11

Naturally Commutating AC to DC Converters

- Uncontrolled Rectifiers

The rectifier converter circuits considered in this chapter have in common an ac voltage supply input and a dc load output. The function of the converter circuit is to convert the ac source energy into fix dc load voltage. Turn-off of converter semiconductor devices is brought about by the ac supply voltage reversal, a process called *line commutation* or *natural commutation.*

Converter circuits employing only diodes are termed *uncontrolled* (or *rectifiers*) while the incorporation of only thyristors results in a (fully) *controlled converter.* The functional difference is that the diode conducts when forward-biased whereas the turn-on of the forward-biased thyristor can be controlled from its gate. An uncontrolled converter provides a fixed output voltage for a given ac supply and load.

Thyristor converters allow an adjustable output voltage by controlling the phase angle at which the forward biased thyristors are turned on. With diodes, converters can only transfer power from the ac source to the dc load, termed rectification and can therefore be described as *unidirectional converters.*

Although rectifiers provide a dc output, they differ in characteristics such as output ripple and mean voltage as well as efficiency and ac supply current harmonics.

An important rectifier characteristic is that of pulse number, which is defined as the repetition rate in the direct output voltage during one complete cycle of the input ac supply.

A useful way to judge the quality of the required dc output, is by the contribution of its superimposed ac harmonics. The harmonic or ripple factor *RF* is defined by

$$RF_v = \frac{V_{ac}}{V_{dc}} = \sqrt{\frac{V_{rms}^2 - V_{dc}^2}{V_{dc}^2}} = \sqrt{\frac{V_{rms}^2}{V_{dc}^2} - 1} = \sqrt{FF^2 - 1}$$

where *FF* is termed the form factor. RF_v is a measure of the voltage harmonics in the output voltage while if currents are used in the equation, RF_i gives a measure of the current harmonics in the output current. Both *FF* and *RF* are applicable to the input and output, and are defined in section 11.6.

The general analysis in this chapter is concerned with single and three phase ac rectifier supplies feeding inductive and resistive dc loads. Purely resistive load equations generally can be derived by setting inductance *L* to zero in the *L-R* load equations. Just as purely inductive load equations generally can be derived by setting resistance *R* to zero in the same *L-R* load equations.

11.1 Single-phase uncontrolled converter circuits – ac rectifiers

11.1.1 Half-wave circuit with a resistive load, R

The simplest meaningful single-phase half-wave load to analyse is the resistive load. The ac supply *V* is impressed across the load every second ac cycle half period, when load current flows.

The load voltage and current shown in figure 11.1a are defined by

$$v_o(\omega t) = i_o R = \begin{cases} \sqrt{2}V\sin\omega t & 0 \le \omega t \le \pi \\ 0 & \pi \le \omega t \le 2\pi \end{cases} \qquad (11.1)$$

The circuit voltage and current equations can be found by substituting *L* = 0, *β* = π and *φ* = 0 in the generalised equations (11.18) to (11.20) in section 11.1.3. The average dc output current I_o and voltage V_o are given by

$$V_o = I_o R = \frac{1}{2\pi}\int_0^\pi \sqrt{2}V\sin\omega t \; d\omega t = \frac{\sqrt{2}}{\pi}V = 0.45V \qquad (11.2)$$

The rms voltage across the load $V_{o\,rms}$, and rms load current $I_{o\,rms}$, are

$$V_{o\,rms} = \left[\frac{1}{2\pi}\int_0^\pi 2V^2\sin^2\omega t \; d\omega t\right]^{½} = I_{o\,rms}R = \frac{1}{\sqrt{2}}V \qquad (11.3)$$

and the power dissipated in the load, specifically the load resistor, is

$$P_o = I_{o\,rms}^2 R = ½\frac{V^2}{R} \qquad (11.4)$$

The ac current in the load is

$$I_{ac} = \sqrt{I_{o\,rms}^2 - I_o^2} = \frac{V}{R}\left[½ - \frac{2}{\pi^2}\right]^{½} \qquad (11.5)$$

The load voltage harmonics are

$$v_o(\omega t) = \frac{\sqrt{2}V}{\pi} + \frac{\sqrt{2}V}{2}\sin\omega t - \frac{2\sqrt{2}V}{\pi}\left[\frac{1}{1\times3}\cos 2\omega t + \frac{1}{3\times5}\cos 4\omega t .. + \frac{1}{n^2-1}\cos n\omega t \,........\right] \qquad (11.6)$$

for n = 2, 4, 6, …

For a resistive load, the load voltage and current ripple factors are both $\sqrt{(½\pi)^2 - 1}$. *FF* = ½π. The poor output voltage form factor can be improved by using a capacitor across the output, viz., the load resistor.

11.1.2 Half-wave circuit with a resistive and back emf R-E load

With an opposing emf *E* in series with the resistive load, the load current and voltage waveforms are as shown in figure 11.1b. Load current commences when the source voltage exceeds the load back emf at

$$\omega t = \alpha = \sin^{-1}\frac{E}{\sqrt{2}V} \qquad (11.7)$$

and ceases when the source voltage falls to the load back emf level at

$$\omega t = \pi - \alpha = \pi - \sin^{-1}\frac{E}{\sqrt{2}V} \qquad (11.8)$$

The diode conducts for a period *θ* = *π* - 2*α*, during which energy is delivered to both the load resistor *R* and load back emf *E*.

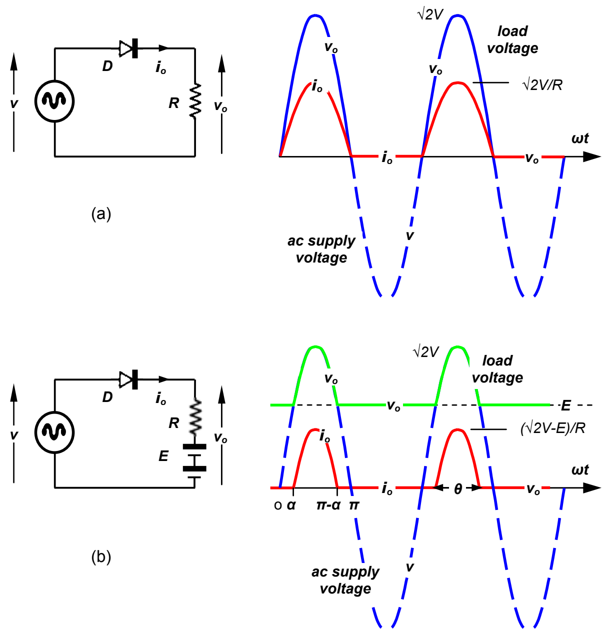


Figure 11.1. *Single-phase half-wave rectifiers:*
(a) purely resistive load, R and (b) resistive load R with back emf, E.

