Power Electronics

The Synchronous Machine Modelling

Lorentz and Reluctance forces

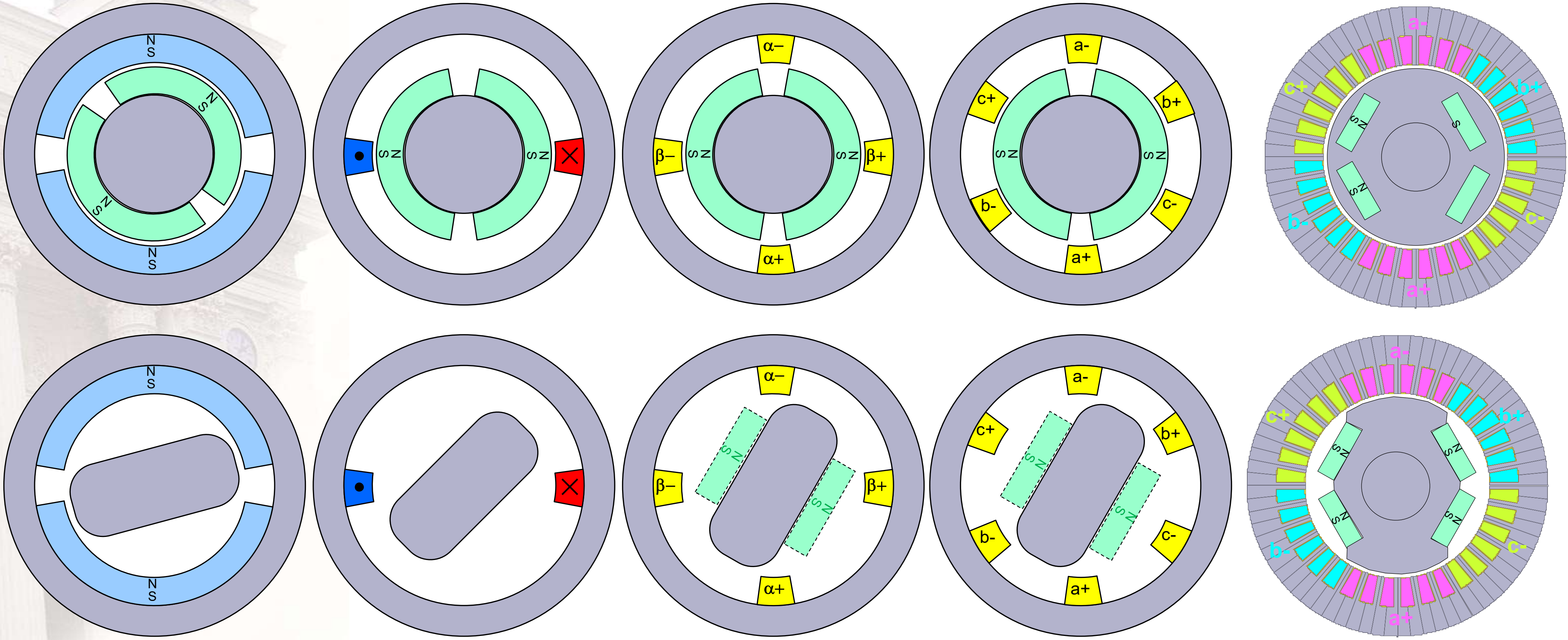
PM
PM rotor

1-phase stator
PM rotor

2-phase stator
PM rotor

3-phase stator
PM rotor

3-phase stator
Reluctance & PM rotor



PM stator
Reluctance rotor

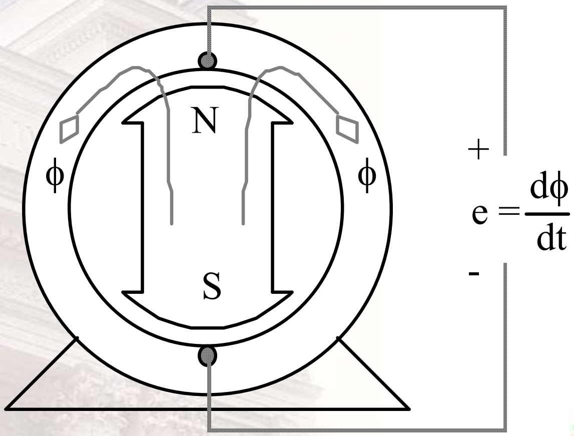
1-phase stator
Reluctance rotor

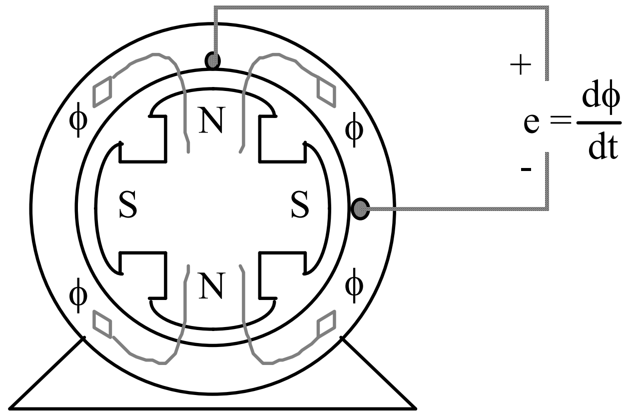
2-phase stator
Reluctance & PM rotor

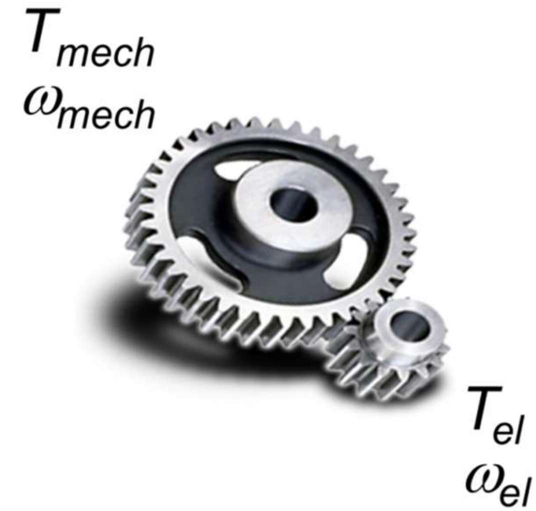
3-phase stator
Reluctance & PM rotor

3-phase stator
Reluctance & PM rotor

of poles = p





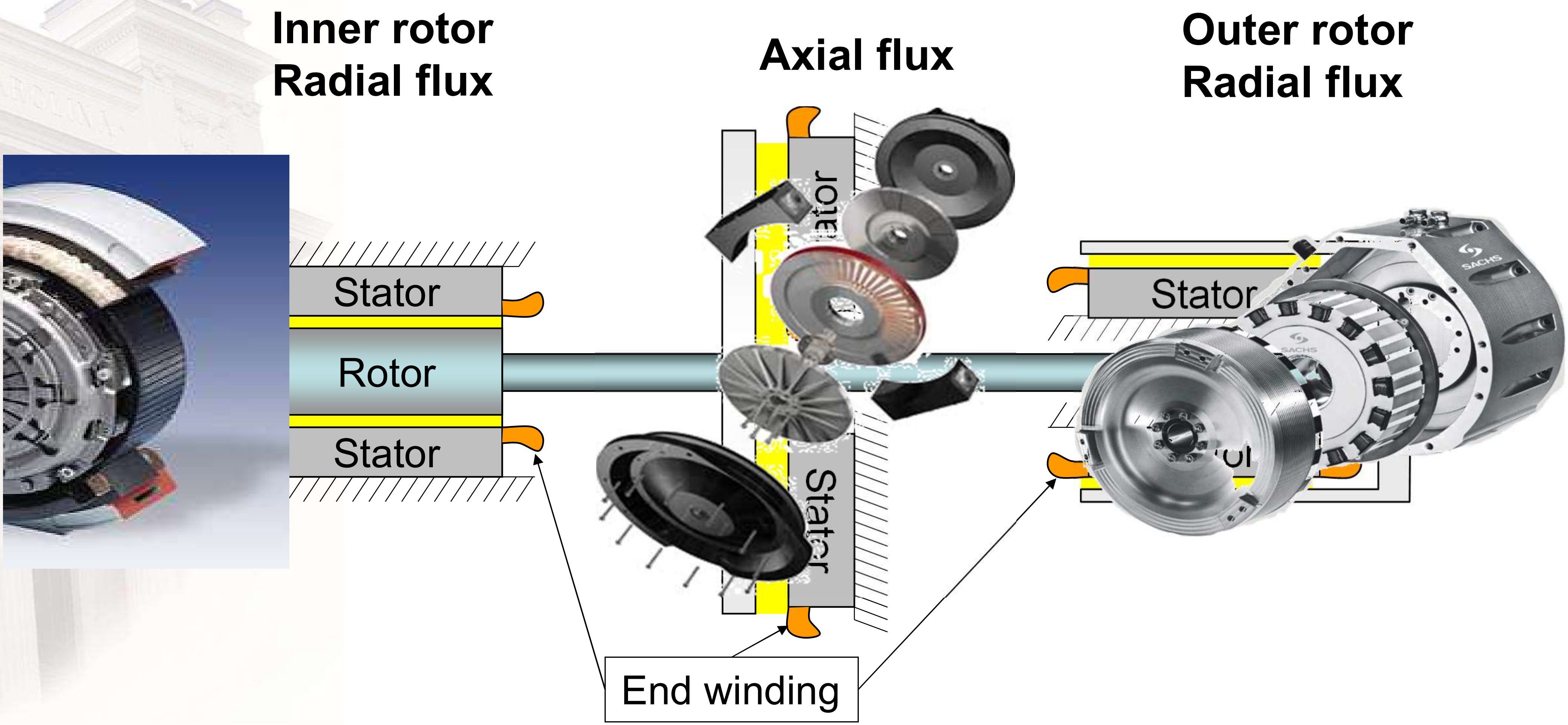


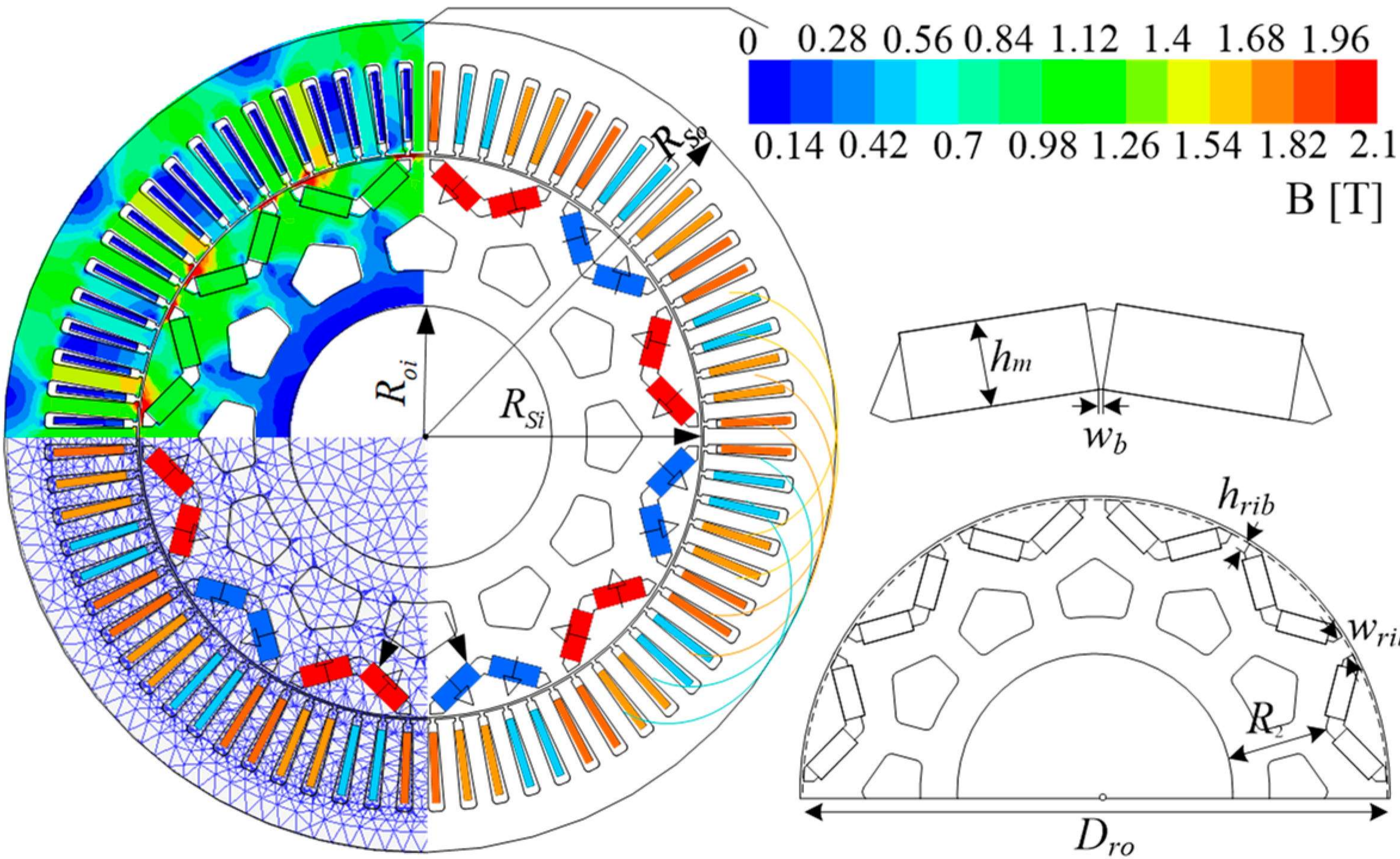
$$\omega_{el} = \frac{p}{2} \cdot \omega_{mech}$$

$$T_{mech} = \frac{p}{2} \cdot T_{el}$$

$$\text{Mechanical Power} = \omega_{mech} \cdot T_{mech} = \frac{p}{2} \cdot \omega_{el} \cdot \frac{2}{p} \cdot T_{el} = \omega_{el} \cdot T_{el}$$

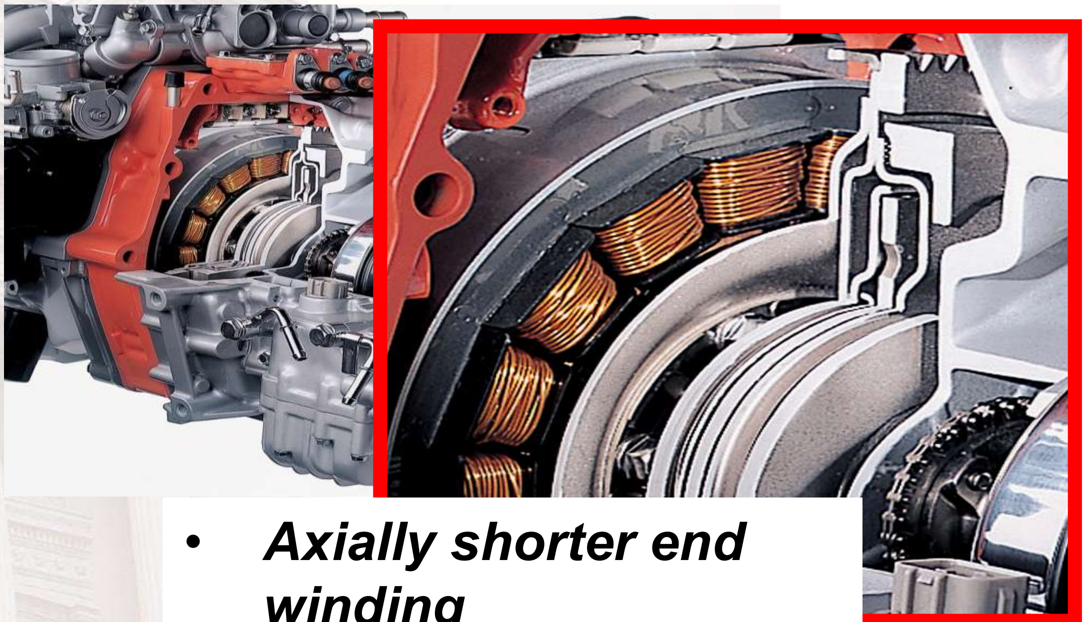
