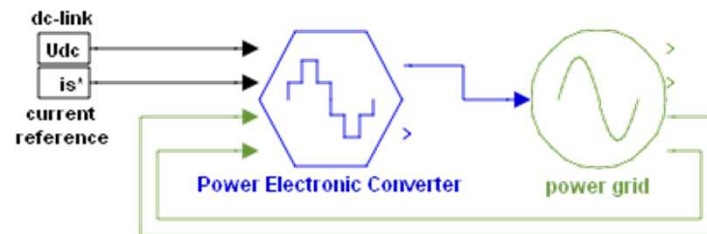


L10 - AC current control



Current vector abc → αβ

- **Symmetric 3-phase**

$$\begin{cases} i_1(t) = \sqrt{2} \cdot \hat{i} \cdot \cos(\omega t - \varphi) \\ i_2(t) = \sqrt{2} \cdot \hat{i} \cdot \cos\left(\omega t - \frac{2\pi}{3} - \varphi\right) \\ i_3(t) = \sqrt{2} \cdot \hat{i} \cdot \cos\left(\omega t - \frac{4\pi}{3} - \varphi\right) \end{cases}$$

- **Transform from abs to αβ**

$$\tilde{i}^{\alpha\beta} = \sqrt{\frac{2}{3}} \cdot \left(i_a \cdot e^{j\frac{0\pi}{3}} + i_b \cdot e^{j\frac{2\pi}{3}} + i_c \cdot e^{j\frac{4\pi}{3}} \right)$$

$$\tilde{i}^{\alpha\beta} = \sqrt{\frac{2}{3}} \cdot \left(\hat{i} \cdot \cos(\omega t - \varphi) + \hat{i} \cdot \cos\left(\omega t - \frac{2\pi}{3} - \varphi\right) \cdot e^{j\frac{2\pi}{3}} + \hat{i} \cdot \cos\left(\omega t - \frac{4\pi}{3} - \varphi\right) \cdot e^{j\frac{4\pi}{3}} \right) \quad \left\{ \cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \right\}$$

$$\tilde{i}^{\alpha\beta} = \sqrt{\frac{2}{3}} \cdot \hat{i} \cdot \left(\frac{e^{j(\omega t - \varphi)} + e^{-j(\omega t - \varphi)}}{2} + \frac{e^{j(\omega t - \frac{2\pi}{3} - \varphi)} + e^{-j(\omega t + \frac{2\pi}{3} - \varphi)}}{2} \cdot e^{j\frac{2\pi}{3}} + \frac{e^{j(\omega t - \frac{4\pi}{3} - \varphi)} + e^{-j(\omega t + \frac{4\pi}{3} - \varphi)}}{2} \cdot e^{j\frac{4\pi}{3}} \right)$$

Current vector $\alpha\beta$

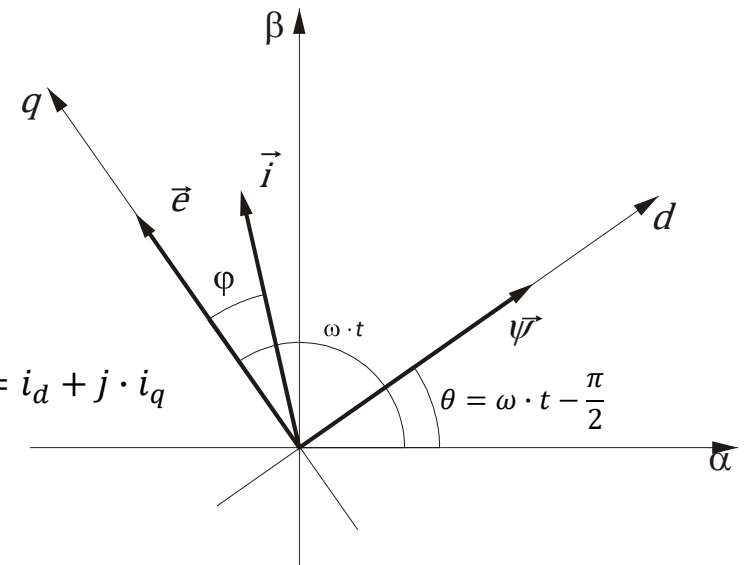
$$\begin{aligned}
 \vec{i}^{\alpha\beta} &= \sqrt{\frac{2}{3}} \cdot \frac{\hat{i}}{2} \cdot \left(\begin{aligned} &e^{j\omega t - j\varphi} + e^{-j\omega t + j\varphi} + \dots \\ &\dots + e^{j\omega t - j\frac{2\pi}{3} - j\varphi + j\frac{2\pi}{3}} + e^{-j\omega t + j\frac{2\pi}{3} + j\varphi + j\frac{2\pi}{3}} + \dots \\ &\dots + e^{j\omega t - j\frac{4\pi}{3} - j\varphi + j\frac{4\pi}{3}} + e^{-j\omega t + j\frac{4\pi}{3} + j\varphi + j\frac{4\pi}{3}} \end{aligned} \right) = \\
 &= \sqrt{\frac{2}{3}} \cdot \frac{\hat{i}}{2} \cdot \left(e^{j\omega t - j\varphi} + e^{-j\omega t + j\varphi} + e^{j\omega t - j\varphi} + e^{-j\omega t + j\varphi + j\frac{4\pi}{3}} + e^{j\omega t - j\varphi} + e^{-j\omega t + j\varphi + j\frac{8\pi}{3}} \right) = \\
 &= \sqrt{\frac{2}{3}} \cdot \frac{\hat{i}}{2} \cdot \left(3 \cdot e^{j\omega t - j\varphi} + \frac{3 \cdot e^{-j\omega t + j\varphi}}{3} \cdot \left(1 + e^{j\frac{4\pi}{3}} + e^{j\frac{8\pi}{3}} \right) \right) = \\
 &= \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot \left(e^{j\omega t - j\varphi} + \frac{e^{-j\omega t + j\varphi}}{3} \cdot \left(1 - \frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right) = \\
 &= \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot e^{j\omega t - j\varphi} = \sqrt{\frac{3}{2}} \cdot \hat{i} (\cos(\omega t - \varphi) + j \sin(\omega t - \varphi)) = i_\alpha + j \cdot i_\beta
 \end{aligned}$$

Current vector $\alpha\beta \rightarrow dq$

- The flux defines d -axis which lags EMF voltage by -90°

$$\begin{aligned}\vec{i}_{dq} &= \vec{i}_{\alpha\beta} \cdot e^{-j(\omega t - \frac{\pi}{2})} = \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot e^{j\omega t - j\varphi} \cdot e^{-j(\omega t - \frac{\pi}{2})} = \\ &= \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot e^{j(\frac{\pi}{2} - \varphi)} = \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot (\cos(\frac{\pi}{2} - \varphi) + j \cdot \sin(\frac{\pi}{2} - \varphi)) = i_d + j \cdot i_q\end{aligned}$$

$$\begin{cases} i_d = \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot \cos(\frac{\pi}{2} - \varphi) \\ i_q = \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot \sin(\frac{\pi}{2} - \varphi) \end{cases}$$



Power from dq

- Using dq -frame, dot product

$$P = e^{dq} \cdot i^{dq} = e_d i_d + e_q i_q$$

- Grid voltage

$$\vec{e}^{\alpha\beta} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t}$$

- Transform $\alpha\beta \rightarrow dq$

$$\begin{aligned} \vec{e}_{dq} &= \vec{e}_{\alpha\beta} \cdot e^{j(\omega t - \frac{\pi}{2} - \phi)} \rightarrow \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t - j\omega t + j\frac{\pi}{2}} = \\ &= j \cdot \sqrt{\frac{3}{2}} \cdot \hat{e} = \vec{e}_q (\vec{e}_d = 0) \end{aligned}$$

- Active power

$$\begin{aligned} P &= e_q i_q = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot \sqrt{\frac{3}{2}} \cdot \hat{i} \sin\left(\frac{\pi}{2} - \phi\right) = \\ &= \frac{3}{2} \hat{e} \cdot \hat{i} \cos(\phi) = \sqrt{3} E_L I \cos(\phi) \end{aligned}$$

- Reactive power

$$\begin{aligned} Q &= e_q i_d = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot \sqrt{\frac{3}{2}} \cdot \hat{i} \cos n\left(\frac{\pi}{2} - \phi\right) = \\ &= \frac{3}{2} \hat{e} \cdot \hat{i} \sin(\phi) = \sqrt{3} E_L I \sin(\phi) \end{aligned}$$

Power from $\alpha\beta$

$$\begin{aligned} P &= \operatorname{Re}(\vec{u} \cdot \vec{i}^*) = \operatorname{Re}\left(\sqrt{\frac{3}{2}}\hat{u} \cdot e^{j\omega} \cdot \sqrt{\frac{3}{2}}\hat{i} \cdot e^{-j(\omega t - \phi)}\right) \\ &= \frac{3}{2}\operatorname{Re}(\hat{u} \cdot \hat{i} \cdot e^{j\omega - j\omega + \phi}) = \frac{3}{2}\hat{u} \cdot \hat{i} \cos(\phi) = \sqrt{3} \cdot U \cdot I_{rms,phase} \cdot \cos(\varphi) \end{aligned}$$