The load average and rms voltages are

$$V_o = \left(½ + \frac{\alpha}{\pi}\right)E + \frac{1}{2\pi}\int_{\alpha}^{\pi-\alpha} \sqrt{2}V \sin\omega t \; d\omega t$$
$$= \left(½ + \frac{\alpha}{\pi}\right)E + \frac{1}{\pi}\sqrt{2}V\cos\alpha \qquad (11.9)$$

$$V_{o\,rms} = \left[E^2\left(½ + \frac{\alpha}{\pi}\right) + V^2\left(½ - \frac{\alpha}{\pi} + \frac{1}{2\pi}\sin 2\alpha\right)^2\right]^{½} \qquad (11.10)$$

The load average and rms currents are

$$I_o = \frac{1}{R}\left[\frac{\sqrt{2}V}{\pi}\cos\alpha - E\left(½ - \frac{\alpha}{\pi}\right)\right] = \frac{1}{R}\left[\frac{\sqrt{2}V}{\pi}\sin½\theta - E\frac{\theta}{2\pi}\right] \qquad (11.11)$$

$$I_{o\,rms} = \frac{1}{R}\left[\frac{V^2}{2\pi}\sin\theta - \frac{2\sqrt{2}}{\pi}V E \sin½\theta + \left(V^2+E^2\right)\frac{\theta}{2\pi}\right]^{½} \qquad (11.12)$$

The total power delivered to the *R-E* load is

$$P_o = P_R + P_E = I^2_{o\,rms}R + E\,I_o \qquad (11.13)$$

Example 11.1: ***Half-wave rectifier with resistive and back emf load***

A dc motor has series armature resistance of 10Ω and is fed via a half-wave rectifier, from the single-phase 230V 50Hz ac mains. Calculate

i. rectifier diode peak current
ii. motor average starting current

If at full speed, the motor back emf is 100V dc, calculate

iii. average and rms motor voltages and currents
iv. motor electrical losses
v. power converted to rotational energy
vi. supply power factor and motor efficiency
vii. diode approximate loss if modelled by $v_D = 0.8 + 0.025\times i_D$.

Solution

Worst case conditions are at standstill when the motor back emf is zero (*E = kΦω*) and the circuit and waveforms in figure 11.1a are applicable.

i. The peak supply and peak load voltage is √2×*V* = √2×230 = 325.3V.
The peak diode and load current is

$$\hat{i}_D = \hat{i}_o = \frac{\hat{v}_o}{R} = \frac{325.3\text{V}}{10\Omega} = 32.5\text{A}$$

ii. The motor average current, at starting, is given by equation (11.2)

$$V_o = I_o R = 0.45\times 230\text{V} = 103.5\text{V}$$
$$I_o = \frac{V_o}{R} = \frac{103.5\text{V}}{10\Omega} = 10.35\text{A}$$

With a 100V back emf, the circuit and waveforms in figure 11.1b are applicable.
The current starts conducting when

$$\omega t = \alpha = \sin^{-1}\frac{E}{\sqrt{2}V} = \sin^{-1}\frac{100\text{V}}{\sqrt{2}\times 230\text{V}} = 17.9°$$

The current conducts for a period *θ = π - 2α* = 180° - 2×17.9 = 144.2°, ceasing at *ωt = π - α* = 162.1°.

iii. The average and rms load currents and voltages are given by equations (11.9) to (11.12).

$$V_o = \left(½ + \frac{\alpha}{\pi}\right)E + \frac{1}{\pi}\sqrt{2}V\cos\alpha$$
$$= \left(½ + \frac{17.9°}{180°}\right)\times 100\text{V} + \frac{1}{\pi}\sqrt{2}\times 230\text{V}\times\cos 17.9° = 158.5\text{V}$$

$$I_o = \frac{1}{R}\left[\frac{\sqrt{2}V}{\pi}\sin½\theta - E\frac{\theta}{2\pi}\right]$$
$$= \frac{1}{10\Omega}\left[\frac{\sqrt{2}\times 230\text{V}}{\pi}\sin½\times 144.2° - 100\text{V}\times\frac{144.2°}{360°}\right] = 5.85\text{A}$$

$$V_{o\,rms} = \left[E^2\left(½ + \frac{\alpha}{\pi}\right) + V^2\left(½ - \frac{\alpha}{\pi} + \frac{1}{2\pi}\sin 2\alpha\right)^2\right]^{½}$$
$$= \left[100^2\left(½ + \frac{17.9°}{180°}\right) + 230^2\left(½ - \frac{17.9°}{180°} + \frac{1}{2\pi}\sin 2\times 17.9°\right)^2\right]^{½} = 179.2\text{V}$$

$$I_{o\,rms} = \frac{1}{R}\left[\frac{V^2}{2\pi}\sin\theta - \frac{2\sqrt{2}}{\pi}V E\sin½\theta + \left(V^2+E^2\right)\frac{\theta}{2\pi}\right]^{½}$$
$$= \frac{1}{10\Omega}\left[\frac{230^2}{2\pi}\sin 144.2° - \frac{2\sqrt{2}}{\pi}\times 230\text{V}\times 100\text{V}\times\sin½\times 144.2° + \left(230^2+100^2\right)\frac{144.2°}{360°}\right]^{½}$$
$$= 10.2\text{A}$$

iv. The motor loss is the loss in the 10Ω resistor in the dc motor equivalent circuit

$$P_R = I^2_{o\,rms}R = 10.2^2\times 10\Omega = 1041.5\text{W}$$

v. The back emf represents the source of electrical energy converted to mechanical energy

$$P_E = E\times I_o = 100\text{V}\times 5.85\text{A} = 585\text{W}$$

vi. The supply power factor is defined as the ratio of the supply power delivered, *P*, to apparent supply power, *S*

$$pf = \frac{P}{S} = \frac{P_R + P_E}{V\times I_{o\,rms}} = \frac{1041.5\text{W} + 585\text{W}}{230\text{V}\times 10.2\text{A}} = 0.69$$

The motor efficiency is

$$\eta = \frac{P_E}{P_R + P_E} = \frac{585\text{W}}{1041.5\text{W} + 585\text{W}}\times 100 = 40.0\%$$

vii. By assuming the diode voltage drop is insignificant in magnitude compared to the 230V ac supply, then the currents and voltages previously calculated involve minimal error. The rectifying diode power loss is

$$P_D = 0.8\times I_o + 0.025\Omega\times I^2_{o\,rms}$$
$$= 0.8\times 5.85\text{A} + 0.025\Omega\times 10.2^2 = 7.3\text{W}$$

♣

11.1.3 Single-phase half-wave rectifier circuit with an R-L load

A single-phase half-wave diode rectifying circuit with an *R-L* load is shown in figure 11.2a, while various circuit electrical waveforms are shown in figure 11.2b. Load current commences when the supply voltage goes positive at *ωt* = 0. It will be seen that load current flows not only during the positive half of the ac supply voltage, 0 ≤ *ωt* ≤ *π,* but also during a portion of the negative supply voltage, *π* ≤ *ωt* ≤ *β*. The load inductor stored energy maintains the load current and the inductor's terminal voltage reverses and is able to overcome the negative supply and keep the diode forward-biased and conducting. This current continues until all the inductor energy, ½*Li*², is released (*i* = 0) at the *current extinction angle* (or *cut-off angle*), *ωt = β*.