Inner or Outer Rotor, Radial or Axial flux





0 0.28 0.56 0.84 1.12 1.4 1.68 1.96
0.14 0.42 0.7 0.98 1.26 1.54 1.82 2.1
B [T]
R_{So}
R_{oi}
R_{Si}
h_m
w_b
h_{rib}
w_{rib}
R_2
D_{ro}

Distributed or Concentrated winding

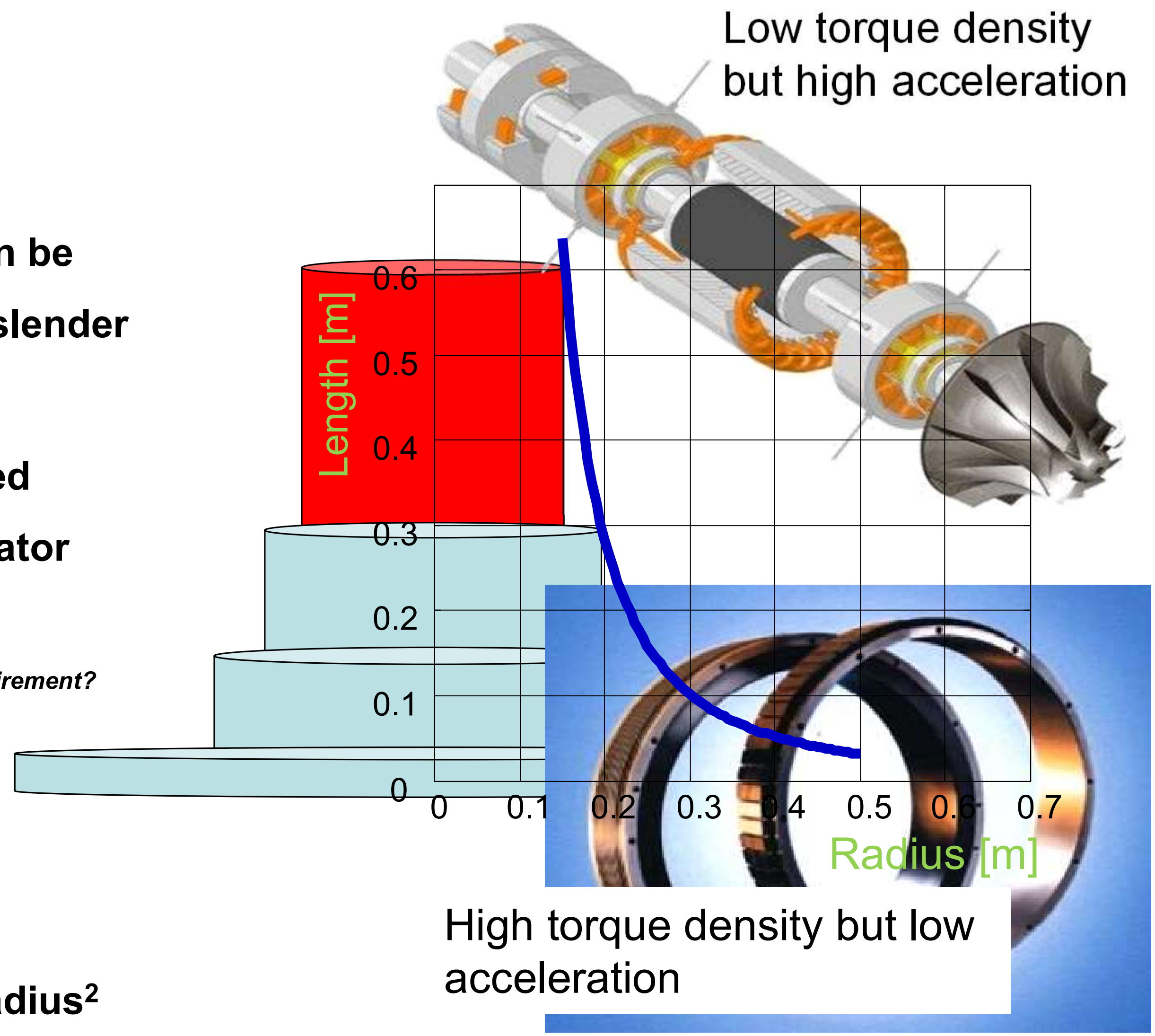


- *Axially shorter end winding*
- *Cheaper assembly*
- *Lower torque quality*

- *Longer end winding*
- *More expensive assembly*
- *Higher torque quality*

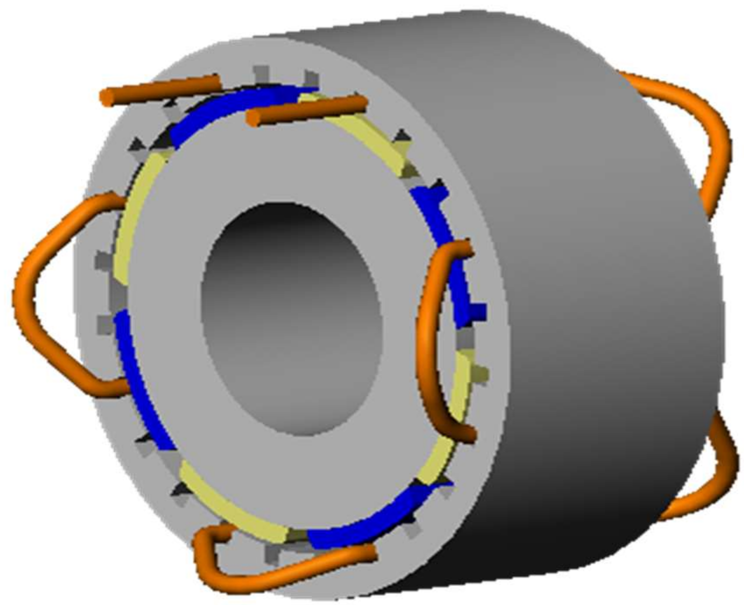
Form Factor

- **For the same torque, a machine can be either short and wide, or long and slender …**
- **Assume 25 000 [N/m²], and a desired torque of 1000 Nm, AND that the stator outer radius is 0.15 – 0.5 meter.**
  - ***How long will the machine be to fulfill the torque requirement?***
- **The long and slender machine will accelerate faster**
  - ***Torque ~ radius²*length***
  - ***Inertia ~ radius⁴*length***
- **Acceleration = Torque/Inertia ~ 1/radius²**