Relation between U_{dc} and U_{grid}

- **Sinusoidal modulation**

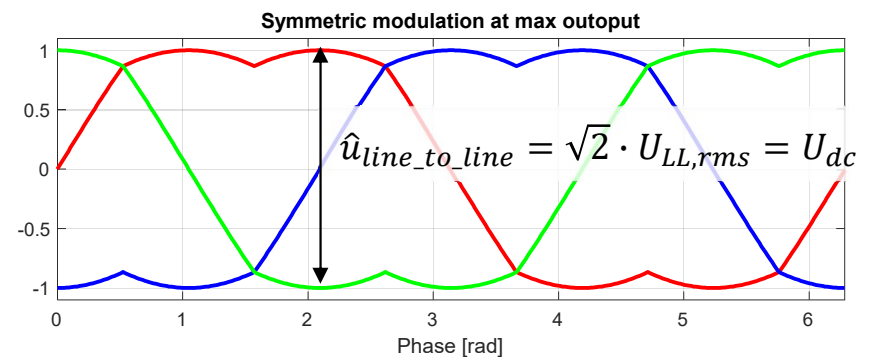
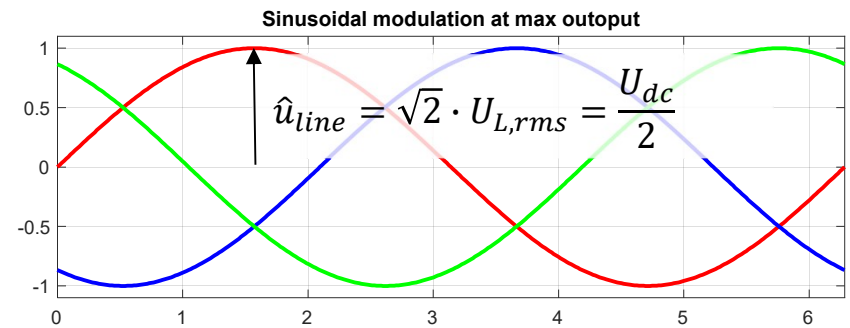
$$U_{LN,rms} = \frac{1}{\sqrt{2}} \frac{U_{dc}}{2} \approx 0.35U_{dc}$$

$$U_{LL,rms} = \sqrt{\frac{3}{2}} \frac{U_{dc}}{2} \approx 0.61U_{dc}$$

- **Symmetrical modulation**

$$U_{LL,rms} = \frac{U_{dc}}{\sqrt{2}} \approx 0.71U_{dc}$$

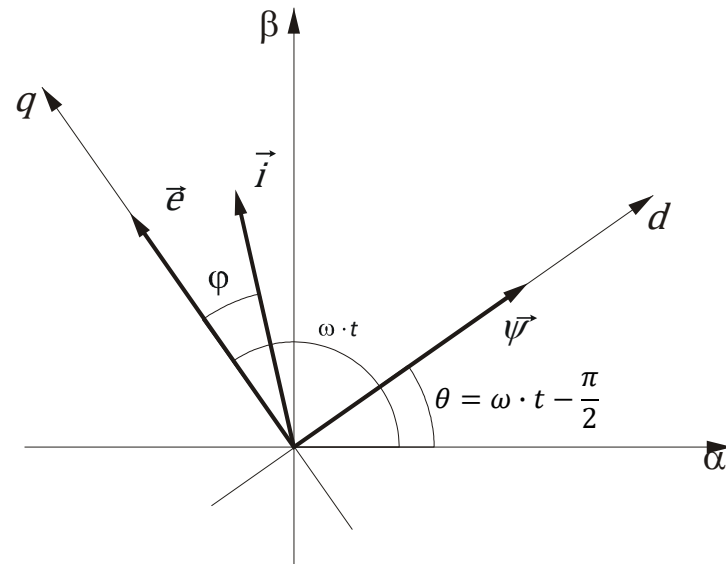
$$U_{LN,rms} = \frac{1}{\sqrt{3}} \frac{U_{dc}}{\sqrt{2}} \approx 0.41U_{dc}$$



Re-introduce the rotating reference frame

- Use the integral of the grid back emf vector:

$$\begin{aligned}\vec{\psi} &= \int_0^t \vec{e} \cdot dt = \int_0^t E \cdot e^{j\omega \cdot t} dt = \frac{\vec{e}}{j \cdot \omega} \\ &= \frac{E}{\omega} e^{j(\omega \cdot t - \frac{\pi}{2})}\end{aligned}$$



Re-introduce the grid flux reference frame (d,q) ...

- Express the grid voltage equation in the grid flux reference frame

$$\vec{u}^{\alpha\beta} = R \cdot \vec{i}^{\alpha\beta} + L \cdot \frac{d\vec{i}^{\alpha\beta}}{dt} + \vec{e}^{\alpha\beta}$$

$$\left\{ \vec{s}^{\alpha\beta} = \vec{s}^{dq} \cdot e^{j\theta} \right\}$$

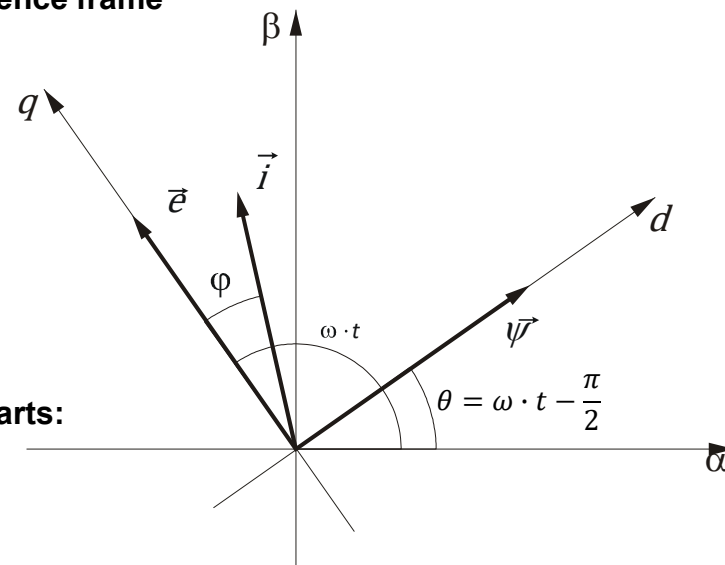
$$\begin{aligned} \vec{u}_s^{dq} \cdot e^{j\theta} &= R \cdot \vec{i}^{dq} \cdot e^{j\theta} + L \cdot \frac{d}{dt} (\vec{i}^{dq} \cdot e^{j\theta}) + \vec{e}^{dq} \cdot e^{j\theta} = \\ &= R \cdot \vec{i}^{dq} \cdot e^{j\theta} + L \cdot \frac{d\vec{i}^{dq}}{dt} \cdot e^{j\theta} + j \cdot \frac{d\theta}{dt} \cdot L \cdot \vec{i}^{dq} \cdot e^{j\theta} + \vec{e}^{dq} \cdot e^{j\theta} = \end{aligned}$$

$$\vec{u}_s^{dq} = R \cdot \vec{i}^{dq} + L \cdot \frac{d\vec{i}^{dq}}{dt} + j \cdot \omega \cdot L \cdot \vec{i}^{dq} + \vec{e}^{dq}$$

- Split up the complex equation in real- and imaginary parts:

$$u_d = R \cdot i_d + L \cdot \frac{di_d}{dt} - \omega \cdot L \cdot i_q$$

$$u_q = R \cdot i_q + L \cdot \frac{di_q}{dt} + \omega \cdot L \cdot i_d + e_q$$



3-phase sampled vector control : 1

- Assume sampled control @ [..., k, k+1, k+2, ...]Ts
- Calculate voltage average over one sample period

$$\frac{\int_{k \cdot T_s}^{(k+1)T_s} \vec{u} \cdot dt}{T_s} = \frac{R \cdot \int_{k \cdot T_s}^{(k+1)T_s} \vec{i} \cdot dt + L \cdot \int_{k \cdot T_s}^{(k+1)T_s} \frac{d\vec{i}}{dt} \cdot dt + j \cdot \omega \cdot L \int_{k \cdot T_s}^{(k+1)T_s} \vec{i} \cdot dt + \int_{k \cdot T_s}^{(k+1)T_s} \vec{e} \cdot dt}{T_s} =$$
$$= \vec{u}(k, k+1) = (R + j \cdot \omega \cdot L) \cdot \vec{i}(k, k+1) + L \cdot \frac{\vec{i}(k+1) - \vec{i}(k)}{T_s} + \vec{e}(k, k+1)$$

3-phase sampled vector control : 2

Assume:

$$\begin{aligned}\bar{u}(k, k+1) &= \bar{u}^*(k) \\ \bar{i}(k+1) &= \bar{i}^*(k) \\ \bar{i}(k, k+1) &= \frac{\bar{i}^*(k) + \bar{i}(k)}{2} \\ \bar{e}(k, k+1) &= \bar{e}(k) \\ \bar{i}(k) &= \sum_{n=0}^{n=k-1} (\bar{i}^*(n) - \bar{i}(n))\end{aligned}$$

Gives:

$$\begin{aligned}\bar{u}^*(k) &= (R + j \cdot \omega \cdot L) \cdot \frac{\bar{i}^*(k) + \bar{i}(k)}{2} + L \cdot \frac{\bar{i}^*(k) - \bar{i}(k)}{T_s} + \bar{e}(k) = \\ &= R \cdot \frac{\bar{i}^*(k) + \bar{i}(k)}{2} + R \cdot \bar{i}(k) + L \cdot \frac{\bar{i}^*(k) - \bar{i}(k)}{T_s} + j \cdot \omega \cdot L \cdot \frac{\bar{i}^*(k) + \bar{i}(k)}{2} + \bar{e}(k) \approx \\ &\approx \left(\frac{L}{T_s} + \frac{R}{2} \right) (\bar{i}^*(k) - \bar{i}(k)) + R \cdot \sum_{n=0}^{n=k-1} (\bar{i}^*(n) - \bar{i}(n)) + j \cdot \omega \cdot L \cdot \bar{i}(k) + \bar{e}(k) = \\ &= \left(\frac{L}{T_s} + \frac{R}{2} \right) \cdot \left(\underbrace{(\bar{i}^*(k) - \bar{i}(k))}_{\text{Proportional}} + \underbrace{\frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2} \right)} \cdot \sum_{n=0}^{n=k-1} (\bar{i}^*(n) - \bar{i}(n))}_{\text{Integral}} \right) + \underbrace{j \cdot \omega \cdot L \cdot \bar{i}(k) + \bar{e}(k)}_{\text{Feedforward}}\end{aligned}$$