During diode conduction the circuit is defined by the Kirchhoff voltage equation

$$v_R + v_L = L\frac{di}{dt} + Ri = v = \sqrt{2}\;V\;\sin\omega t \qquad \text{(V)} \qquad (11.14)$$

where *V* is the rms ac supply voltage. Solving equation (11.14) yields the load (and diode) current

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\left\{\sin\left(\omega t - \phi\right) + \sin\phi\;e^{-\omega t/\tan\phi}\right\} \qquad \text{(A)}$$
$$0 \le \omega t \le \beta \ge \pi \qquad \text{(rad)} \qquad (11.15)$$

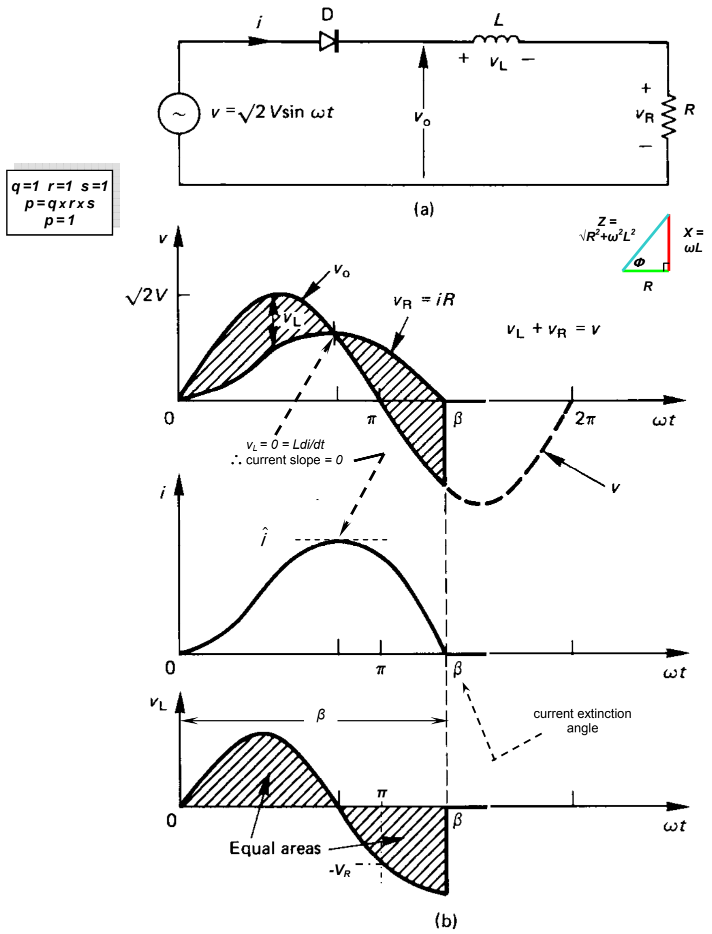


Figure 11.2. *Half-wave rectifier with an R-L load:*
(a) circuit diagram and (b) waveforms, illustrating the equal area and zero current slope criteria.

where $Z = \sqrt{(R^2 + \omega^2 L^2)}$ (ohms)

$\tan\phi = \omega L / R = Q$ and $R = Z\cos\phi$

$$i(\omega t) = 0 \qquad \text{(A)} \qquad\qquad \beta \le \omega t \le 2\pi \qquad \text{(rad)} \tag{11.16}$$

The current extinction angle β is determined solely by the load impedance Z and can be solved from equation (11.15) when the current, $i = 0$ with $\omega t = \beta$, such that $\beta > \pi$, that is

$$\sin(\beta - \phi) + \sin\phi\; e^{-\beta/\tan\phi} = 0 \tag{11.17}$$

This is a transcendental equation which can be solved by iterative techniques. Figure 11.3a can be used to determine the extinction angle β, given any load impedance (power factor) angle $\phi = \tan^{-1}\omega L / R$.

The mean value of the rectified current, the output current, $\bar{I}_o$, is given by integration of equation (11.15)

$$\bar{I}_o = \tfrac{1}{2\pi}\int_0^\beta i(\omega t)\; d\omega t \qquad \text{(A)}$$
$$\bar{I}_o = \tfrac{\sqrt{2}V}{2\pi R}(1 - \cos\beta) \qquad \text{(A)} \tag{11.18}$$

while the mean output voltage V_o is given by

$$V_o = \tfrac{1}{2\pi}\int_0^\beta \sqrt{2}\; V\sin\omega t\;\; d\omega t = \bar{I}_o R = \tfrac{\sqrt{2}V}{2\pi}(1 - \cos\beta) \qquad \text{(V)} \tag{11.19}$$

Since the mean voltage across the load inductance is zero, $V_o = \bar{I}_o R$ (see the equal area criterion to follow). Figure 11.3b shows the normalised output voltage V_o / V as a function of $\omega L / R$.

The rms output (load) voltage and current are given by

$$V_{rms} = \left[\tfrac{1}{2\pi}\int_0^\beta \left(\sqrt{2}V\right)^2 \sin^2\omega t\; d\omega t\right]^{1/2} = V\left[\tfrac{1}{2\pi}\{\beta - \tfrac{1}{2}\sin 2\beta\}\right]^{1/2}$$
$$i_{rms} = \frac{V\cos\phi}{R}\left[\frac{1}{2\pi}\left\{\beta - \frac{\sin\beta\cos(\beta+\phi)}{\cos\phi}\right\}\right]^{1/2} = \frac{V}{Z}\left[\frac{1}{2\pi}\left\{\beta - \frac{\sin\beta\cos(\beta+\phi)}{\cos\phi}\right\}\right]^{1/2} \tag{11.20}$$

From equations (11.19) and (11.20) the harmonic content in the output voltage is indicated by the voltage form factor.

$$FF_v = \frac{V_{rms}}{V_o} = \frac{\left[\pi\{\beta - \tfrac{1}{2}\sin 2\beta\}\right]^{1/2}}{1 - \cos\beta} \tag{11.21}$$

For a resistive load, when $\beta = \pi$, the form factor reduces to a value of 1.57. The ripple factor is therefore $\sqrt{FF_v^2 - 1} = 1.21$. For a purely resistive load the voltage and current form factors are equal.