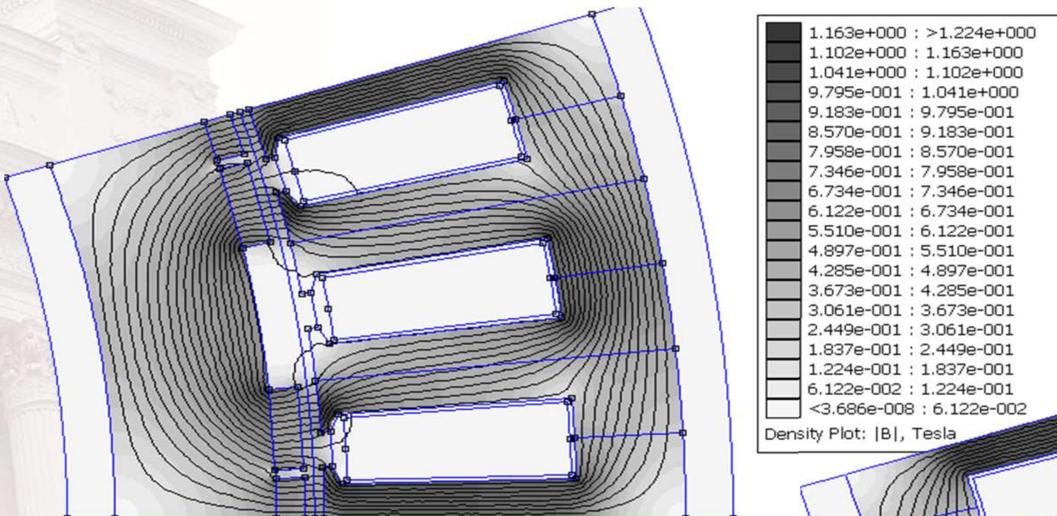


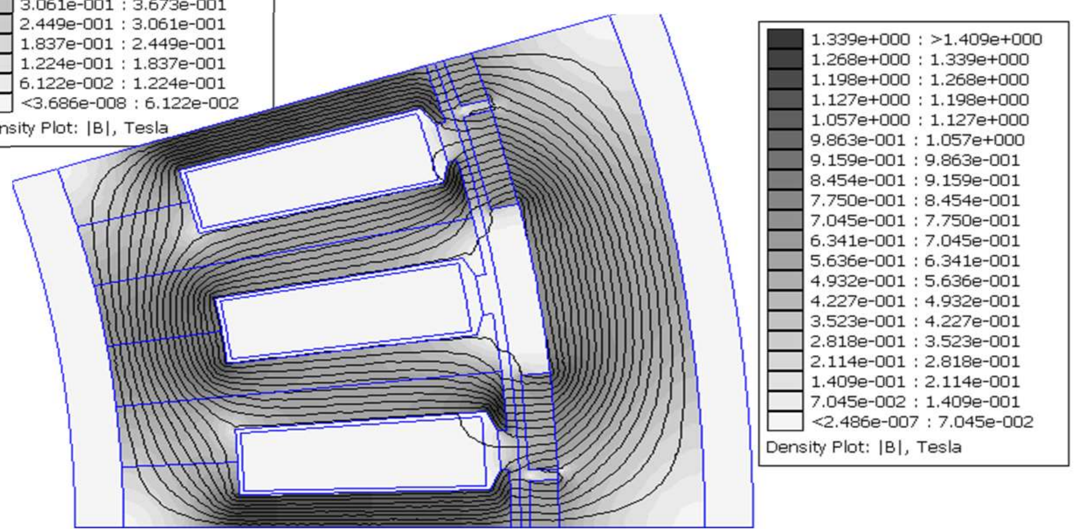
Permanent Magnet Synchronous Machines

- Same as the generic machine.
- Voltage and frequency proportional to speed.
- Current proportional to torque.
- High continuous torque density:
  - *1...10 Nm/kg*
  - *Compare to ICE 1...2 Nm/kg*
- High efficiency:
  - *Up to 97%*
- Higher efficiency, higher torque density and more expensive than other machines:
  - *Due to the permanent magnets.*