Current Controllers split on d - and q -

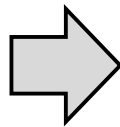
- **Components**

$$u_d^*(k) = \left(\frac{L}{T_s} + \frac{R}{2} \right) \cdot \left((i_d^*(k) - i_d(k)) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2} \right)} \cdot \sum_{n=0}^{n=k-1} (i_d^*(n) - i_d(n)) \right) - \omega \cdot L \cdot i_q(k)$$

$$u_q^*(k) = \left(\frac{L}{T_s} + \frac{R}{2} \right) \cdot \left((i_q^*(k) - i_q(k)) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2} \right)} \cdot \sum_{n=0}^{n=k-1} (i_q^*(n) - i_q(n)) \right) + \omega \cdot L \cdot i_d(k) + e_q(k)$$

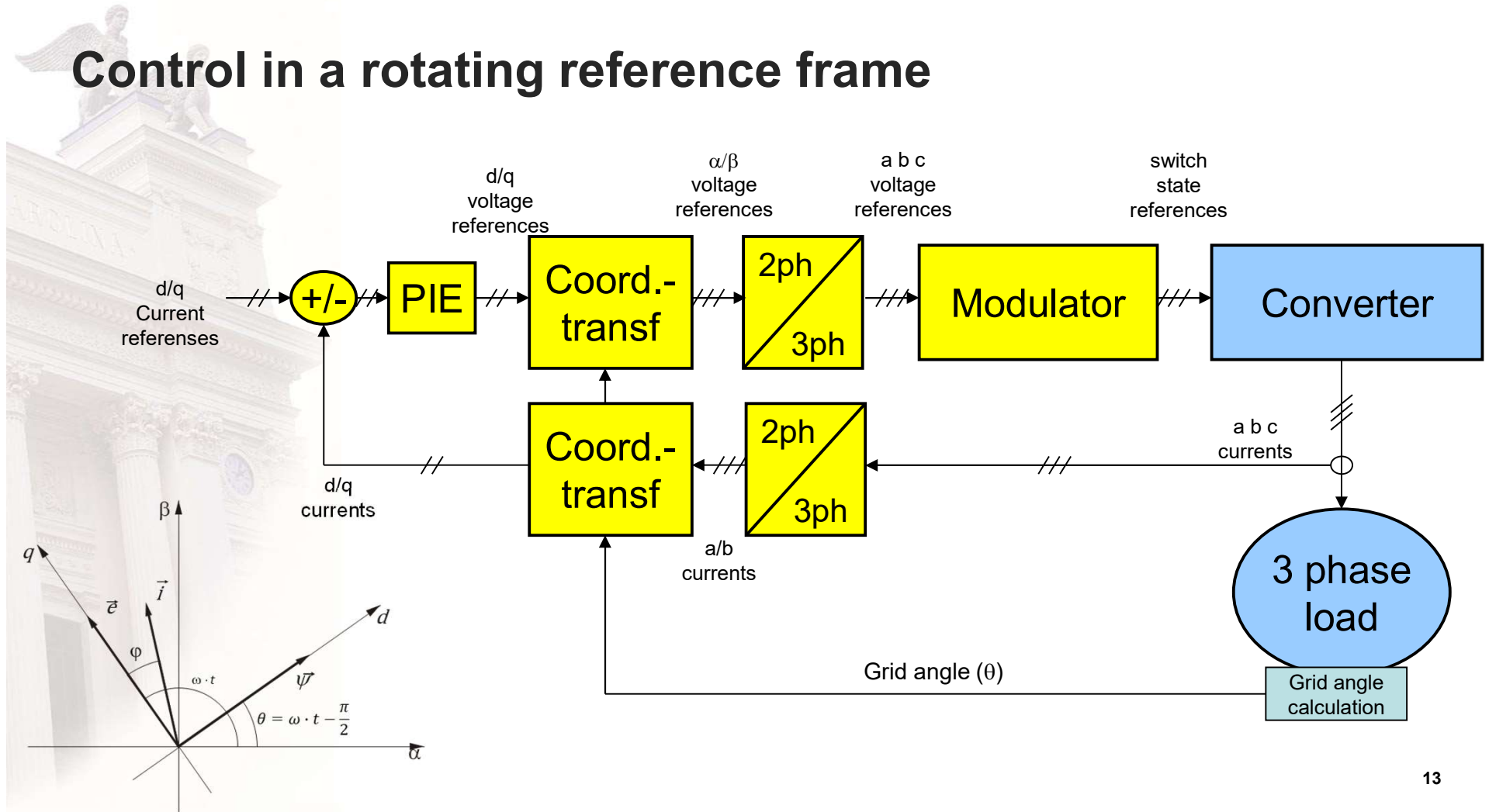
- **Some evaluation of values ...**

- >> $L=1e-3$;
- >> $R=0.05$;
- >> $T_s=100e-6$;
- >> $[L/T_s \ R/2] = [10.0000 \ 0.0250]$
- >> $[L/R \ T_s/2] = [0.0200 \ 0.00005]$



- The inductance defines the gain
- The electric time constant defines the Integral gain

Control in a rotating reference frame



Example

- Corresponding to a traction machine at 100 Hz_

– $L_a=0.001$;

– $R_a=0.1$;

– $T_s=0.1e-3$;

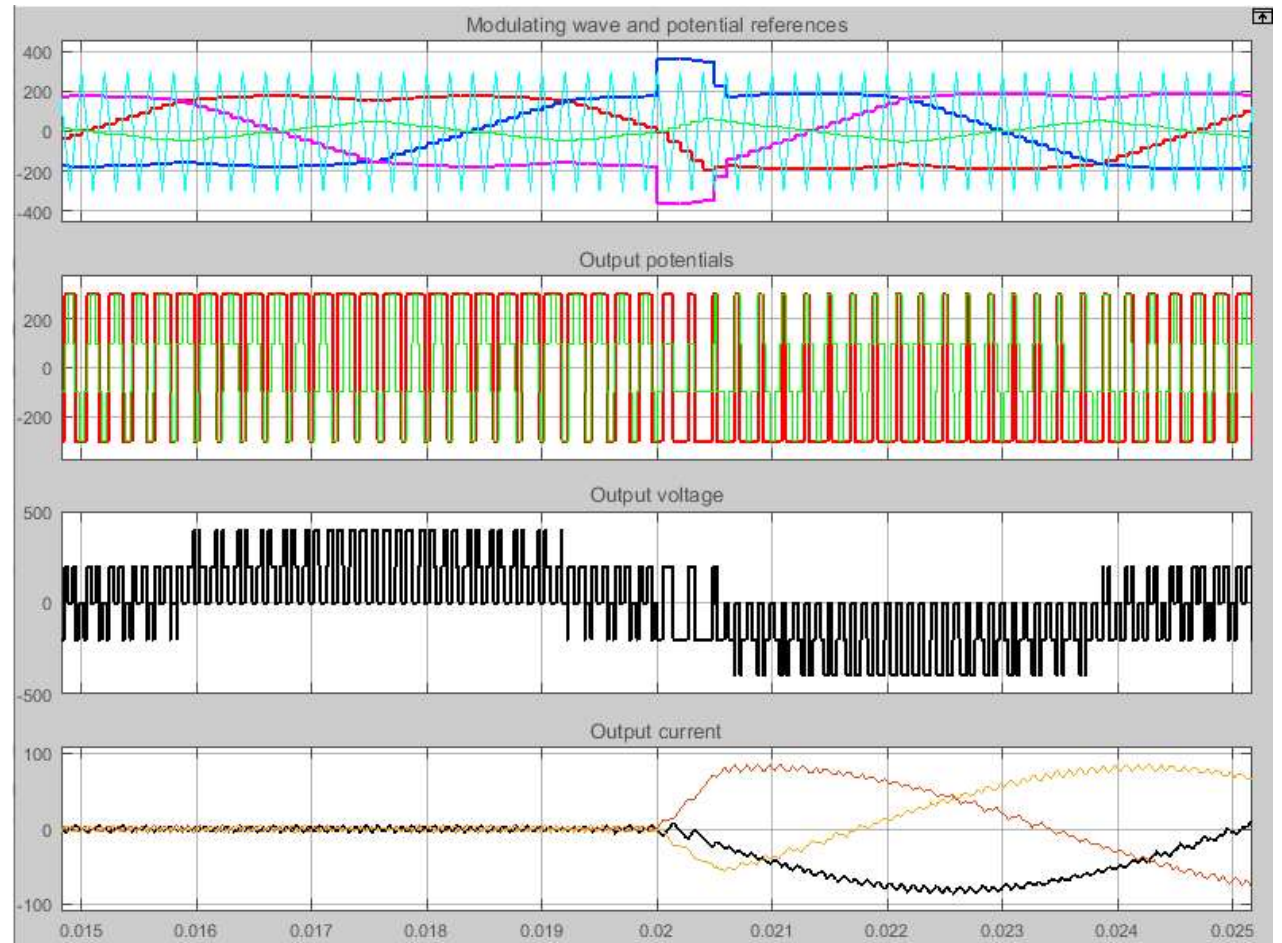
– $U_{dc}=600$;

