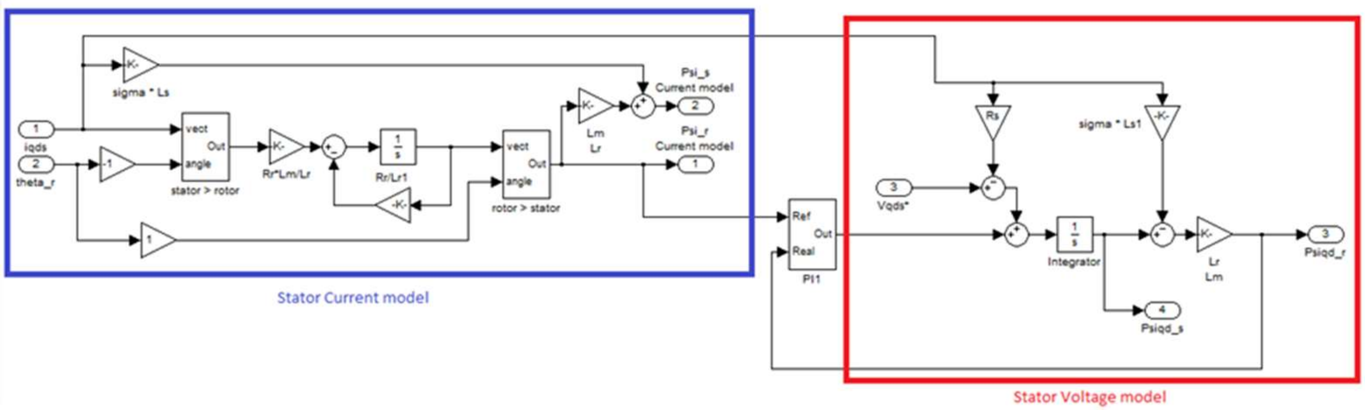


Gopinath Style Flux Observer

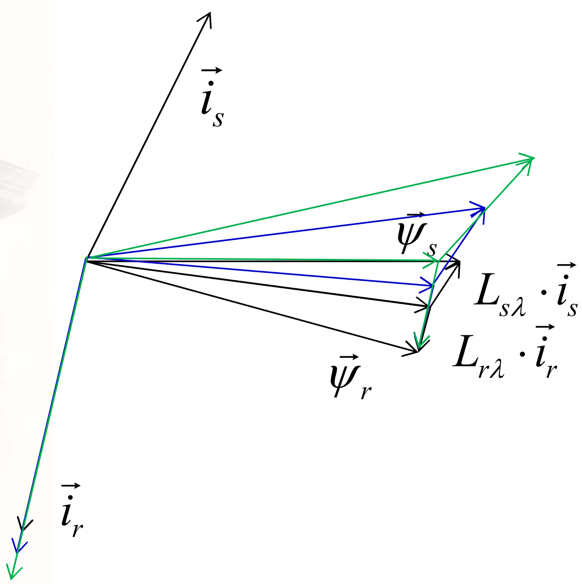
$$\frac{\mathrm{d}\Psi_r^r}{dt} = \frac{R_r L_m I_s^r}{L_r} - \frac{R_r}{L_r}\Psi_r^r$$

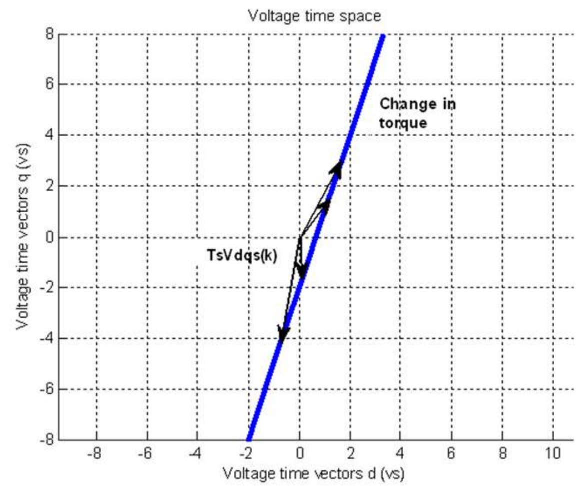
$$\frac{d\Psi_s^s}{dt} = V_s^s - R_s I_s^s$$

$$\frac{d\Psi_s^s}{dt} = V_s^s - R_s I_s^s$$

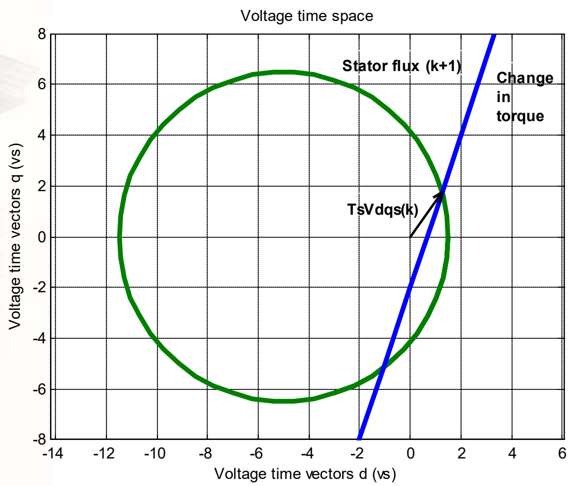


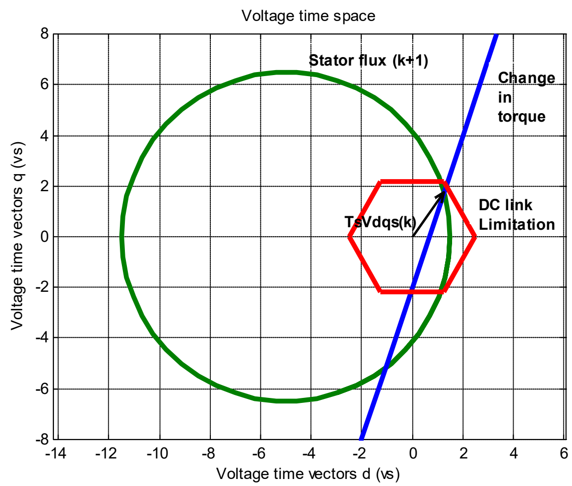
DB-DTFC


$\vec{i}_s$
$\vec{\psi}_s$
$L_{s\lambda} \cdot \vec{i}_s$
$L_{r\lambda} \cdot \vec{i}_r$
$\vec{\psi}_r$
$\vec{i}_r$


Voltage time space
Change in torque
TsVdqs(k)
Voltage time vectors q (vs)
Voltage time vectors d (vs)
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
10

DB-DTFC


Voltage time space
Stator flux (k+1)
Change in torque
TsVdqs(k)
Voltage time vectors q (vs)
Voltage time vectors d (vs)


Voltage time space
Stator flux (k+1)
Change in torque
TsVdqs(k)
DC link Limitation
Voltage time vectors q (vs)
Voltage time vectors d (vs)