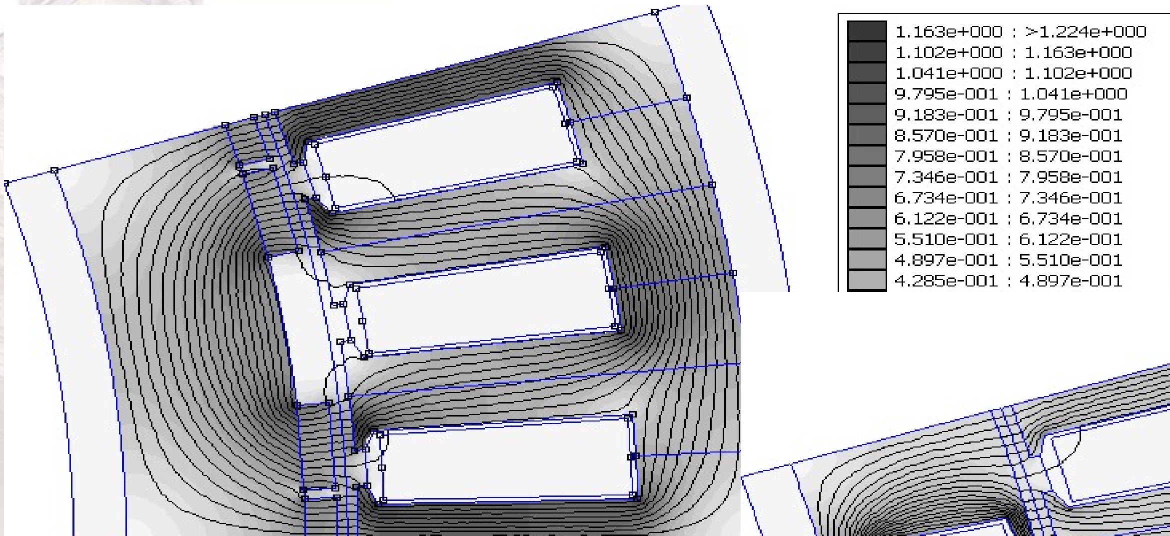


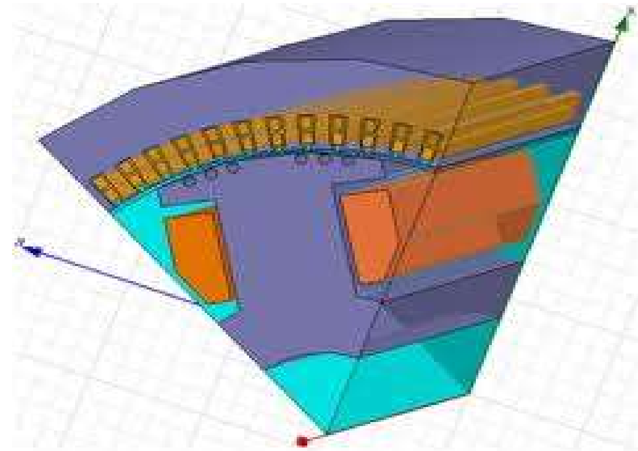
PMSM - Inner and outer rotor

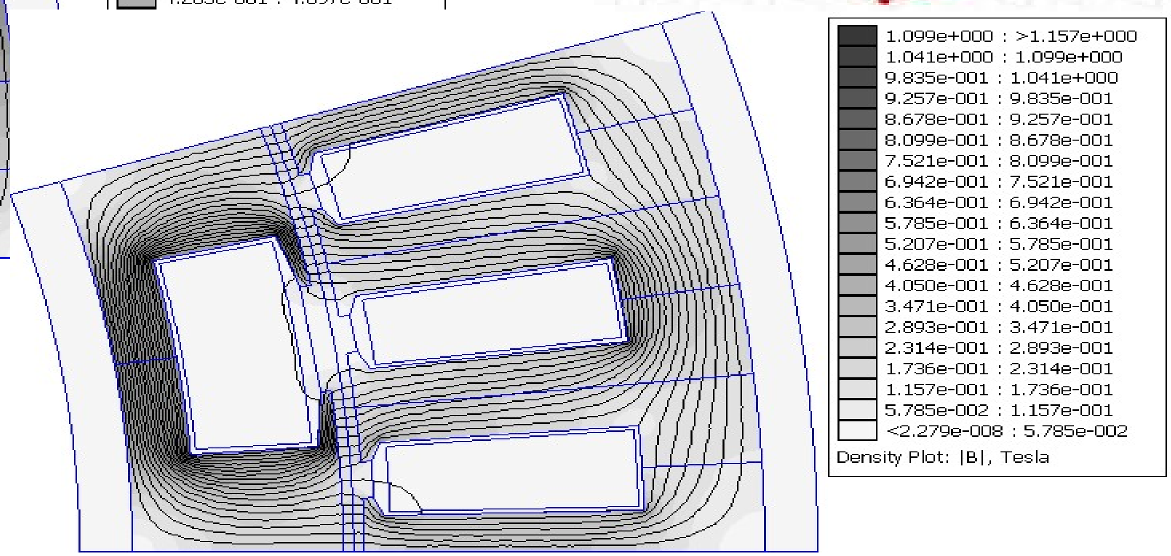




Electrically magnetized



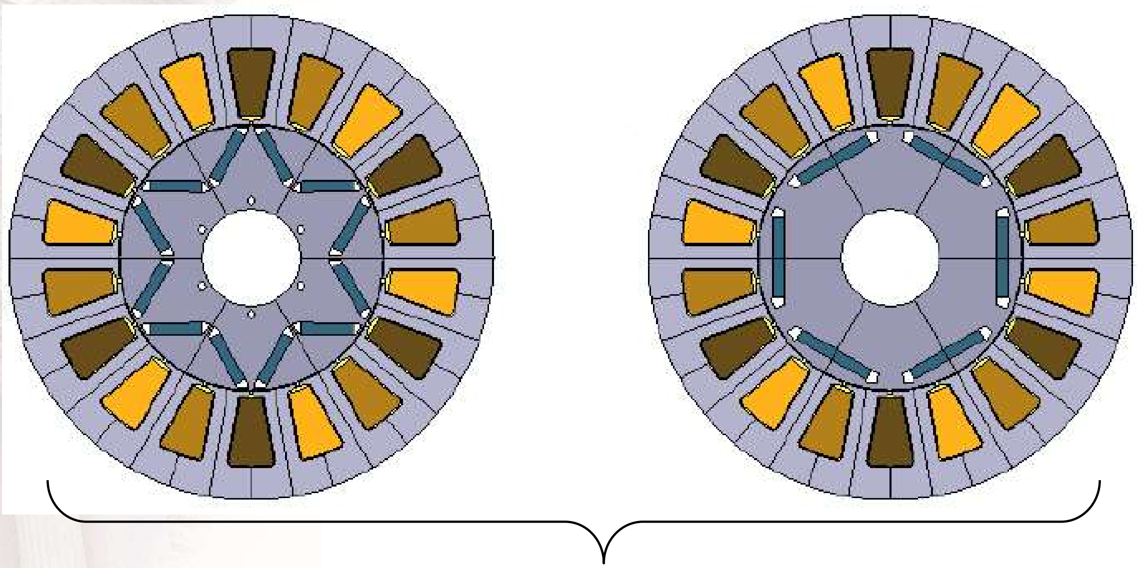




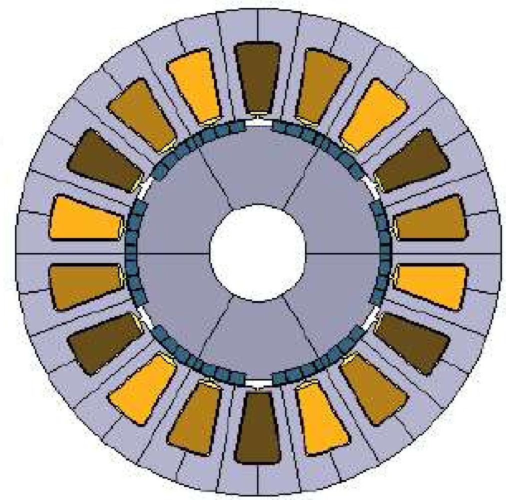
Can be made in 2 ways:

- Conductive
- Inductive

Different Magnet arrangements



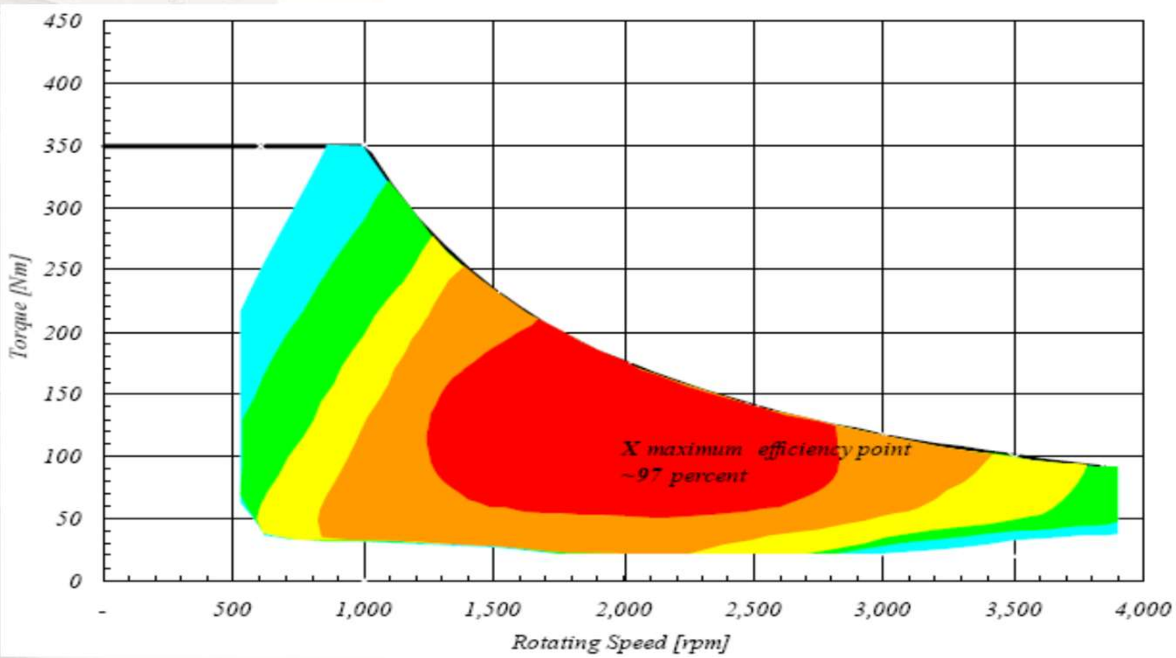
IPM
(Interior Permanent Magnet)

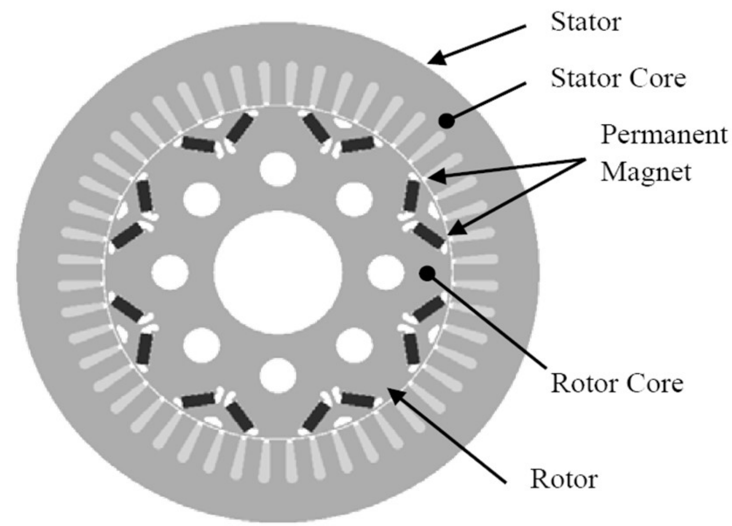


SPM
(Surface Mounted Permanent Magnet)

Example from Toshiba

- **"Large Torque and High Efficiency Permanent Magnet Reluctance Motor for A Hybrid Truck" - Masanori Arata et. Al, EVS-22**