– $E_a=250$;

– $f_{el}=100$;

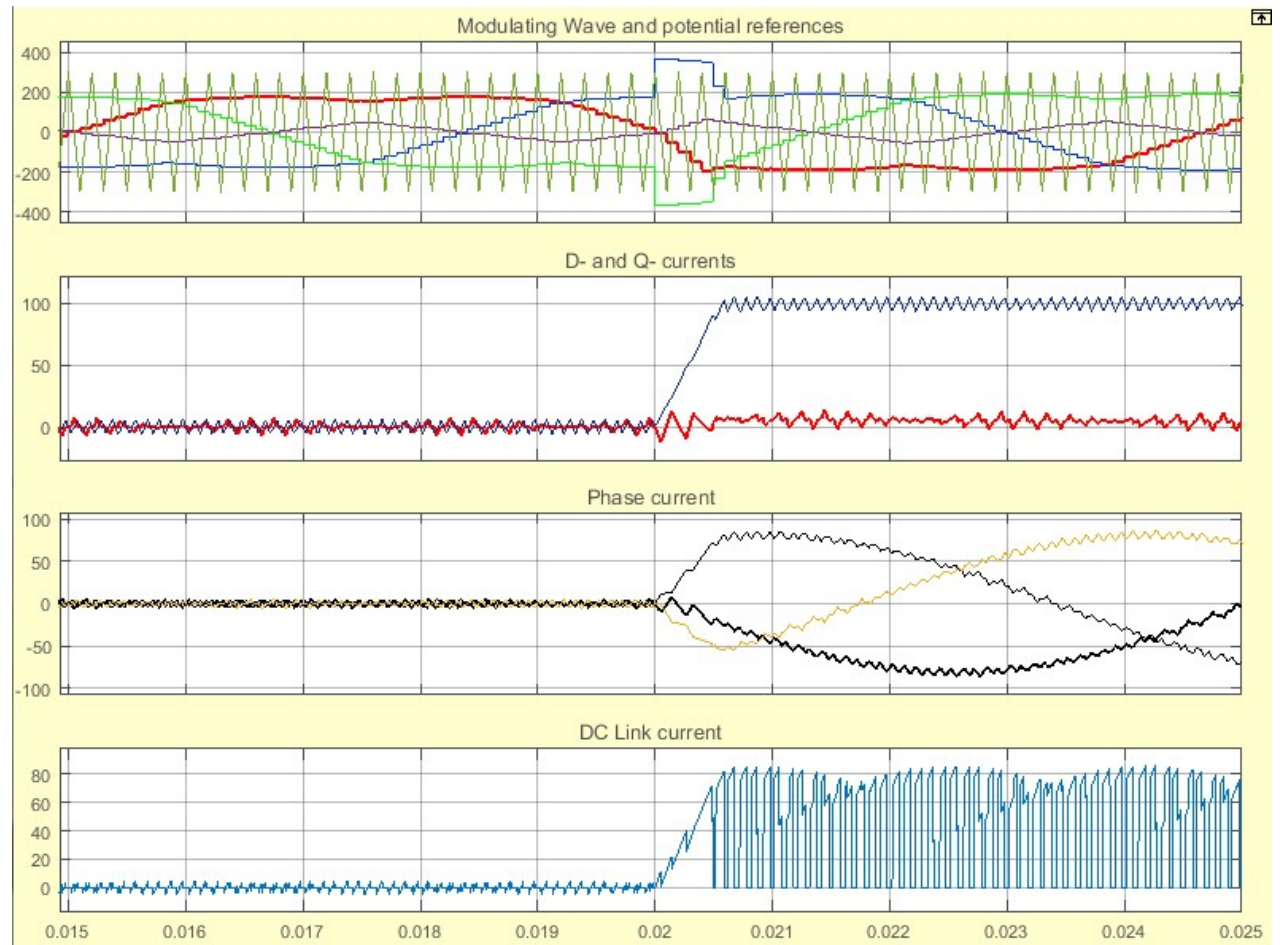
– $I_{sxREF}=0$;

– $I_{syREF}=100$ [A] @ 20 [ms]

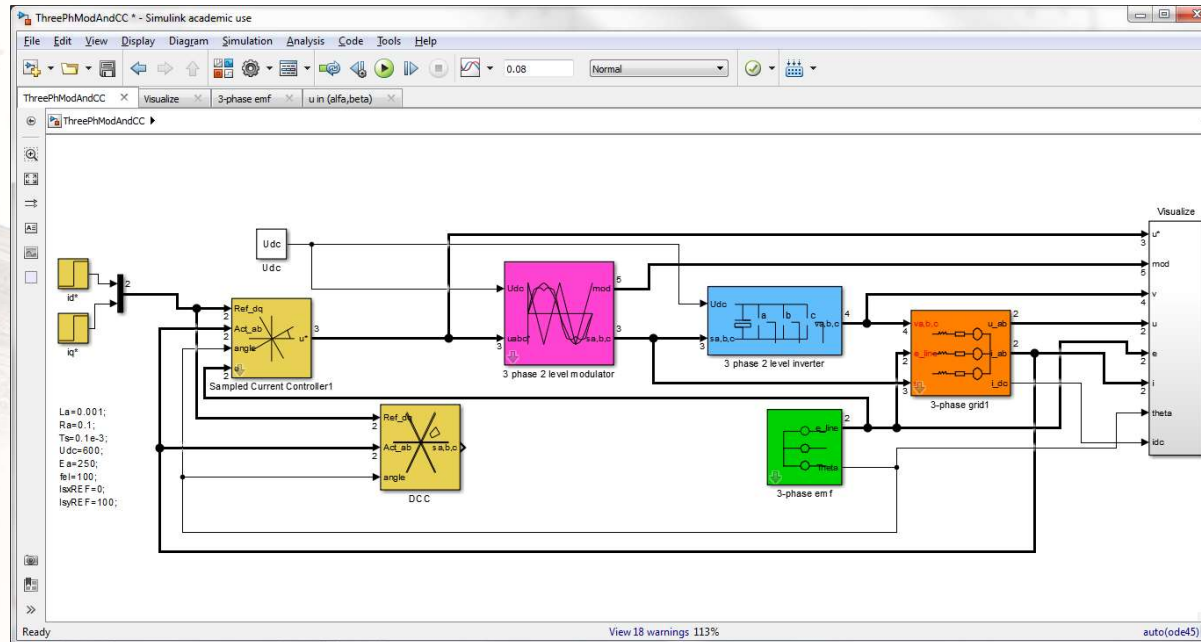


Example

- Corresponding to a traction machine at 100 Hz_
 - $L_a=0.001$;
 - $R_a=0.1$;
 - $T_s=0.1e-3$;
 - $U_{dc}=600$;
 - $E_a=250$;
 - $f_{el}=100$;
 - $I_{sxREF}=0$;
 - $I_{syREF}=100$ [A] @ 20 [ms]
- Notice:
 - DC link ripple current



To Simulink for more ...



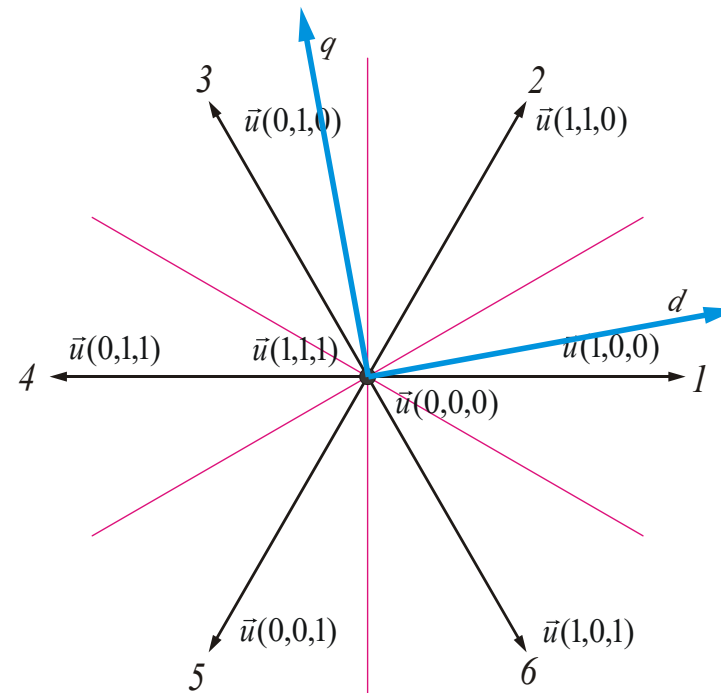
3-phase Direct Current Control

- Where is the current vector moving?

$$\vec{u}^{\alpha\beta} = R \cdot \vec{i}^{\alpha\beta} + L \cdot \frac{d\vec{i}^{\alpha\beta}}{dt} + \vec{e}^{\alpha\beta}$$

$$\frac{d\vec{i}^{\alpha\beta}}{dt} = \frac{\vec{u}^{\alpha\beta} - R \cdot \vec{i}^{\alpha\beta} - \vec{e}^{\alpha\beta}}{L}$$

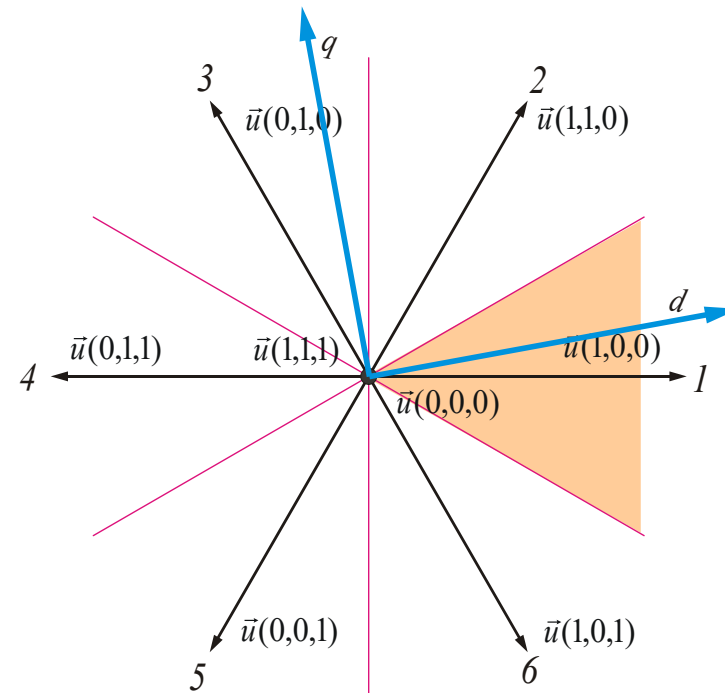
- The, which vetors have the best chance to move the current in a certain direction?