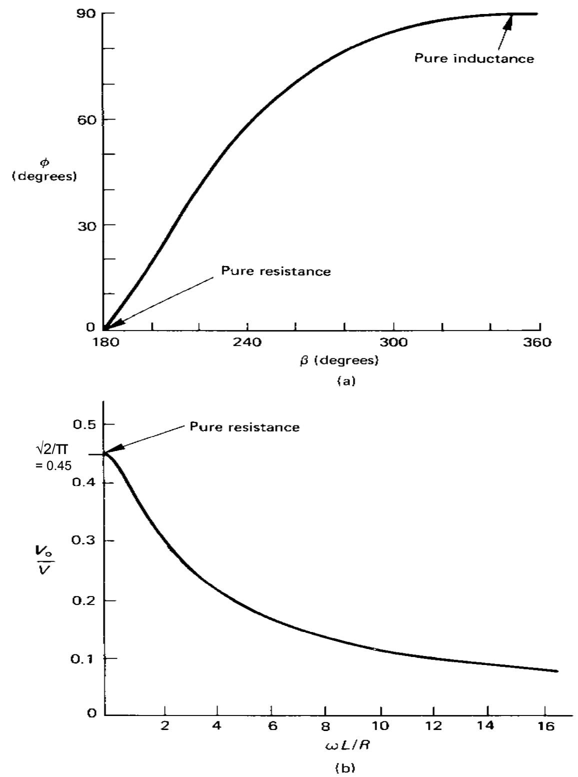


Figure 11.3. *Single-phase half-wave converter characteristics: (a) load impedance angle ø versus current extinction angle β and (b) variation in normalised mean output voltage V_o/ V versus ωL/R.*

The power delivered to the load, which is the power delivered to the load resistance R, is

$$P_L = i_{rms}^2 R \tag{11.22}$$

The supply power factor, using the rms current in equation (11.20), is

$$pf = \frac{\text{power, } P_L}{\text{apparent power}}$$

$$= \frac{i_{rms}^2 R}{i_{rms} V} = \frac{i_{rms} R}{V} = \frac{V_{R\,rms}}{V} = \left[\frac{1}{2\pi}\left\{\beta - \frac{\sin\beta \, \cos(\beta+\phi)}{\cos\phi}\right\}\right]^{1/2} \cos\phi = \mu \times \cos\phi \tag{11.23}$$

The characteristics for an *R-L-E* load can be determined by using $\alpha = 0$ in the case of the half-wave controlled converter in section 12.2.1iii.

For a purely inductive load, L, $\beta = 2\pi$ is substituted into the appropriate equations. The average output voltage tends to zero and the current is given by

$$i(\omega t) = \frac{\sqrt{2}\,V}{\omega L}\{1 - \cos\omega t\} \qquad \text{(A)}$$

which has a mean current value of $\sqrt{2}\,V/\omega L$.

11.1.3i – Inductor equal voltage area criterion

The average output voltage V_o, given by equation (11.19), is based on the fact that the average voltage across the load inductance, in steady state, is zero. The inductor voltage is given by

$$v_L = L\,di/dt \qquad \text{(V)}$$

which for the circuit in figure 11.2a can be expressed as

$$\int_0^{\beta/\omega} v_L(t)\;dt \;= \int_{i_o}^{i_\beta} L\;di = L(i_\beta - i_0) \tag{11.24}$$

If the load current is in steady state then $i_\beta = i_o$, which is zero here, and in general

$$\int v_L\,dt = 0 \qquad \text{(Vs)} \tag{11.25}$$

The inductor voltage waveform for the circuit in figure 11.2a is shown in the last plot in figure 11.2b. The inductor equal voltage area criterion implies that the shaded positive area must equal the shaded negative area, in order to satisfy equation (11.25). The net inductor energy at the end of the cycle is zero (specifically, unchanged since $i_o = i_\beta$), that is, the energy into the inductor equals the energy transferred from the inductor. This area aspect is a useful aid in predicting and drawing the load current waveform.

It is useful to superimpose the supply voltage v, the load voltage v_o, and the resistor voltage v_R waveforms on the same time axis, ωt. The load resistor voltage, $v_R = Ri$, is directly related to the load current, i. The inductor voltage v_L will be the difference between the load voltage and the resistor voltage, and this bounded net area must be zero. Thus the average output voltage is $V_o = \bar{I}_o R$. The equal voltage areas associated with the load inductance are shown shaded in two plots in figure 11.2b.

11.1.3ii - Load current zero slope criterion

The load inductance voltage polarity changes from positive to negative as energy initially transferred into the inductor, is released. The stored energy in the inductor allows current to continue to flow after the input ac voltage has reversed. At the instant when the inductor voltage reverses, its terminal voltage is zero, and

$$v_L = L di/dt = 0$$
$$\text{that is} \quad di/dt = 0 \tag{11.26}$$

The current slope changes from positive to negative, whence the voltage across the load resistance ceases to increase and starts to decrease, as shown in figure 11.2b. That is, the *Ri* waveform crosses the supply voltage waveform with zero slope, whence when the inductor voltage is zero, the current begins to decrease. The fact that the resistor voltage slope is zero when $v_L = 0$, aids prediction and sketching of the various circuit waveforms in figure 11.2b, and subsequent waveforms in this chapter.

11.1.4 Half-wave rectifier circuit with an R load and capacitor filter

The output voltage ripple factor of a half-wave rectifier with a resistive load can be improved by adding decoupling capacitance across the load output, as shown in figure 11.4.

In the period $\alpha \le \omega t \le \beta$

$$v_o(t) = \sqrt{2}V_s \sin\omega t$$

and

$$i_s = i_C + i_o = C\frac{dv_o}{dt} + \frac{v_o}{R_L}$$

Solving for the source current i_s

$$i_s(t) = \frac{\sqrt{2}V_s}{R_L}\left[\omega R_L C\cos\omega t + \sin\omega t\right]$$

$$= \frac{\sqrt{2}V_s}{R_L}\sqrt{1+(\omega R_L C)^2}\,\cos(\omega t - \theta) \tag{11.27}$$

$$\text{where } \theta = \tan^{-1}\frac{1}{\omega R_L C}$$

Since $i_s = 0$ when $\omega t = \beta$, $\beta - \theta = \tfrac{1}{2}\pi$. That is

$$\beta = \tfrac{1}{2}\pi + \theta = \tfrac{1}{2}\pi + \tan^{-1}\frac{1}{\omega R_L C}$$

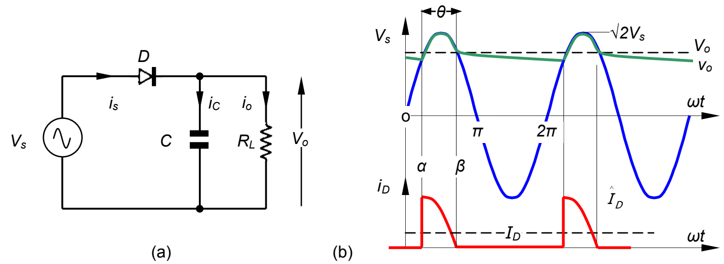


Figure 11.4. *Single-phase half-wave rectifier:*
(a) circuit with a capacitively filtered resistive load and (b) waveforms.