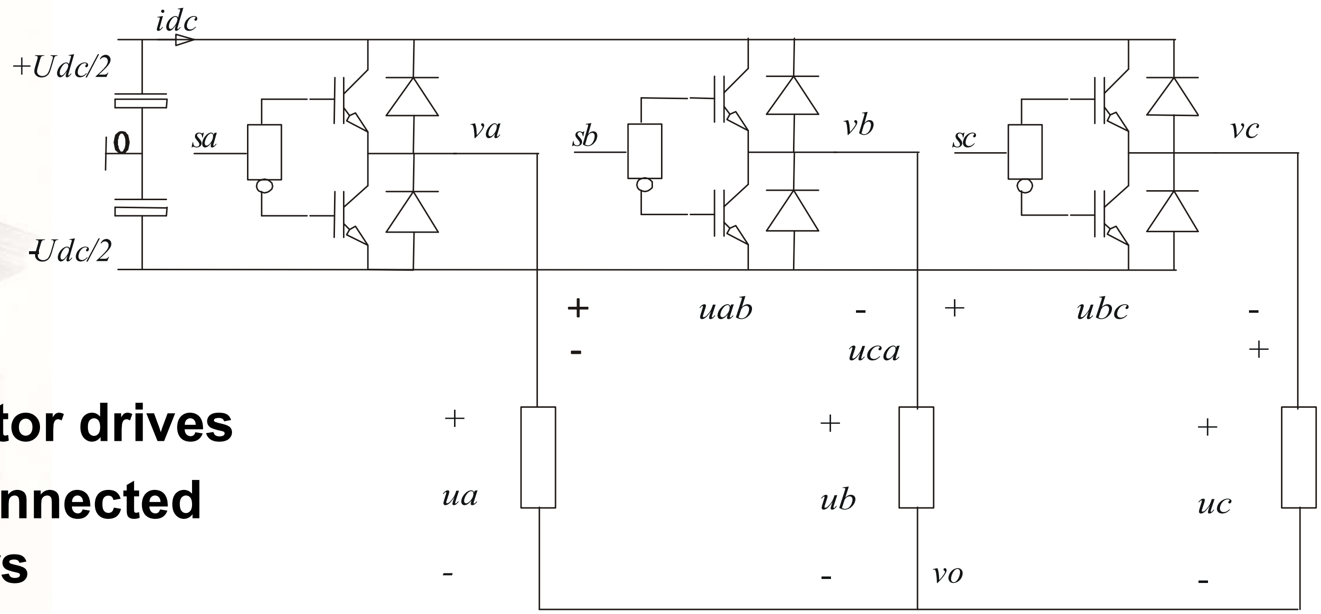


(a) An example of cross section of the PRM

Whiteboard …

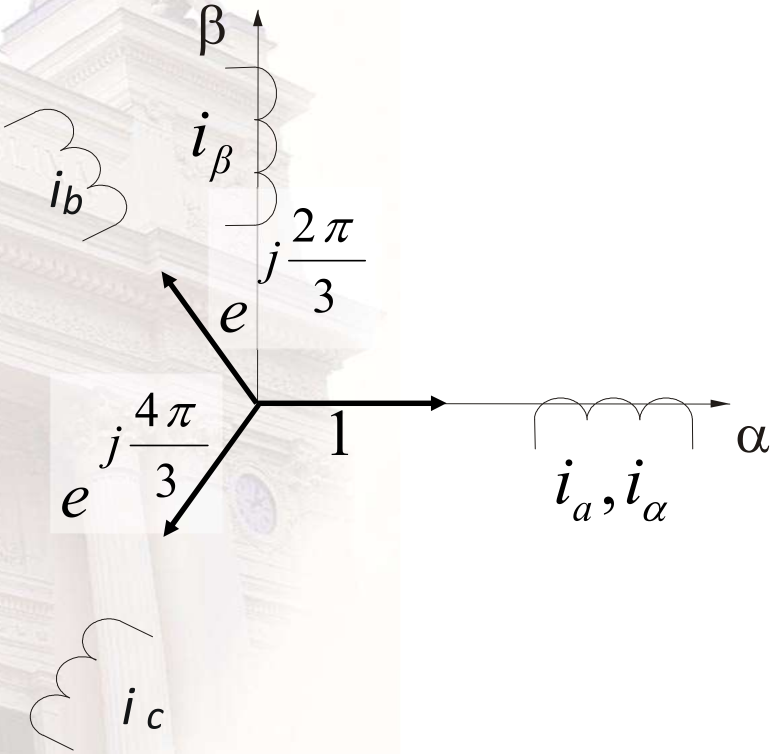
Current control of a 3-phase load

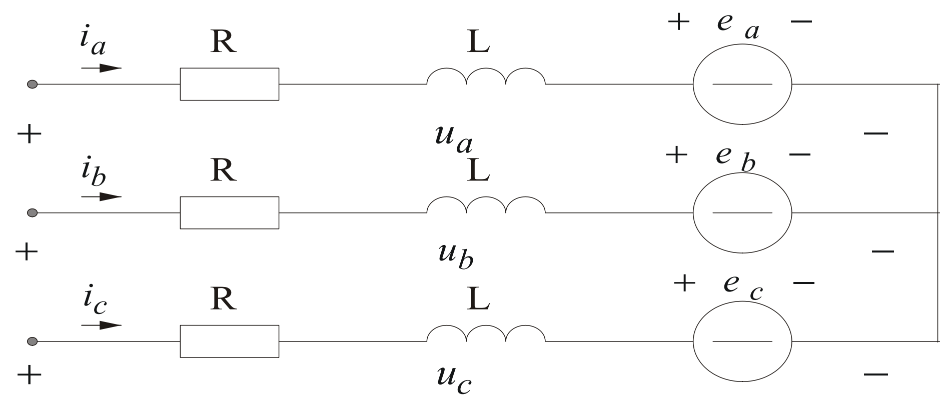
3 – phase converters



- In AC motor drives
- In grid connected converters

The generic 3-phase load





$$\sqrt{\frac{2}{3}}\left(u_a = R \cdot i_a + L \cdot \frac{di_a}{dt} + e_a\right)$$

$$\sqrt{\frac{2}{3}} \cdot e^{j\frac{2\pi}{3}} \cdot \left(u_b = R \cdot i_b + L \cdot \frac{di_a}{dt} + e_b\right)$$

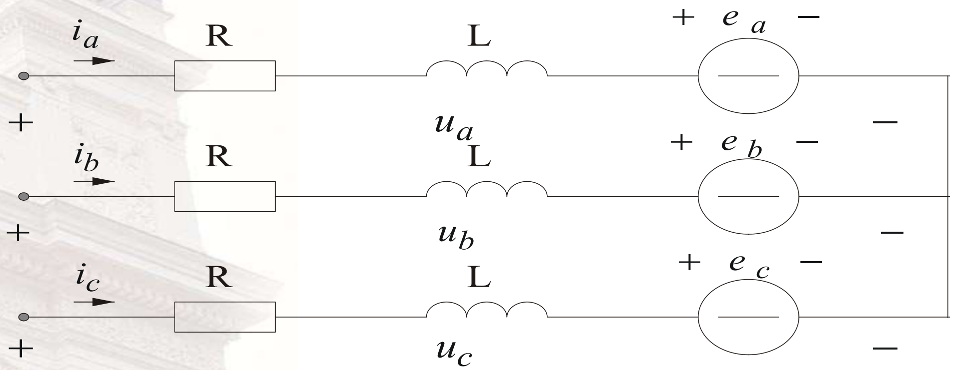
$$+ \sqrt{\frac{2}{3}} \cdot e^{j\frac{4\pi}{3}} \cdot \left(u_c = R \cdot i_c + L \cdot \frac{di_a}{dt} + e_c\right)$$

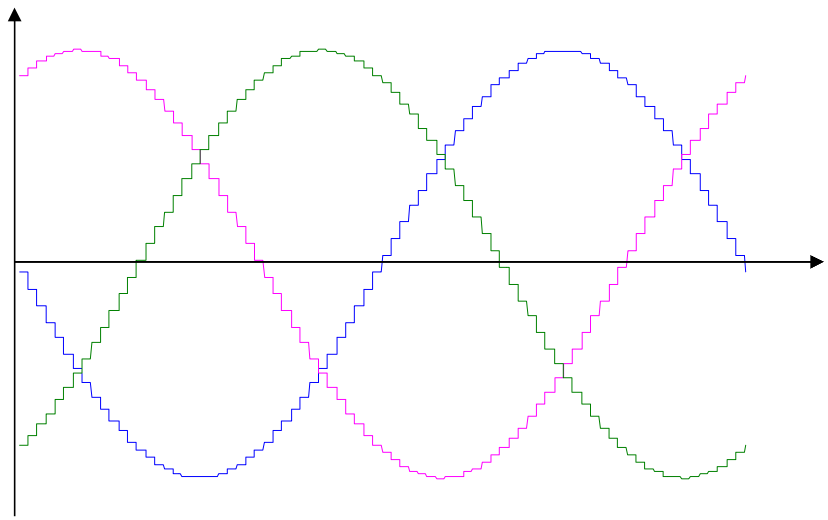
$$\vec{u} = R \cdot \vec{i} + L \cdot \frac{di}{dt} + \vec{e}$$

Power-invarians

$$p(t) = u_a \cdot i_a + u_b \cdot i_b + u_c \cdot i_c = u_\alpha \cdot i_\alpha + u_\beta \cdot i_\beta$$

Symmetric emf





$$
\begin{cases}
e_a = \hat{e} \cdot \cos(\omega \cdot t) \\
e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \dfrac{2\pi}{3}\right) \\
e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \dfrac{4\pi}{3}\right)
\end{cases}
$$

Example, grid voltage vector

$$\vec{e} = \sqrt{\frac{2}{3}} \cdot \left(e_a + e_b \cdot e^{j\frac{2\pi}{3}} + e_c \cdot e^{j\frac{4\pi}{3}} \right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\cos(\omega \cdot t) + \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\begin{array}{l} \cos(\omega \cdot t) + \left(\cos(\omega \cdot t) \cdot \cos\left(\frac{2\pi}{3}\right) + \sin(\omega \cdot t) \cdot \sin\left(\frac{2\pi}{3}\right)\right) \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) \\ + \left(\cos(\omega \cdot t) \cdot \cos\left(\frac{4\pi}{3}\right) + \sin(\omega \cdot t) \cdot \sin\left(\frac{4\pi}{3}\right)\right) \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \end{array} \right) =$$

$$= \sqrt{\frac{2}{3}} \cdot \hat{e} \cdot \left(\cos(\omega \cdot t) \cdot \left(1 + \frac{1}{4} + \frac{1}{4}\right) + j \cdot \sin(\omega t \cdot t) \cdot \left(\frac{3}{4} + \frac{3}{4}\right)\right) =$$

$$= \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot \left(\cos(\omega \cdot t) + j \cdot \sin(\omega t \cdot t)\right) = E \cdot e^{j\omega t}$$

Voltage equation in the (α,β) and the (*x,y*)-frame

- **Define the rotating reference frame (*d,q*) by E and its Integral**

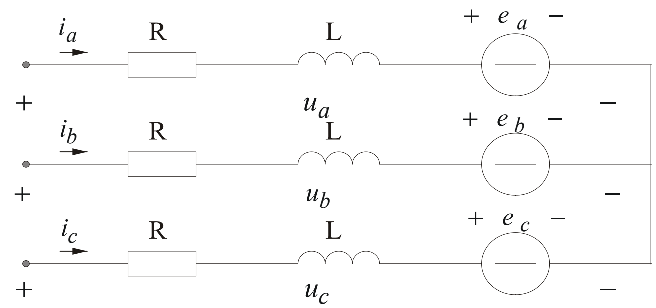
$$\vec{\psi} = \int_0^t \vec{e}\cdot dt = \int_0^t E\cdot e^{j\omega\cdot t}dt = \frac{\vec{e}}{j\cdot\omega} = \frac{E}{\omega}e^{j\left(\omega\cdot t-\frac{\pi}{2}\right)}$$

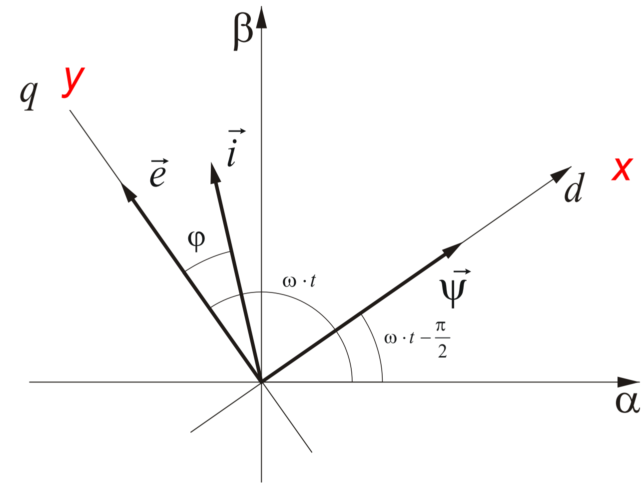
- **Express the stator equation in either the stator reference frame (α,β)**

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{d\,\vec{i}}{dt} + \vec{e}$$

- **Or in the rotor reference frame (d,q)**

$$\vec{u} = R\cdot\vec{i} + L\cdot\frac{d\,\vec{i}}{dt} + j\cdot\omega\cdot L\cdot\vec{i} + \vec{e}$$





Derive the Control in the rotor reference frame

Assume sampled control @ [..., k, k+1, k+2, ...]Ts

Calculate voltage average over one sample period

$$\frac{\int_{k\cdot T_s}^{(k+1)T_s} \vec{u}\cdot dt}{T_s} = \frac{R\cdot \int_{k\cdot T_s}^{(k+1)T_s} \vec{i}\cdot dt + L\cdot \int_{k\cdot T_s}^{(k+1)T_s} \frac{d\vec{i}}{dt}\cdot dt + j\cdot\omega\cdot L \int_{k\cdot T_s}^{(k+1)T_s} \vec{i}\cdot dt \quad \int_{k\cdot T_s}^{(k+1)T_s} \vec{e}\cdot dt}{T_s} =$$

$$= \overline{\vec{u}}(k,k+1) = \left(R + j\cdot\omega\cdot L\right)\cdot \overline{\vec{i}}(k,k+1) + L\cdot\frac{\vec{i}(k+1)-\vec{i}(k)}{T_s} + \overline{\vec{e}}(k,k+1)$$

Make some assumptions ...