Selecting the right vector

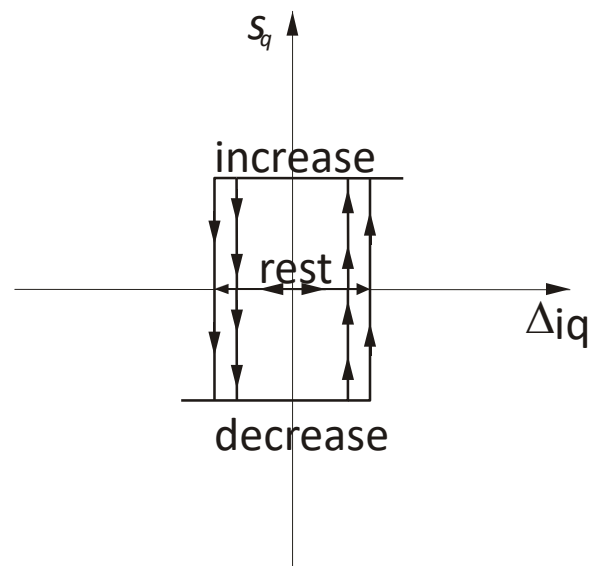
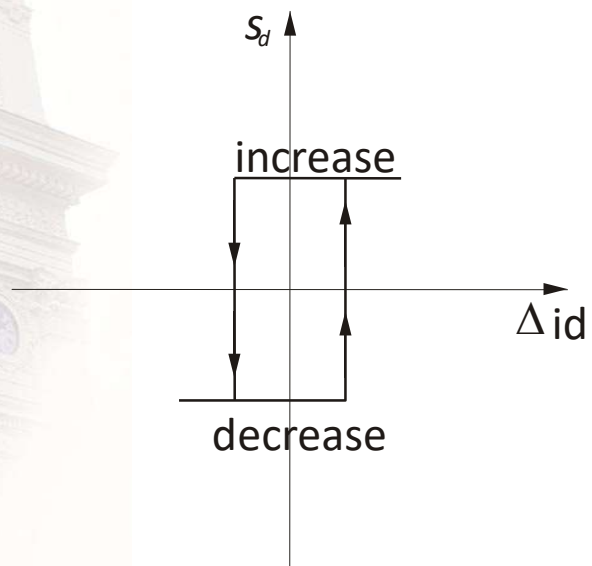
$$\text{vector} = \text{sector} + s_{\text{offset}}$$

s_{offset}	Decrease i_q	Increase i_q
Decrease i_d	4	2
Increase i_d	5	1

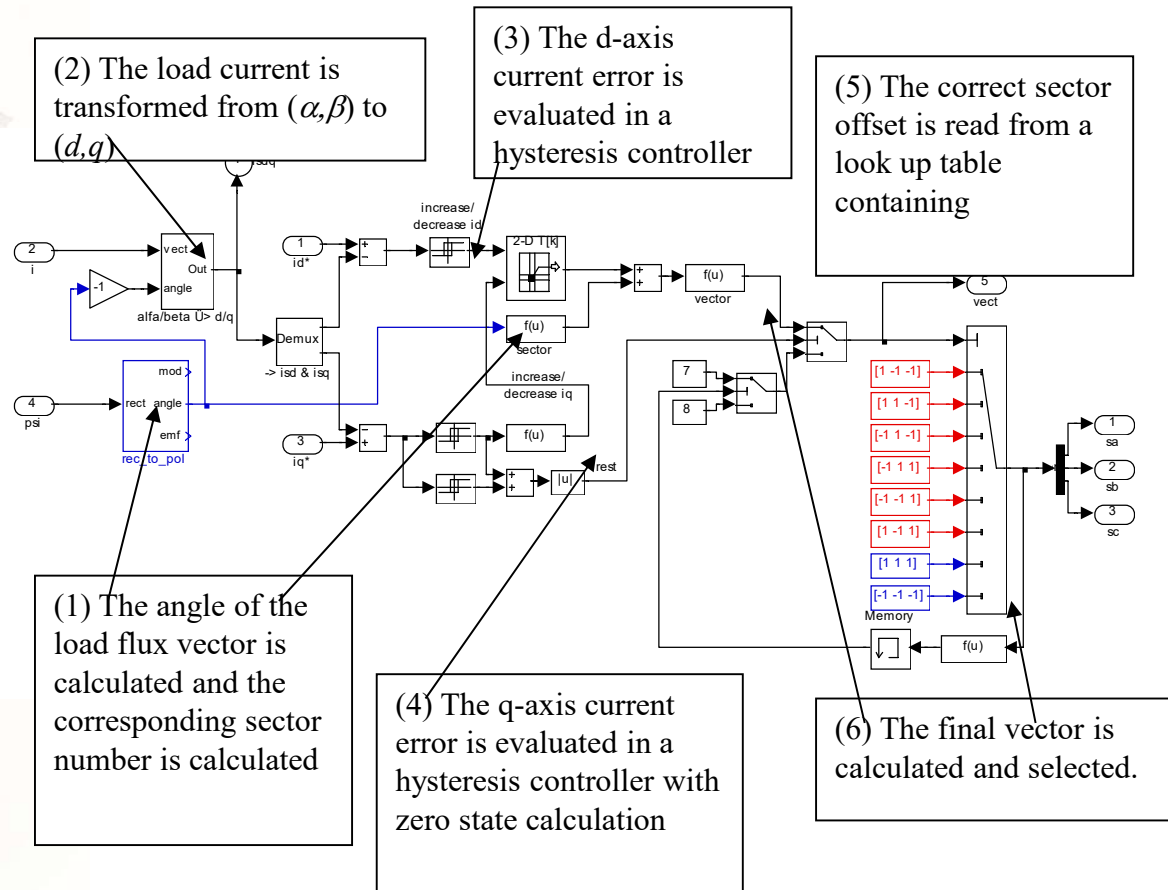




Tolerance bands in d - and q -



The Direct Current Controller



SCC for slow computer

- Predict current sample ahead when defining voltage reference

$$\vec{u} = R \cdot \vec{i} + L \cdot \frac{d\vec{i}}{dt} + j \cdot \omega \cdot \vec{i}$$

$$\vec{u}^*(k) = \vec{u}(k, k+1) = R \cdot \vec{i}_{sp}(k) + j \cdot \omega \cdot L \cdot \vec{i}_{sp}(k) + \frac{L}{T_s} \cdot (\vec{i}_{sp}(k+1) - \vec{i}_{sp}(k))$$

$$\vec{i}_{sp}(k+1) = \vec{i}_{sp}(k) \cdot \left(1 - \frac{R \cdot T_s}{L} - j \cdot \omega \cdot T_s\right) + \frac{T_s}{L} \cdot \vec{u}^*(k)$$

$$\hat{\vec{i}}(k+1) = \vec{i}(k) + (\vec{i}_{sp}(k+1) - \vec{i}_{sp}(k))$$

SCC PIE parameters

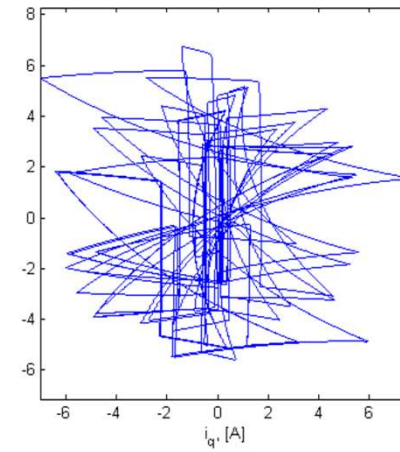
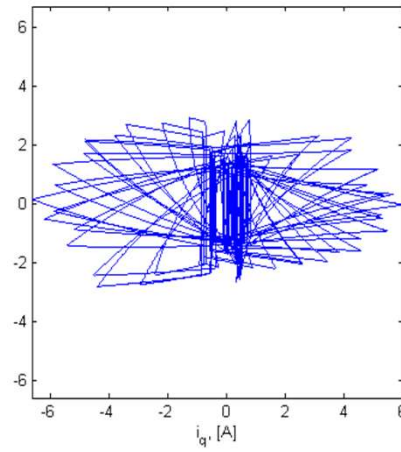
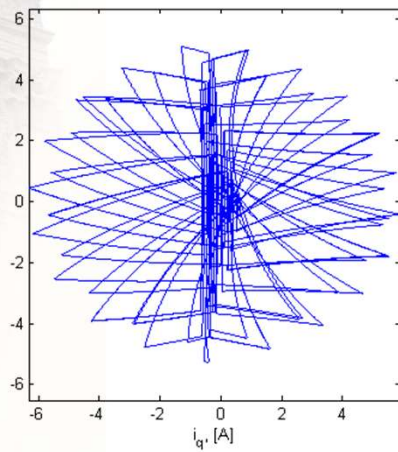
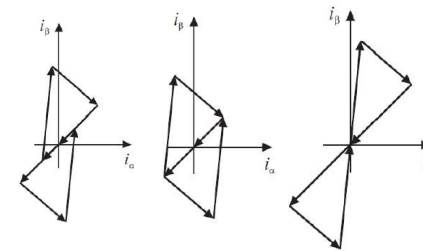
$$\begin{cases} u_{d,ref,k} = K \cdot \left((i_{d,ref,k} - i_{d,k}) + \frac{1}{T_i} \cdot \sum_{n=0}^{k-1} (i_{d,ref,n} - i_{d,n}) \right) - K_c \cdot \frac{i_{q,ref,k} + i_{q,k}}{2} + e_{d,k} \\ u_{q,ref,k} = K \cdot \left((i_{q,ref,k} - i_{q,k}) + \frac{1}{T_i} \cdot \sum_{n=0}^{k-1} (i_{q,ref,n} - i_{q,n}) \right) + K_c \cdot \frac{i_{d,ref,k} + i_{d,k}}{2} + e_{q,k} \end{cases}$$