For the period $\beta \le \omega t \le 2\pi + \alpha$

$$i_s = 0$$

and

$$C\frac{dv_o}{dt} + \frac{v_o}{R_L} = 0 \quad \text{where} \quad v_o(\omega t = \beta) = \sqrt{2}V_s\cos\theta$$

Therefore

$$v_o(t) = \sqrt{2}V_s e^{-(\omega t - \beta)\tan\theta}\cos\theta \tag{11.28}$$

Since $v_o = \sqrt{2}V_s \sin\alpha$ when $\omega t = 2\pi + \alpha$

$$\sin\alpha = e^{-\alpha\tan\theta}e^{-(\frac{3}{2}\pi - \theta)\tan\theta}\cos\theta$$

where α can be solved iteratively.

The peak to peak ripple voltage is $2V_s(1 - \sin\alpha)$, which decreases as C increases for which $\alpha \rightarrow \tfrac{1}{2}\pi$. The peak inverse voltage rating of the diode is approximately $2\sqrt{2}V_s$.

The full-wave rectified case is considered in section 11.1.8iv, where the period boundary $\omega t = \pi + \alpha$ is used.

Example 11.2: ***Half-wave rectifier with source resistance***

In the dc supply half-wave rectifier circuit of figure 11.5, the source voltage is $230\sqrt{2}\,\sin(2\pi\,50t)$ V with an internal resistance $R_i = 1$ Ohm, $R_L = 10$ Ohms, and the filter capacitor C is very large. Calculate

i. the mean value of the load voltage, V_o
ii. the diode average and peak currents, I_D, $\hat{I}_D$
iii. the capacitor peak charging and discharging currents
iv. the diode reverse blocking voltage, V_{DR}

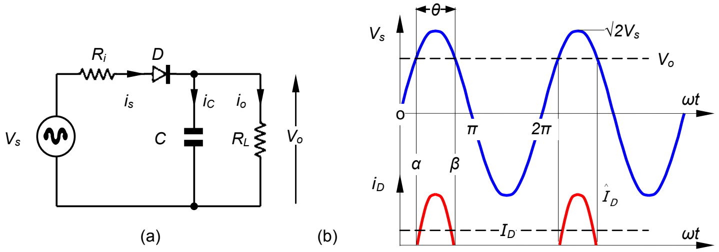


Figure 11.5. *Single-phase half-wave rectifier: (a) circuit with a resistive load and (b) waveforms.*

Solution

i. Because the load filter capacitor is large, it is assumed that the dc output voltage is ripple free and constant. The capacitor provides the load current when the ac supply level is less that the dc output. The load current and peak diode (hence supply) current are therefore

$$I_o = \frac{V_o}{R_L} \qquad \hat{I}_D = \frac{\sqrt{2}V_s - V_o}{R_i}$$

The ac supply provides current, through the rectifying diode, during the period

$$i_s = \frac{1}{R_i}\left(\sqrt{2}V_s \sin\omega t - V_o\right) \qquad \alpha \le \omega t \le \beta$$

If the capacitor voltage is to be maintained constant, the charge into the capacitor must equal the charge delivered by the capacitor when the rectifying diode is not conducting, that is

$$\int_\alpha^\beta (i_s - i_o)\, d\omega t = \int_\beta^{\alpha+2\pi} i_o\, d\omega t$$

Also

$$V_o = \sqrt{2}V_s \sin\alpha$$

$$\alpha = \frac{\pi - \theta}{2} \qquad \beta = \frac{\pi + \theta}{2}$$

Manipulation yields

$$\tan\tfrac{1}{2}\theta - \tfrac{1}{2}\theta = \pi\frac{R_i}{R_L} = \pi\frac{1\Omega}{10\Omega} = 0.1\pi$$

An iterative solution yields *θ* = 99.6º, that is, the diode conducts for a period of 5.53ms (10ms×99.6º/180º), every cycle of the ac supply, 20ms. The capacitor, hence output voltage, is

$$V_o = \sqrt{2}V_s \sin\alpha = \sqrt{2}V_s \sin\frac{\pi - \theta}{2}$$

$$= \sqrt{2}\times 230\text{V}\times\sin\frac{180^\circ - 99.6^\circ}{2} = 209.95\text{V}$$

ii. The average diode current is given by

$$\bar{I}_D = \frac{1}{2\pi}\int_\alpha^\beta \frac{1}{R_i}\left(\sqrt{2}V_s\sin\omega t - V_o\right) d\omega t = \frac{1}{2\pi R_i}\left(\sqrt{2}V_s\times 2\times\cos\frac{\pi-\theta}{2} - V_o\times\theta\right)$$

$$= \frac{1}{2\pi\times 1\Omega}\left(\sqrt{2}\times 230\text{V}\times 2\times\cos\frac{180^\circ - 99.6^\circ}{2} - 209.95\text{V}\times\pi\times\frac{99.6^\circ}{180^\circ}\right) = 21.0\text{A}$$

Alternatively, as would be expected, the average diode current is the average load current:

$$\bar{I}_D = \bar{I}_o = \frac{V_o}{R_L} = \frac{209.95\text{V}}{10\Omega} = 21.0\text{A}$$

The peak diode current is

$$\hat{I}_D = \frac{\sqrt{2}V_s - V_o}{R_i} = \frac{\sqrt{2}\times 230\text{V} - 210\text{V}}{1\Omega} = 115.3\text{A}$$

iii. The capacitor peak charging current is the difference between the peak diode current and the load current, viz., 115A - 21A = 94A, while the peak discharging current is the average load current of 21A.

iv. The diode reverse voltage is the difference between the instantaneous supply voltage and the output voltage 210V. This is a maximum at the negative peak of the ac supply, when the diode voltage is √2×230V + 210V = 535.3V. During any period when the load is disrupted, the output capacitor can charge up to √2×230V, hence the diode can experience, worst case, 2×√2×230V = 650.5V.