Assume:

$$\bar{\vec{u}}(k,k+1) = \vec{u}^*(k) \qquad (a)$$

$$\vec{i}(k+1) = \vec{i}^*(k) \qquad (b)$$

$$\bar{\vec{i}}(k,k+1) = \frac{\vec{i}^*(k) + \vec{i}(k)}{2} \qquad (c)$$

$$\bar{\vec{e}}(k,k+1) = \vec{e}(k) \qquad (d)$$

$$\vec{i}(k) = \sum_{n=0}^{n=k-1} \left(\vec{i}^*(n) - \vec{i}(n)\right) \qquad (e)$$

Gives:

$$\vec{u}^*(k) = \left(R + j\cdot\omega\cdot L\right)\cdot\frac{\vec{i}^*(k)+\vec{i}(k)}{2} + L\cdot\frac{\vec{i}^*(k)-\vec{i}(k)}{T_s} + \vec{e}(k) =$$

$$= R\cdot\frac{\vec{i}^*(k)-\vec{i}(k)}{2} + R\cdot\vec{i}(k) + L\cdot\frac{\vec{i}^*(k)-\vec{i}(k)}{T_s} + j\cdot\omega\cdot L\cdot\frac{\vec{i}^*(k)+\vec{i}(k)}{2} + \vec{e}(k) \approx$$

$$\approx \left(\frac{L}{T_s}+\frac{R}{2}\right)\left(\vec{i}^*(k)-\vec{i}(k)\right) + R\cdot\sum_{n=0}^{n=k-1}\left(\vec{i}^*(n)-\vec{i}(n)\right) + j\cdot\omega\cdot L\cdot\vec{i}(k) + \vec{e}(k) =$$

$$= \left(\frac{L}{T_s}+\frac{R}{2}\right)\cdot\left(\underbrace{\left(\vec{i}^*(k)-\vec{i}(k)\right)}_{\text{Proportional}} + \underbrace{\frac{T_s}{\left(\frac{L}{R}+\frac{T_s}{2}\right)}\cdot\sum_{n=0}^{n=k-1}\left(\vec{i}^*(n)-\vec{i}(n)\right)}_{\text{Integral}}\right) + \underbrace{j\cdot\omega\cdot L\cdot\vec{i}(k)+\vec{e}(k)}_{\textit{Feed forward}}$$

Current Controllers split on *d*- and *q*-

- **Components**

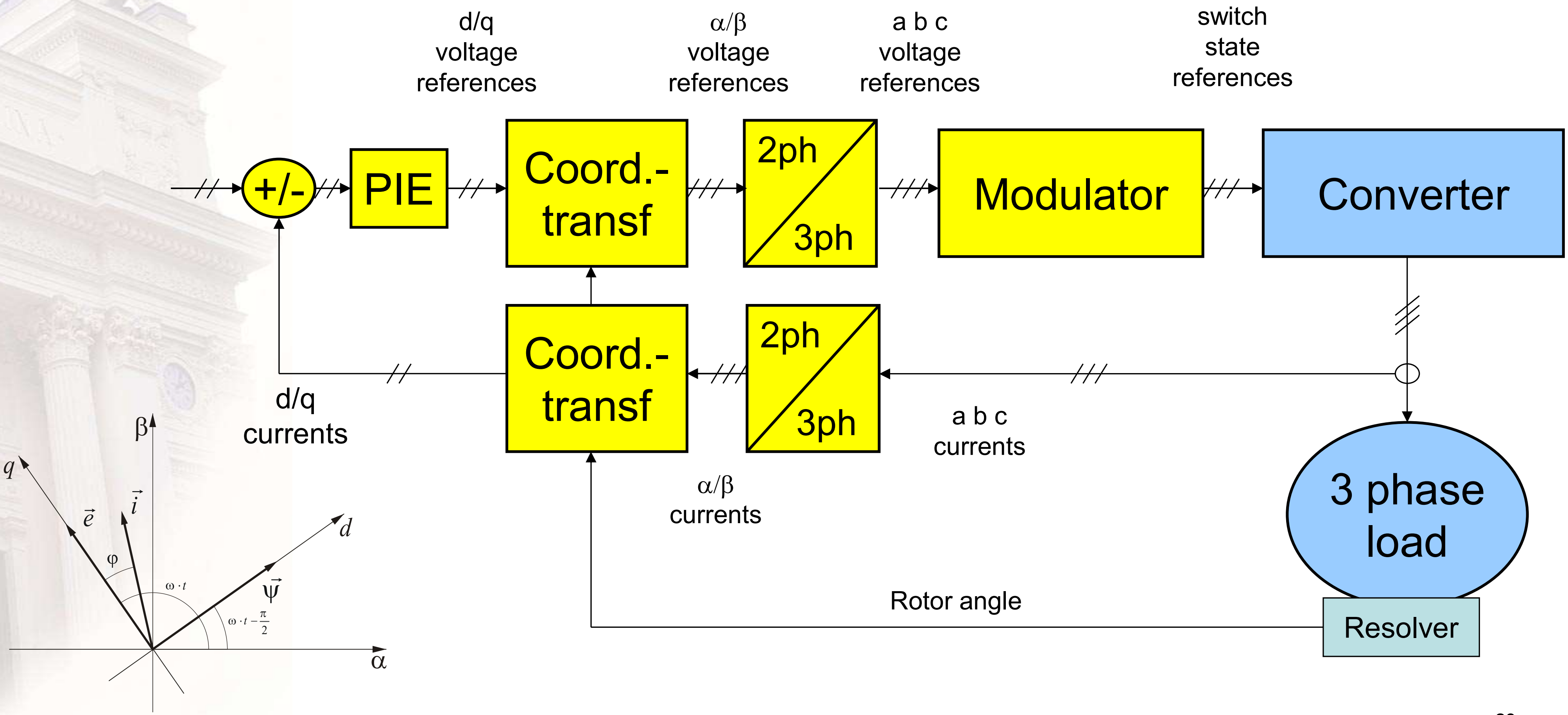
$$u_d^*(k) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot \left(\left(i_d^*(k) - i_d(k)\right) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2}\right)} \cdot \sum_{n=0}^{n=k-1} \left(i_d^*(n) - i_d(n)\right) \right) - \omega \cdot L \cdot i_q(k)$$

$$u_q^*(k) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot \left(\left(i_q^*(k) - i_q(k)\right) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2}\right)} \cdot \sum_{n=0}^{n=k-1} \left(i_q^*(n) - i_q(n)\right) \right) + \omega \cdot L \cdot i_d(k) + e_q(k)$$

- **Some evaluation of values ...**
  - ***>> L=1e-3;***
  - ***>> R=0.05;***
  - ***>> Ts=100e-6;***
  - ***>> [L/Ts R/2] = [10.0000   0.0250]***
  - ***>> [L/R Ts/2] = [0.0200   0.0001]***

- The inductance defines the gain
- The electric time constant defines the Integral gain

Control in a rotating reference frame



Modelling and control of a PMSM

3-phase winding currents as vectors

- **Assign each winding a direction**
- **Scale each equations contribution with a unity vector in each direction**
- **Add the equations, vectorially …**

$$\sqrt{\frac{2}{3}}\cdot\left(u_a = R_s\cdot i_a + \frac{d\psi_a}{dt} = R_s\cdot i_a + \frac{d}{dt}(\psi_{\delta a} + L_{s\lambda}\cdot i_a)\right)$$

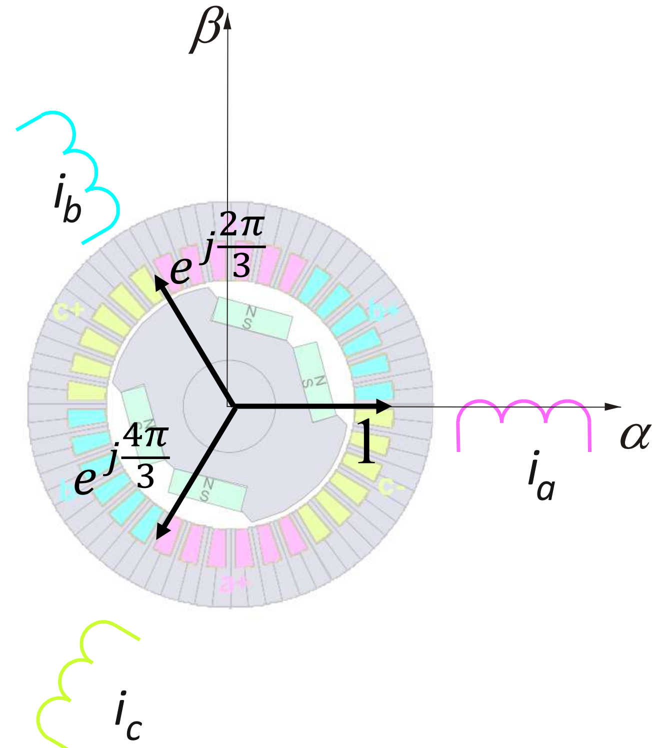
$$e^{j\cdot\frac{2\pi}{3}}\cdot\sqrt{\frac{2}{3}}\cdot\left(u_b = R_s\cdot i_b + \frac{d\psi_b}{dt} = R_s\cdot i_b + \frac{d}{dt}(\psi_{\delta b} + L_{s\lambda}\cdot i_b)\right)$$

$$+\; e^{j\cdot\frac{4\pi}{3}}\cdot\sqrt{\frac{2}{3}}\cdot\left(u_c = R_s\cdot i_c + \frac{d\psi_c}{dt} = R_s\cdot i_c + \frac{d}{dt}(\psi_{\delta c} + L_{s\lambda}\cdot i_c)\right)$$

$$\vec{u}_s^{\alpha\beta} = R_s\cdot\vec{i}_s^{\alpha\beta} + \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = R_s\cdot\vec{i}_s^{\alpha\beta} + \frac{d}{dt}\left(\vec{\psi}_\delta^{\alpha\beta} + L_{s\lambda}\cdot\vec{i}_s^{\alpha\beta}\right)$$

PM-flux linkage + the stator currents own contribution to the air gap flux linkage