$$\begin{cases} K = \left(\frac{L}{T_s} + \frac{R}{2} \right) \\ T_i = R / \left(\frac{L}{T_s} + \frac{R}{2} \right) = 1 / \left(\frac{L}{RT_s} + \frac{1}{2} \right) \\ K_c = \frac{\omega_1 L}{2} \end{cases}$$

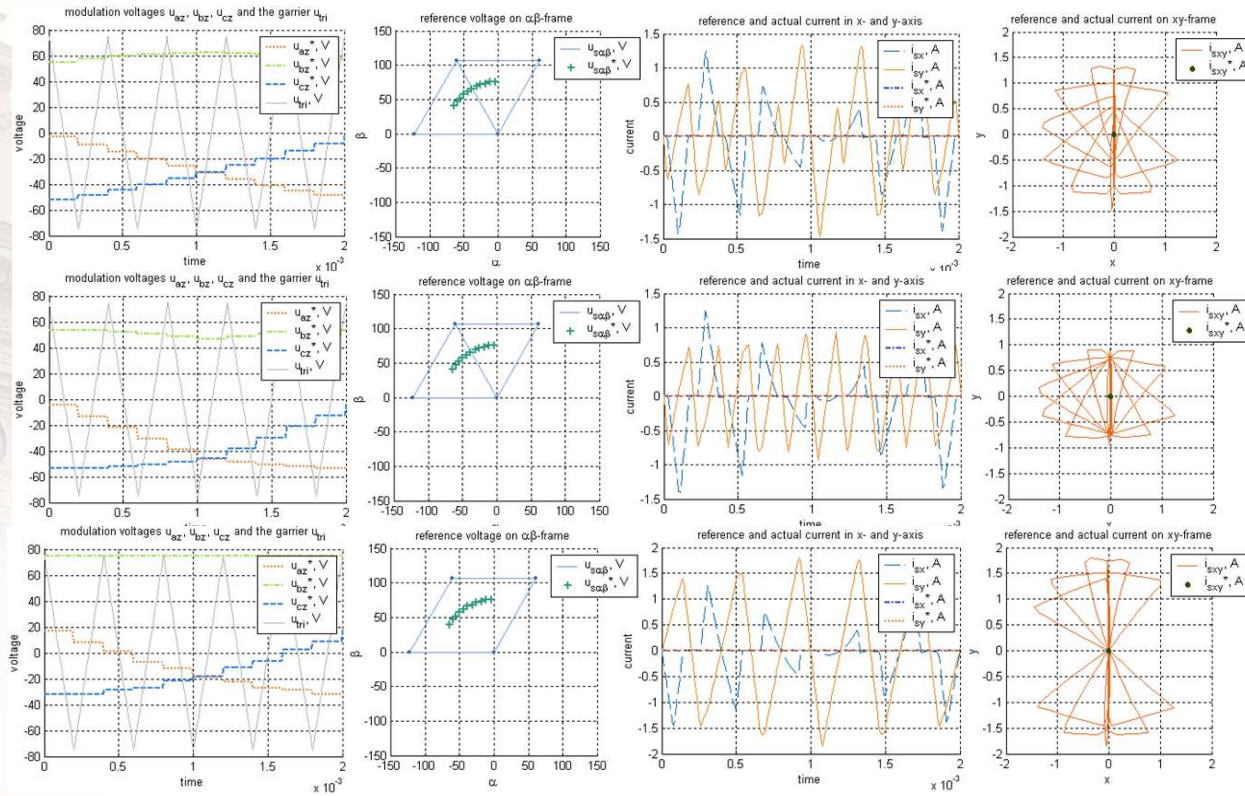
SCC current ripple

- Sine, symmetric and bus clamped
- $U_{dc}=650V$, $L=10mH$, $R=1\Omega$

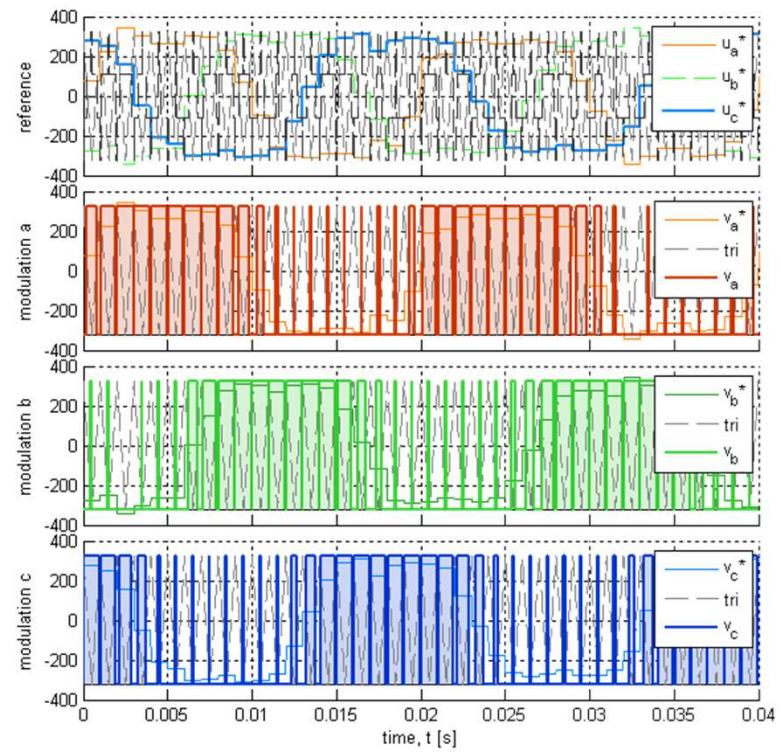
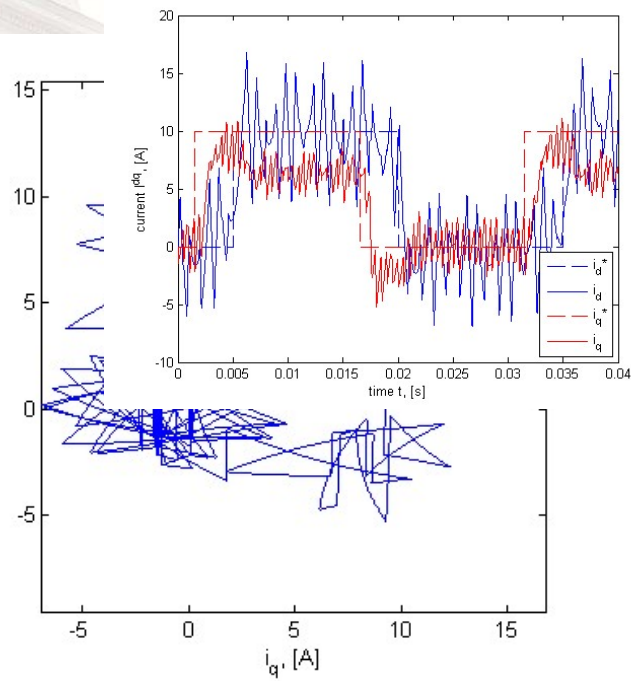
2.7