♣

11.1.5 Half-wave circuit with an R-L load and freewheel diode

The circuit in figure 11.2a, which has an *R-L* load, is characterised by discontinuous current (*i* = 0) and high ripple current. Continuous load current can result when a diode D_f is added across the load as shown in figure 11.6a. This freewheel diode prevents the voltage across the load from reversing during the negative half-cycle of the ac supply voltage. The inductor energy is not returned to the ac supply, rather is retained in the load circuit. The stored energy in the inductor cannot reduce to zero instantaneously, so the current is forced to find an alternative path whilst decreasing towards zero. When the rectifier diode D_1 ceases to conduct at zero volts it blocks, and diode D_f provides an alternative load current freewheeling path, as indicated by the waveforms in figure 11.6b.

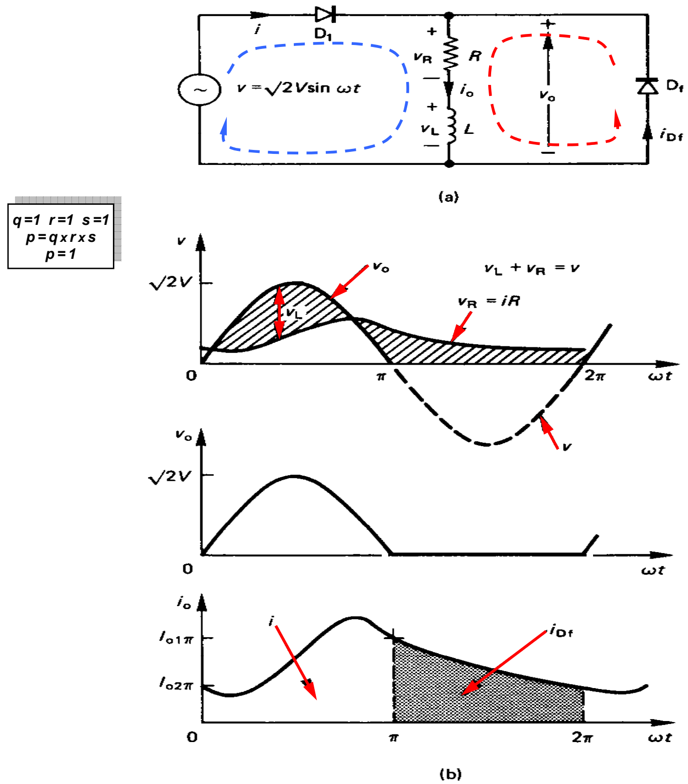


Figure 11.6. *Half-wave rectifier with a load freewheel diode and an R-L load: (a) circuit diagram and parameters and (b) circuit waveforms.*

The output voltage is the positive half of the sinusoidal input voltage. The mean output voltage (thence mean output current) is

$$V_o = \bar{I}_o R = \frac{1}{2\pi}\int_0^\pi \sqrt{2}V\sin\omega t\, d\omega t \qquad (11.29)$$

$$V_o = \sqrt{2}V/\pi = 0.45\times V = \bar{I}_o R \qquad \text{(V)}$$

The rms value of the load circuit voltage v_0 is given by

$$V_{rms}=\sqrt{\tfrac{1}{2\pi}\int_0^{\pi}\left(\sqrt{2}V\sin\omega t\right)^2 d\omega t} \qquad (11.30)$$
$$= V/\sqrt{2} = 0.71\times V \qquad \text{(V)}$$

The output ripple (ac) voltage is defined as

$$V_{Ri}\triangleq\sqrt{V_{rms}^2-V_o^2} \qquad (11.31)$$
$$=\sqrt{\left(\sqrt{2}V/2\right)^2-\left(\sqrt{2}V/\pi\right)^2}=V\sqrt{½-2/\pi^2} = 0.545\times V$$

hence the load voltage form and ripple factors are defined as

$$FF_v=V_{rms}/V_o=½\pi=1.57 \qquad (11.32)$$
$$RF_v\triangleq V_{Ri}/V_o=\sqrt{\left(\tfrac{V_{rms}}{V_o}\right)^2-1}=\sqrt{FF_v^{\,2}-1} = \sqrt{¼\pi^2-1} = 1.211$$

After a large number of ac supply cycles, steady-state load current conditions are established, and from Kirchhoff's voltage law, the load current is defined by

$$L\frac{di}{dt}+Ri=\sqrt{2}V\sin\omega t \qquad \text{(A)} \qquad 0\le\omega t\le\pi \qquad (11.33)$$

and when the freewheel diode conducts

$$L\frac{di}{dt}+Ri=0 \qquad \text{(A)} \qquad \pi\le\omega t\le 2\pi \qquad (11.34)$$

During the period 0 ≤ ωt ≤ π, when the freewheel diode current is given by $i_{Df} = 0$, the supply current, which is the load current, are given by

$$i(\omega t)=i_o(\omega t)=\frac{\sqrt{2}V}{Z}\sin(\omega t-\phi)+\left(I_{o2\pi}+\frac{\sqrt{2}V}{Z}\sin\phi\right)e^{-\omega t/\tan\phi} \qquad \text{(A)} \qquad (11.35)$$
$$0\le\omega t\le\pi$$

for

$$I_{o2\pi}=\frac{\sqrt{2}V}{Z}\sin\phi\,\frac{1+e^{-\pi/\tan\phi}}{e^{\pi/\tan\phi}-e^{-\pi/\tan\phi}} \qquad \text{(A)}$$

where $Z = \sqrt{R^2+(\omega L)^2}$ (ohms)

$\tan\phi = \omega L/R$

During the period π ≤ ωt ≤ 2π, when the supply current $i = 0$, the freewheel diode current and hence load current are given by

$$i_o(\omega t)=i_{Df}(\omega t)=I_{o1\pi}e^{-(\omega t-\pi)/\tan\phi} \qquad \text{(A)} \qquad \pi\le\omega t\le 2\pi \qquad (11.36)$$

for

$$I_{o1\pi}=I_{o2\pi}e^{\pi/\tan\phi} \qquad \text{(A)}$$

For discontinuous load current (the freewheel diode current i_{Df} falls to zero before the rectifying diode D_1 recommences conduction), the appropriate integration gives the average diode currents as

$$\overline{I}_{D1}=\frac{V}{\sqrt{2}\,\pi R}\left(2-\left(1+e^{-\pi/\tan\phi}\right)\times\sin^2\phi\right) \qquad (11.37)$$
$$\overline{I}_{Df}=\overline{I}_o-\overline{I}_{D1}=\frac{V}{\sqrt{2}\,\pi R}\left(1+e^{-\pi/\tan\phi}\right)\times\sin^2\phi$$

In figure 11.6b it will be seen that although the load current can be continuous, the supply current is discontinuous and therefore has a high harmonic content.