The stator currents own leakage flux linkage



Introduce the rotor reference frame (x,y) …

- **Express the stator equation in the rotor reference frame**

$$\vec{u}_s^{\alpha\beta} = R_s \cdot \vec{i}_s^{\alpha\beta} + \frac{d}{dt}\left(\vec{\psi}_\delta^{\alpha\beta} + L_{s\lambda} \cdot \vec{i}_s^{\alpha\beta}\right)$$

$$\left\{\vec{s}_s^{\alpha\beta} = \vec{s}_s^{xy} \cdot e^{j\theta_r}\right\}$$

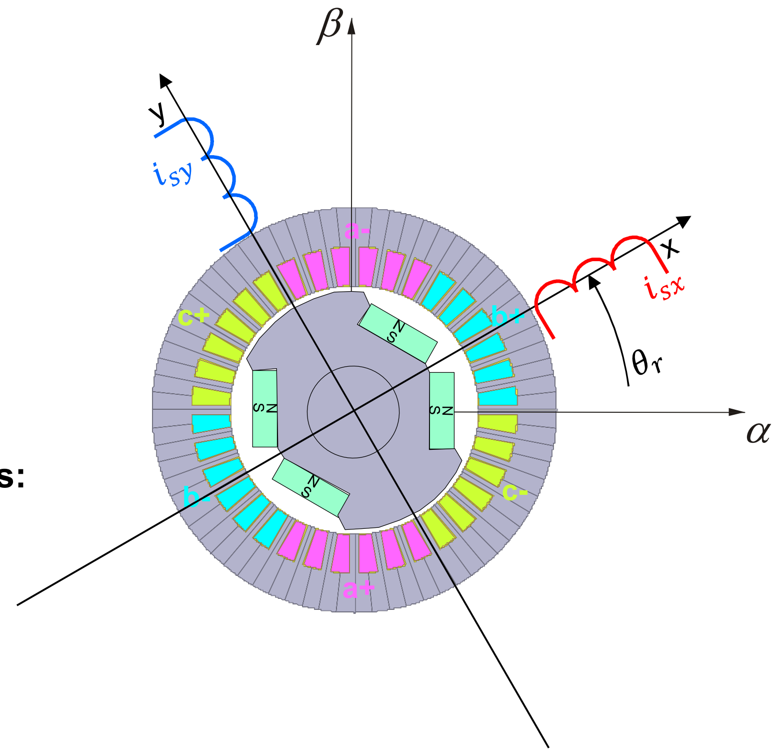
$$\vec{u}_s^{xy} \cdot e^{j\theta_r} = R_s \cdot \vec{i}_s^{xy} \cdot e^{j\theta_r} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} \cdot e^{j\theta_r} + L_{s\lambda} \cdot \vec{i}_s^{xy} \cdot e^{j\theta_r}\right)$$
$$= R_s \cdot \vec{i}_s^{xy} \cdot e^{j\theta_r} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{dq}\right) \cdot e^{j\theta_r} + j \cdot \frac{d\theta_r}{dt} \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right) \cdot e^{j\theta_r}$$

$$\vec{u}_s^{xy} = R_s \cdot \vec{i}_s^{xy} + \frac{d}{dt}\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right) + j \cdot \omega_r \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)$$

- **Split up the complex equation in real- and imaginary parts:**

$$u_{sx} = R_s \cdot i_{sx} + \frac{d}{dt}\left(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}\right) - \omega_r \cdot \left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) =$$
$$= R_s \cdot i_{sx} + \frac{d}{dt}\left(\psi_m + L_{sx} \cdot i_{sx}\right) - \omega_r \cdot L_{sy} \cdot i_{sy}$$

$$u_{sy} = R_s \cdot i_{sy} + \frac{d}{dt}\left(L_{my} \cdot i_{sy} + L_{s\lambda} \cdot i_{sy}\right) + \omega_r \cdot \left(\psi_m + L_{mx} \cdot i_{sx} + L_{s\lambda} \cdot i_{sx}\right) =$$
$$= R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot \left(\psi_m + L_{sx} \cdot i_{sx}\right)$$



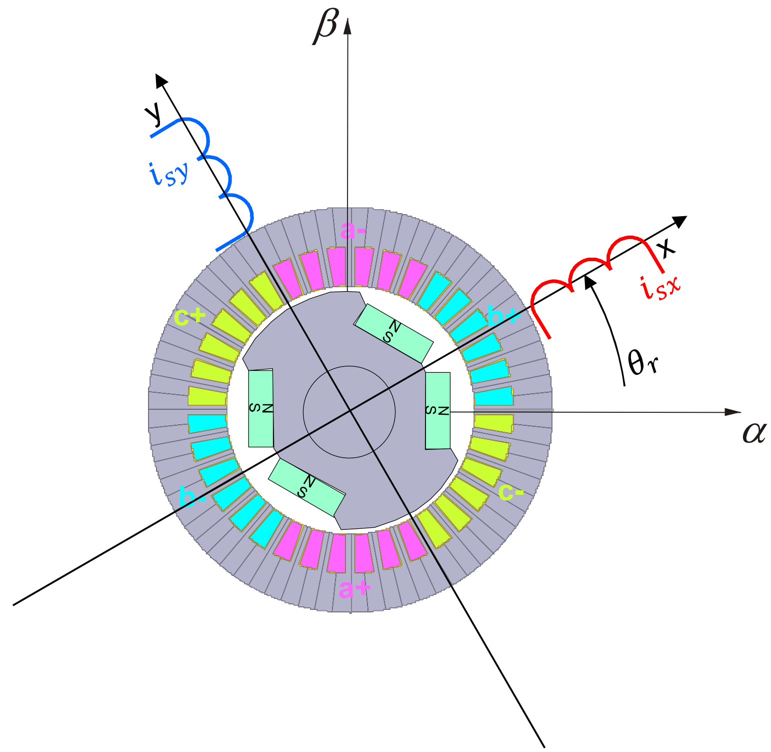
The 3-phase winding is electronically commutated

- **The i_{sx} and i_{sy} currents cannot be supplied directly!**
- **Instead, they have to be supplied as 3 phase currents**
- **The translation is made in two steps:**
- **First, from *xy* to $\alpha\beta$:**

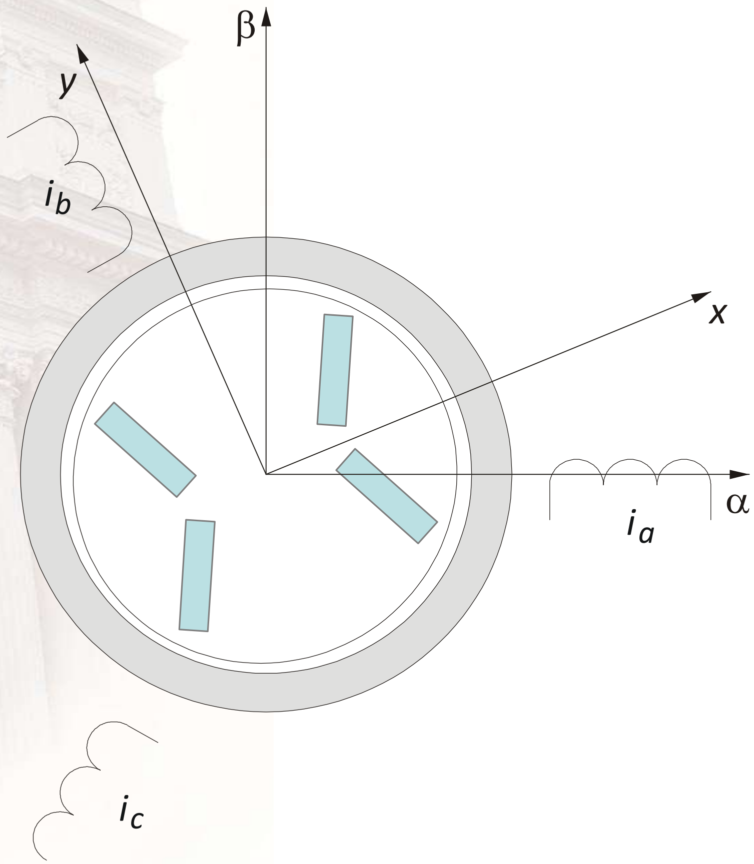
$$\vec{i}_s^{xy} = i_{sx} + j i_{sy} = i_s e^{j\gamma}$$
$$\vec{i}_s^{\alpha\beta} = \vec{i}_s^{xy} e^{j\theta_r} = i_s e^{j(\gamma+\theta_r)} = i_{s\alpha} + j i_{s\beta}$$
$$i_{s\alpha} + j i_{s\beta} = i_s \cos(\omega_r t + \gamma) + j i_s \sin(\omega_r t + \gamma)$$

- **Then, from $\alpha\beta$ to *abc* :**

$$i_a = \sqrt{\frac{2}{3}} i_{s\alpha}$$
$$i_b = \sqrt{\frac{2}{3}} \left(-\frac{1}{2} i_{s\alpha} + \frac{\sqrt{3}}{2} i_{s\beta}\right)$$
$$i_c = \sqrt{\frac{2}{3}} \left(-\frac{1}{2} i_{s\alpha} - \frac{\sqrt{3}}{2} i_{s\beta}\right)$$



The PMSM – like the ideal 3-phase load



$$\sqrt{\frac{2}{3}}\left(u_{sa} = R_s \cdot i_{sa} + \frac{d\psi_{sa}}{dt}\right)$$

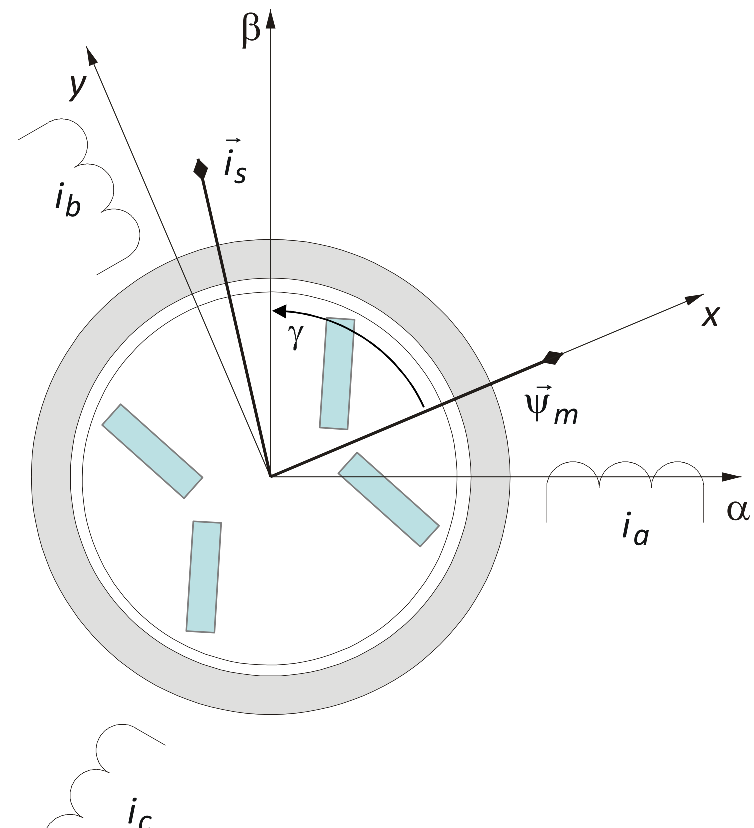
$$\sqrt{\frac{2}{3}} \cdot e^{j\frac{2\pi}{3}} \cdot \left(u_{sb} = R_s \cdot i_{sb} + \frac{d\psi_{sb}}{dt}\right)$$

$$+ \quad \sqrt{\frac{2}{3}} \cdot e^{j\frac{2\pi}{3}} \cdot \left(u_{sc} = R_s \cdot i_{sc} + \frac{d\psi_{sc}}{dt}\right)$$