SCC current ripple

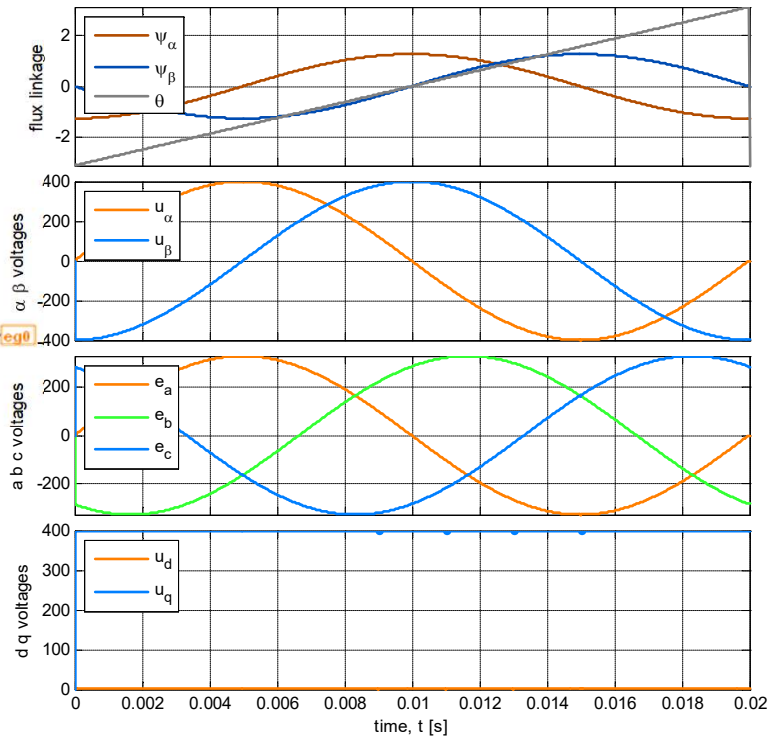
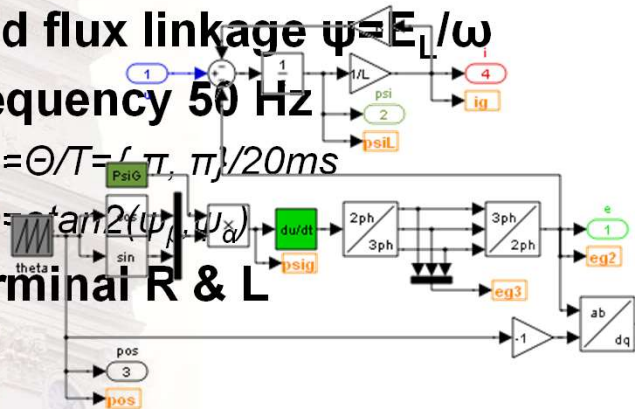


SCC step response

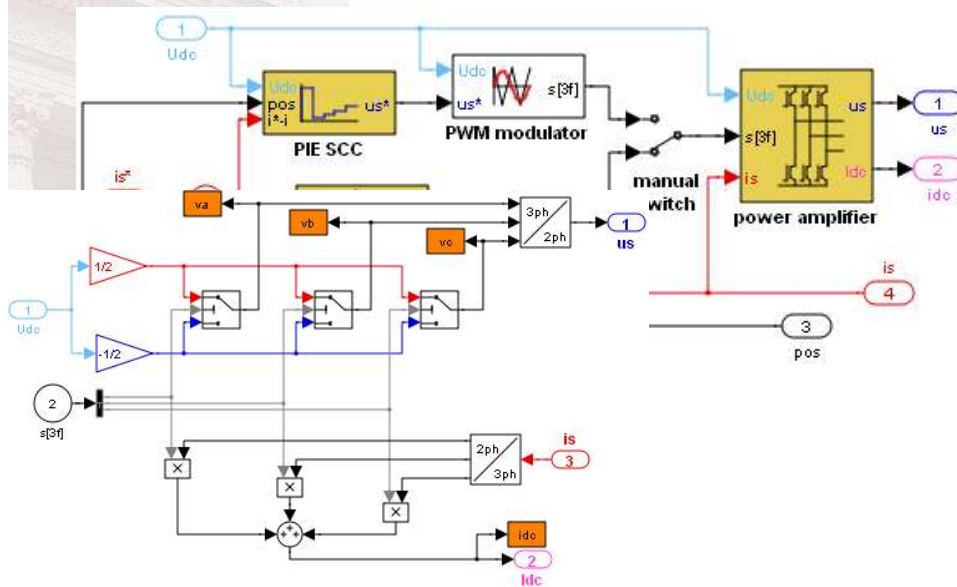


Grid voltage

- Grid flux linkage $\psi = E_L / \omega$
- Frequency 50 Hz
 - $\omega = \Theta / T = f \cdot 2\pi, \pi / 20ms$
 - $\Theta = \int \omega dt = \int 2\pi f dt = 2\pi f t$
- Terminal R & L



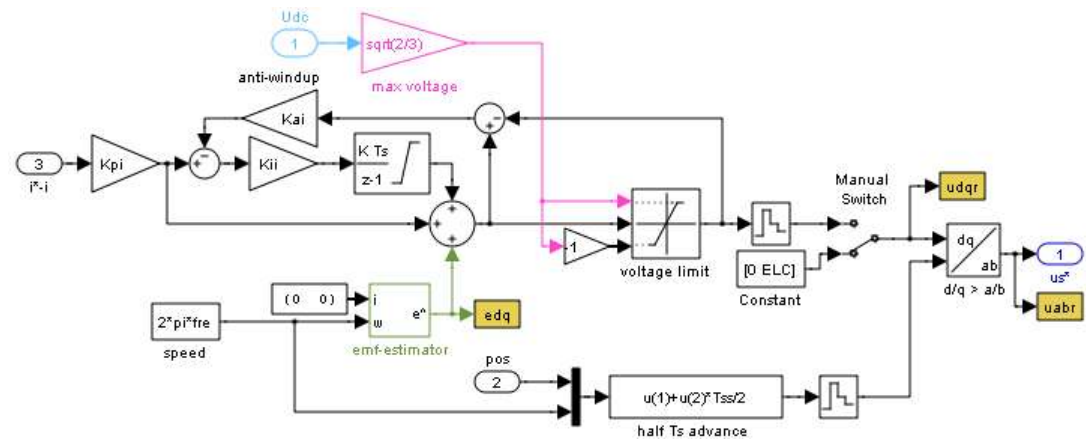
3 ϕ power electronic converter



- **Two different current controllers**
 - *Sampled current control*
 - *Direct current control*
- **Voltages and currents mainly in $\alpha\beta$ frame,**
- **dq used for control**
- **Field rotation angle used instead of flux vector**
 - *Grid flux!*
- **Switch states $\{0,1\}$**

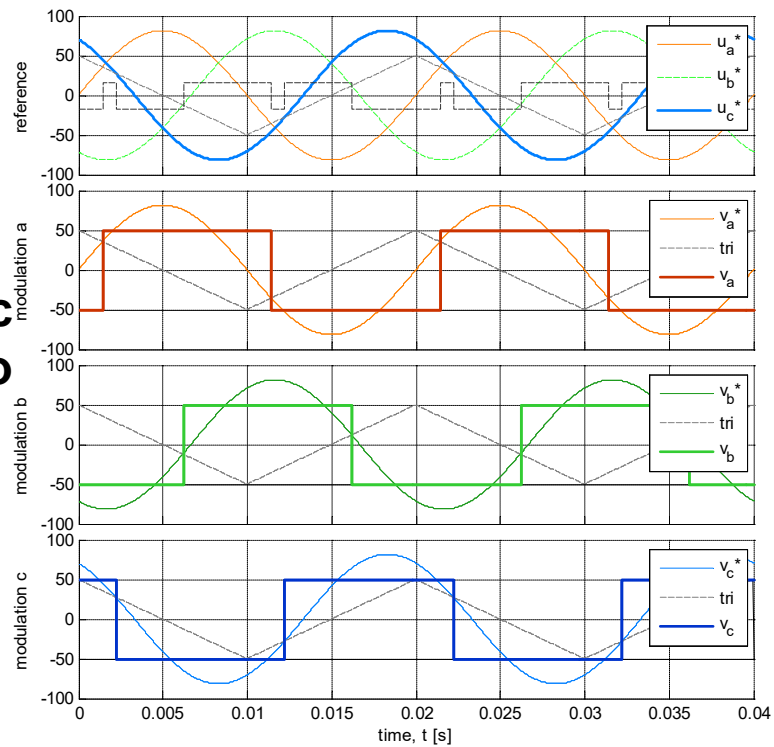
SCC block

- Vector control – dq vector quantities of voltages and currents but same circuit and control parameters
- Feed forward EMF included, cross-coupled ωL excluded
- Current controller calculate voltage references, no current delays and estimators presented
- Advanced angle to compensate rotation
- dq transformed back to $\alpha\beta$



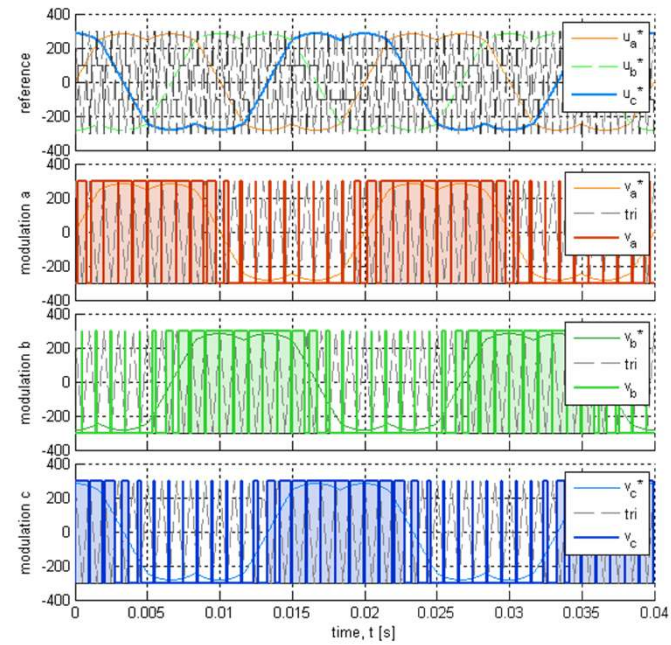
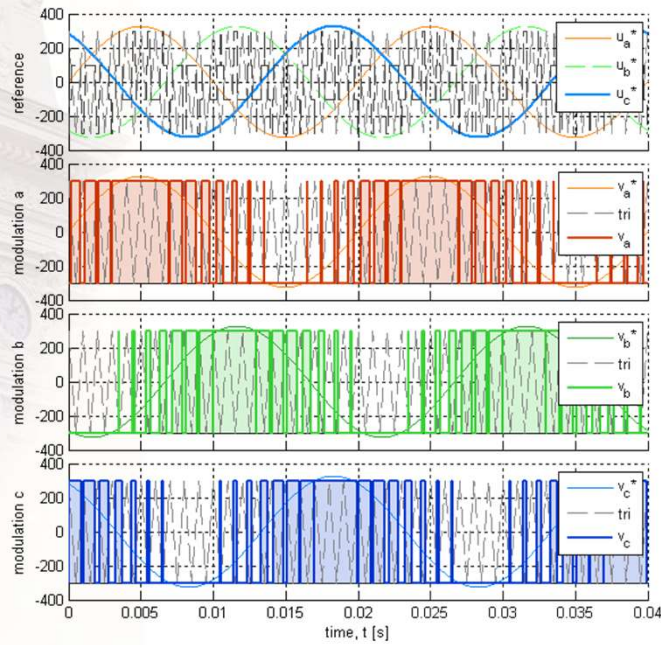
SCC open

- Current controller gives u_q^*
- T_s equals to fundamental period
- Unsampld references
- 180 voltage pulses
- What sampling frequency and dc link voltage has to be selected to match the grid?