The output voltage Fourier series ($V_o + V_1 + V_{n=2,4,6..}$) is (see equation (11.6))

$$v_o(t)=\frac{\sqrt{2}V}{\pi}+\frac{\sqrt{2}V}{2}\sin\omega t - \frac{\sqrt{2}V}{\pi}\sum_{n=2,4,6}^{\infty}\frac{2}{(n^2-1)}\cos n\omega t \qquad (11.38)$$

Dividing each harmonic output voltage component by the corresponding load impedance at that frequency gives the harmonic output current, whence rms current. That is

$$I_n=\frac{V_n}{Z_n}=\frac{V_n}{|R+jn\omega L|}=\frac{V_n}{\sqrt{R^2+(n\omega L)^2}} \qquad (11.39)$$

and

$$I_{rms}=\sqrt{I_o^2+\sum_{n=1,2,4,6..}½I_n^2} \qquad (11.40)$$

Example 11.3: *Half-wave rectifier – with load freewheel diode*

In the circuit of figure 11.6, the source voltage is 240√2 sin(2π 50t) V, R = 10 ohms, and L = 50 mH. Calculate

i. the mean and rms values of the load voltage, V_o and V_{rms}
ii. the mean value of the load current, $\overline{I}_o$
iii. the current boundary conditions, namely $I_{o1\pi}$ and $I_{o2\pi}$
iv. the average freewheel diode current, hence average rectifier diode current
v. the rms load current, hence load power and supply rms current
vi. the supply power factor

If the freewheel diode is removed from across the load, determine

vii. an expression for the current hence the current extinction angle
viii. the average load voltage hence average load current
ix. the rms load voltage and current
x. the power delivered to the load and supply power factor

From the rms and average output voltages and currents, determine the load form and ripple factors.

Solution

i. From equation (11.29), the mean output voltage is given by

$$V_o=\sqrt{2}V/\pi=\sqrt{2}\times 240\text{V}/\pi=108\text{V}$$

From equation (11.30) the load rms voltage is

$$V_{rms}=V/\sqrt{2} = 240\text{V}/\sqrt{2} = 169.7\text{V}$$

ii. The mean output current, equation (11.29), is

$$\overline{I}_o=V_o/R = \sqrt{2}V/\pi R = \sqrt{2}\times 240\text{V}/\pi\times 10\Omega = 10.8\text{A}$$

iii. The load impedance is characterised by

$$Z = \sqrt{R^2+(\omega L)^2}$$
$$=\sqrt{10^2+(2\pi\times 50\text{Hz}\times 0.05)^2} = 18.62\ \Omega$$
$$\tan\phi = \omega L/R$$
$$=2\pi\times 50\text{Hz}\times 0.05\text{H}/10\Omega=1.57 \text{ or } \phi=57.5^\circ\equiv 1\text{rad}$$

From section 11.1.5, equation (11.35)

$$I_{o2\pi}=\frac{\sqrt{2}V}{Z}\sin\phi\,\frac{1+e^{-\pi/\tan\phi}}{e^{\pi/\tan\phi}-e^{-\pi/\tan\phi}}$$
$$I_{o2\pi}=\frac{\sqrt{2}\times 240\text{V}}{18.62\Omega}\times\sin(\tan^{-1}1.57)\times\frac{1+e^{-\pi/1.57}}{e^{\pi/1.57}-e^{-\pi/1.57}}=3.41\text{A}$$

Hence, from equation (11.36)

$$I_{o1\pi}=I_{o2\pi}e^{\pi/\tan\phi}=3.41\times e^{\pi/1.57}=25.22\text{A}$$

Since $I_{o2\pi}=3.41\text{A}>0$, continuous load current flows.

iv. Integration of the diode current given in equation (11.36) yields the average freewheel diode current.

$$\overline{I}_{Df}=\frac{1}{2\pi}\int_0^{\pi}i_{Df}(\omega t)\,d\omega t=\frac{1}{2\pi}\int_0^{\pi}I_{o1\pi}e^{-\omega t/\tan\phi}\,d\omega t$$
$$=\frac{1}{2\pi}\int_0^{\pi}25.22\text{A}\times e^{-\omega t/1.57\text{rad}}\,d\omega t=\frac{25.22\text{A}}{2\pi}\times 1.57\text{rad}\times\left[1-e^{-\frac{\pi}{1.57}}\right]=5.46\text{A}$$

The average input current, which is the rectifying diode mean current, is given by

$$\overline{I}_s=\overline{I}_{D1}=\overline{I}_o-\overline{I}_{Df}=10.8\text{A}-5.46\text{A}=5.34\text{A}$$

v. The load voltage harmonics given by equation (11.38) can be used to evaluate the load current at the load impedance for that frequency harmonic.

$$v(t)=\frac{\sqrt{2}V}{\pi}+\frac{\sqrt{2}V}{2}\sin\omega t-\sum_{n=2,4,6}^{\infty}\frac{2\sqrt{2}V}{(n^2-1)\pi}\cos(n\omega t)$$

The following table shows the calculations for each frequency component.

harmonic n	$V_n = \dfrac{2\sqrt{2}V}{(n^2-1)\pi}$ (V)	$Z_n = \sqrt{R^2 + (n\omega L)^2}$ (Ω)	$I_n = \dfrac{V_n}{Z_n}$ (A)	$\tfrac{1}{2}I_n^2$
0	***(108.04)****	10.00	10.80	***(116.72)***
1	***(169.71)****	18.62	9.11	41.53
2	72.03	32.97	2.18	2.39
4	14.41	63.62	0.23	0.03
6	6.17	94.78	0.07	0.00
8	3.43	126.06	0.03	0.00
see equation (11.38) for first two terms			$I_o^2 + \sum \tfrac{1}{2}I_n^2 =$	160.67

The rms load current is

$$I_{rms} = \sqrt{I_o^2 + \sum_{n=1,2,4..} \tfrac{1}{2}I_n^2} = \sqrt{160.7} = 12.68\text{A}$$

The power dissipated in the load resistance is therefore

$$P_{10\Omega} = I_{rms}^2 R = 12.68\text{A}^2 \times 10\Omega = 1606.7\text{W}$$

The freewheel diode rms current is

$$I_{Df} = \sqrt{\frac{1}{2\pi}\int_0^\pi \left(I_{o1\pi} e^{-\omega t/\tan\phi}\right)^2 d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^\pi \left(25.22\text{A} \times e^{-\omega t/1.57\text{rad}}\right)^2 d\omega t} = 8.83\text{A}$$

Thus the input (and rectifying diode) rms current is given by

$$I_{D1_{rms}} = I_{s_{rms}} = \sqrt{I_{rms}^2 - I_{Df\,rms}^2}$$

$$= \sqrt{12.68^2 - 8.83^2} = 9.09\text{A}$$

vi. The input ac supply power factor is

$$pf = \frac{P_{out}}{V_{rms} I_{rms}} = \frac{1606.7\text{W}}{240\text{V} \times 9.09\text{A}} = 0.74$$

vii. If the freewheel diode D_f is removed, the current is given by equation (11.15), that is