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}$$

Mathematical Model – Voltage and Current

- **The PMSM has:**
  - ***a 3-phase winding, with resistance and inductance***
  - ***A back emf***
- **It can be described with the same equation as the ideal 3-phase load – in the ROTOR reference frame (*x,y*)**



$$\vec{u}_s = R_s \cdot \vec{i} + L_s \cdot \frac{d\vec{i}}{dt} + j \cdot \omega \cdot L_s \cdot \vec{i} + \vec{e}_s$$

$$\vec{e}_a = j \cdot \omega_{el} \cdot \psi_m$$

$$\vec{u}_s^{\,xy} = R_s \cdot \vec{i}_s^{\,xy} + L_s \cdot \frac{d\,\vec{i}_s^{\,xy}}{dt} + j\omega_r \cdot \left(\vec{\psi}_m^{\,xy} + L_s \cdot \vec{i}_s^{\,xy}\right)$$

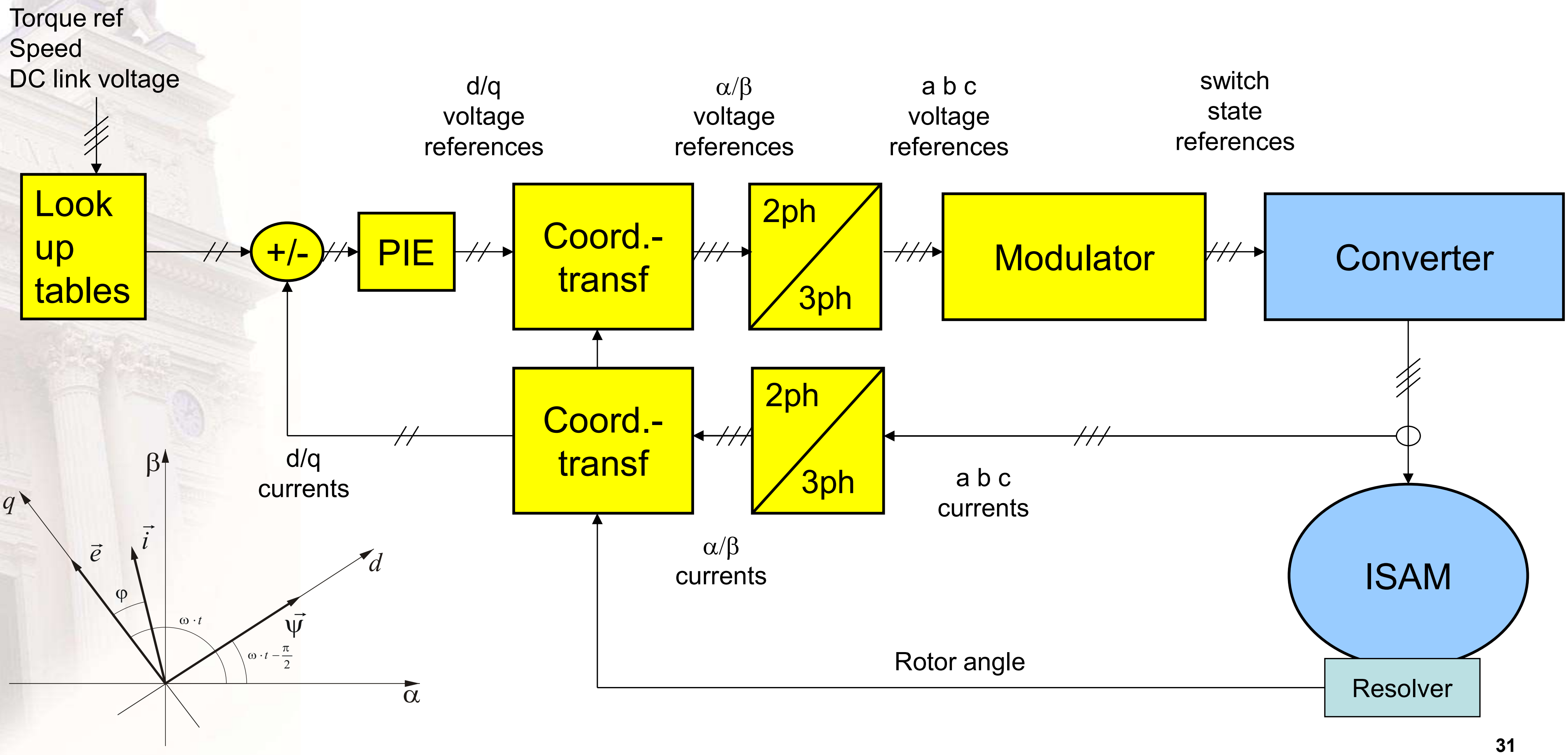
3-phase sampled vector control : 4

Components:

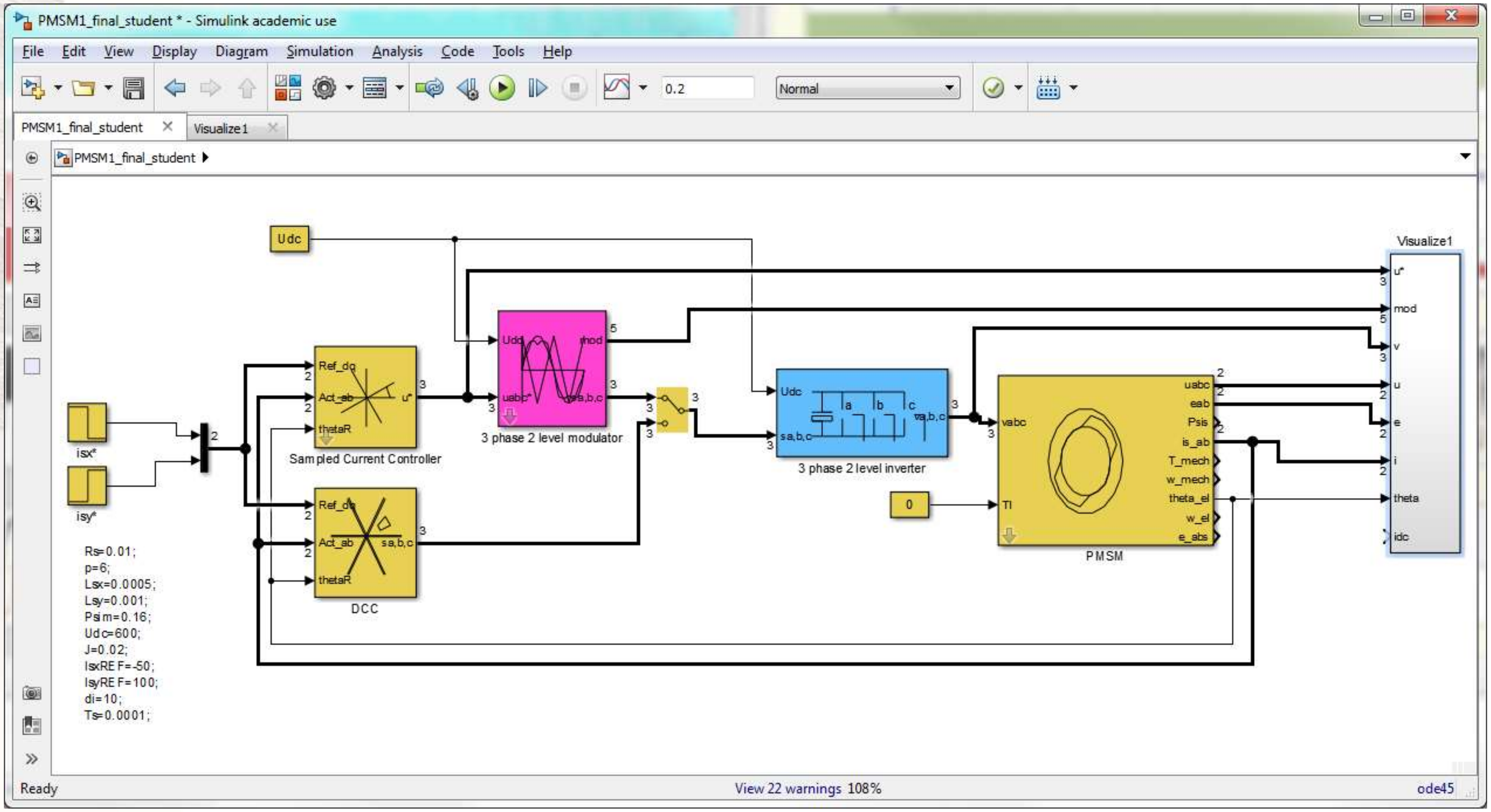
$$u_{sx}^{*}(k)=\left(\frac{L_s}{T_s}+\frac{R}{2}\right)\cdot\left(\left(i_{sx}^{*}(k)-i_{sx}(k)\right)+\frac{T_s}{\left(\frac{L_s}{R_s}+\frac{T_s}{2}\right)}\cdot\sum_{n=0}^{n=k-1}\left(i_{sx}^{*}(n)-i_{sx}(n)\right)\right)-\omega_{el}\cdot L\cdot i_{sy}(k)$$

$$u_{sy}^{*}(k)=\left(\frac{L_s}{T_s}+\frac{R}{2}\right)\cdot\left(\left(i_{sy}^{*}(k)-i_{sy}(k)\right)+\frac{T_s}{\left(\frac{L_s}{R_s}+\frac{T_s}{2}\right)}\cdot\sum_{n=0}^{n=k-1}\left(i_{sy}^{*}(n)-i_{sy}(n)\right)\right)+\omega_{el}\cdot L\cdot i_{sx}(k)+e_{sy}(k)$$

Control in a rotating reference frame



Some Simulink



End of modelling lecture