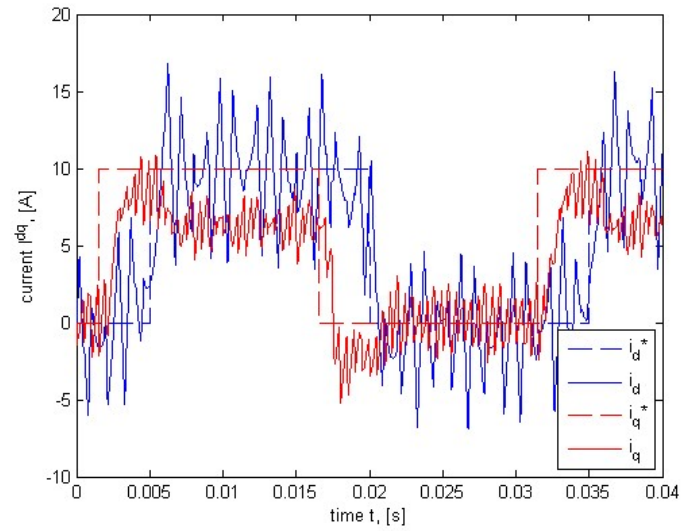
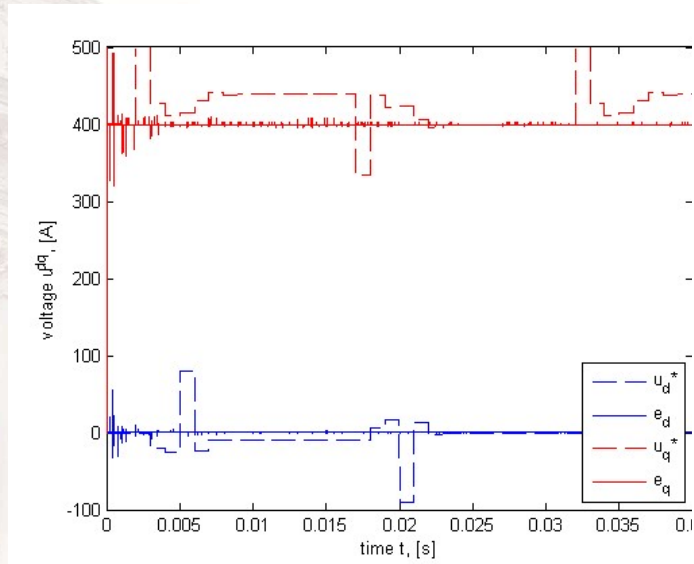


Connecting PEC to Grid?

- $U_{LL}=400V$ $U_{dc}=600V$ $T_s=1ms$

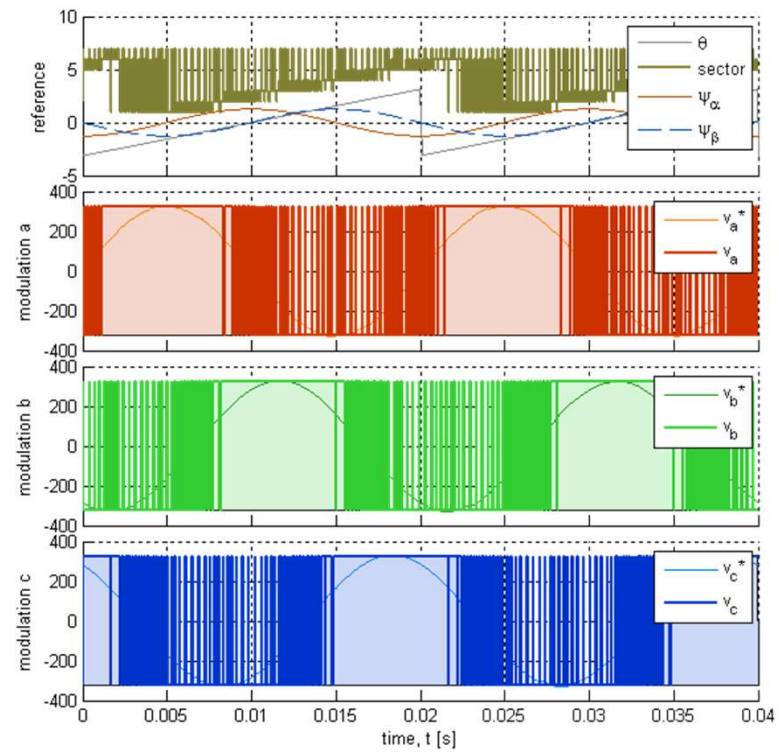
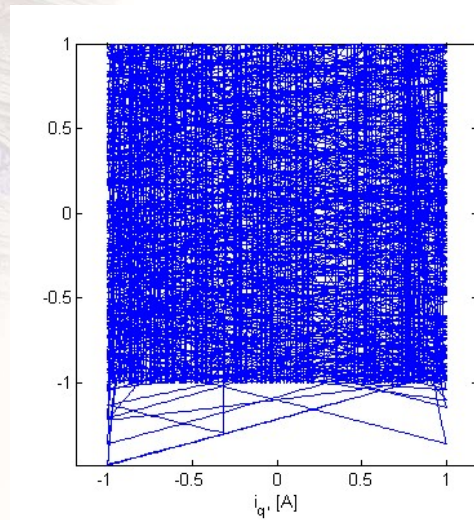


Voltage demand

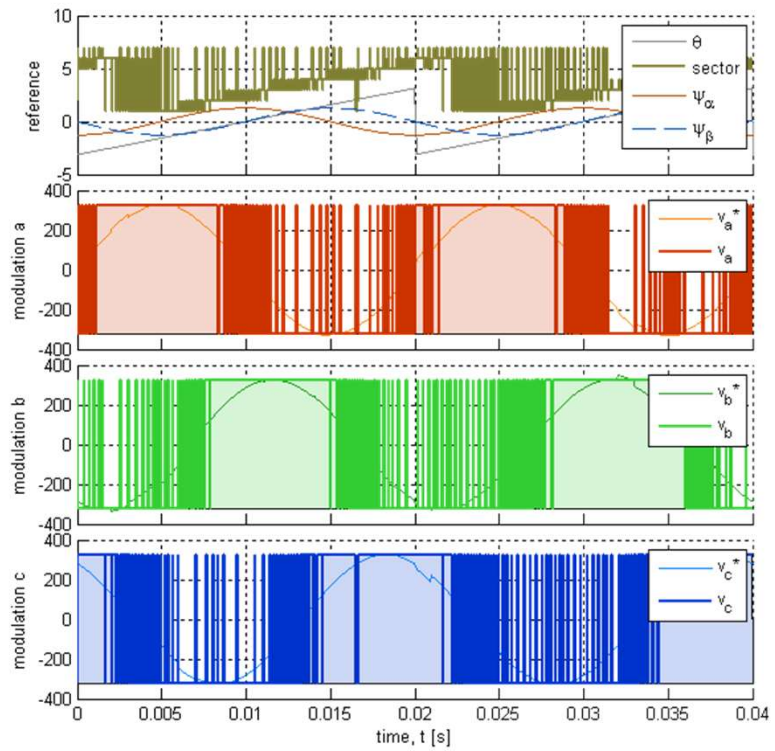
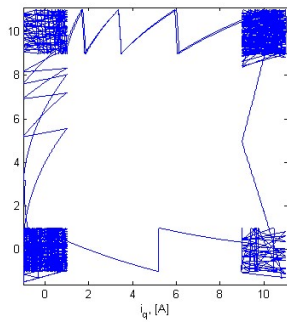
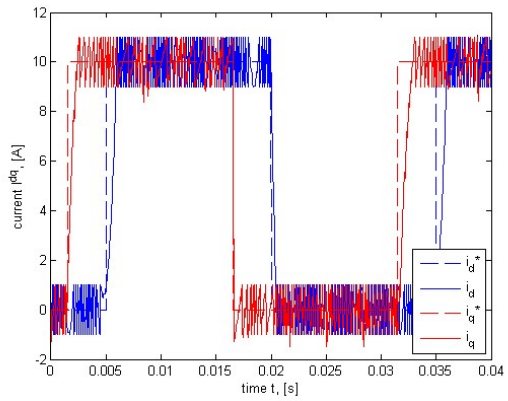


DCC current ripple

- Select $di=2$ A
- Switching intensity & frequency?



DCC step response





Exercises on 3 ϕ current control (1)

- **PE ExercisesWithSolutions2019b vers 190206**
- **Vector representation of 3 ϕ system**
 - *Relations between quantities*
 - *Coordinate transformation*
- **Control methods principles and schematics**
 - *Sampled current control controller and parameters*
 - *Direct current control controller and parameters*
 - *Waveform presentation of control action over carrier period*