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\left\{\sin(\omega t - \phi) + \sin\phi\ e^{-\omega t/\tan\phi}\right\}$$

$$= \frac{\sqrt{2} \times 240\text{V}}{18.62\Omega}\left\{\sin(\omega t - 1.0) + 0.841 \times e^{-\omega t/1.57}\right\}$$

$$= 18.23 \times \left\{\sin(\omega t - 1.0) + 0.841 \times e^{-\omega t/1.57}\right\} \quad (\text{A}) \qquad 0 \le \omega t \le \beta \quad (\text{rad})$$

The current extinction angle β is found by setting $i = 0$ and solving iteratively for β. Figure 11.3a gives an initial estimate of 240° (4.19 rad) when ϕ = 57.5° (1 rad). That is

$$0 = \sin(\beta - 1.0) + 0.841 \times e^{-\beta/1.57}$$

gives β = 4.08 rad or 233.8°, after iteration.

viii. The average load voltage from equation (11.19) is

$$V_o = \frac{\sqrt{2}V}{2\pi}(1 - \cos\beta) = \frac{\sqrt{2}\times 240\text{V}}{2\pi}(1 - \cos 4.08) = 86.0\text{V}$$

The average load current is

$$\bar{I}_o = V_o / R = 86.0\text{V}/10\Omega = 8.60\text{A}$$

ix. The load rms voltage is 169.7V with the freewheel diode and increases without the diode to, as given by equation (11.20)

$$V_{rms} = V\left[\tfrac{1}{2\pi}\left\{\beta - \tfrac{1}{2}\sin 2\beta\right\}\right]^{1/2}$$

$$= 240\text{V}\left[\tfrac{1}{2\pi}\left\{4.08 - \tfrac{1}{2}\sin 2 \times 4.08\right\}\right]^{1/2} = 181.6\text{V}$$

The rms load current from equation (11.20) is decreased to

$$i_{rms} = \frac{V}{Z}\left[\frac{1}{2\pi}\left\{\beta - \frac{\sin\beta\cos(\alpha+\beta+\phi)}{\cos\phi}\right\}\right]^{1/2}$$

$$= \frac{240\text{V}}{18.62\Omega} \times \left[\frac{1}{2\pi}\left\{4.08 - \frac{\sin 4.08\cos(4.08+1.57)}{\cos 1.57}\right\}\right]^{1/2} = 9.68\text{A}$$

Removal of the freewheel diode decreases the rms load current from 12.68A to 9.68A.

x. The load power is reduced without a load freewheel diode, from 1606.7W with a load freewheel diode, to

$$P_{10\Omega} = i_{rms}^2 R = 9.68^2 \times 10\Omega = 937\text{W}$$

The supply power factor is also reduced, from 0.74 to

$$pf = \frac{P_{out}}{V_{rms} I_{rms}} = \frac{937\text{W}}{240\text{V} \times 9.68\text{A}} = 0.40$$

Load factor	circuit with freewheel diode		circuit without freewheel diode	
	form factor	ripple factor	form factor	ripple factor
	$FF = {}^{rms}/_{ave}$	$RF = \sqrt{FF^2 - 1}$	$FF = {}^{rms}/_{ave}$	$RF = \sqrt{FF^2 - 1}$
Voltage factor	169.7V/108V = 1.57	1.21	181.6V/86V = 2.1	1.86
Current factor	12.68A/10.8A = 1.17	0.615	9.68A/8.60A = 1.12	0.517

♣

11.1.6 Single-phase full-wave bridge rectifier circuit with a resistive load, R

The simplest meaningful single-phase full-wave load to analyse is the resistive load. The supply is impressed across the load every ac cycle half period, when load current flows.
The load voltage and current shown in figure 11.7a are defined by

$$v_o(\omega t) = i_o R = \left|\sqrt{2}V\sin\omega t\right| \qquad 0 \le \omega t \le 2\pi \tag{11.41}$$

The average dc output current and voltage are double the half-wave case and are given by

$$V_o = I_o R = \frac{1}{\pi}\int_0^\pi \sqrt{2}V\sin\omega t\ d\omega t = \frac{2\sqrt{2}}{\pi}V = 0.90V \tag{11.42}$$

The rms voltage across the load, and rms load current, are √2 greater than the half-wave case, specifically

$$V_{o\,rms} = \left[\frac{1}{\pi}\int_0^\pi 2V^2\sin^2\omega t\ d\omega t\right]^{1/2} = I_{o\,rms}R = V \tag{11.43}$$

and the power dissipated in the load, specifically the load resistor R, is

$$P_o = I_{o\,rms}^2 R = \frac{V^2}{R} \tag{11.44}$$

The ac current in the load is

$$I_{ac} = \sqrt{I_{o\,rms}^2 - I_o^2} = \frac{V}{R}\left[1 - \frac{8}{\pi^2}\right]^{1/2} \tag{11.45}$$

The load voltage harmonics are (twice the half-wave case, without the supply frequency component)

$$v_o(\omega t) = \frac{2\sqrt{2}V}{\pi} - \frac{4\sqrt{2}V}{\pi}\left[\frac{1}{1\times 3}\cos 2\omega t + \frac{1}{3\times 5}\cos 4\omega t .. + \frac{1}{n^2-1}\cos n\omega t ....\right] \tag{11.46}$$

for n = 2, 4, 6, …

11.1.7 Single-phase full-wave bridge rectifier circuit with a resistive and back emf load, R-E

With an opposing emf E in the load circuit, the load current and voltage waveforms are as shown in figure 11.7b. Load current commences when

$$\omega t = \alpha = \sin^{-1}\frac{E}{\sqrt{2}V} \tag{11.47}$$

and ceases when

$$\omega t = \pi - \alpha = \pi - \sin^{-1}\frac{E}{\sqrt{2}V} \tag{11.48}$$

Diodes conduct every ac half cycle for a period $\theta = \pi - 2\alpha$, during which energy is delivered to both the load resistor R and load back emf E.
The load average and rms voltages are

$$V_o = 2E\frac{\alpha}{\pi} + \frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} \sqrt{2}V \sin\omega t \; d\omega t \qquad (11.49)$$

$$= 2E\frac{\alpha}{\pi} + \frac{2}{\pi}\sqrt{2}V\cos\alpha$$

$$V_{o\,rms} = \left[2\frac{\alpha}{\pi}E^2 + V^2\left(1 - 2\frac{\alpha}{\pi} + \frac{1}{\pi}\sin 2\alpha\right)^2\right]^{½} \qquad (11.50)$$

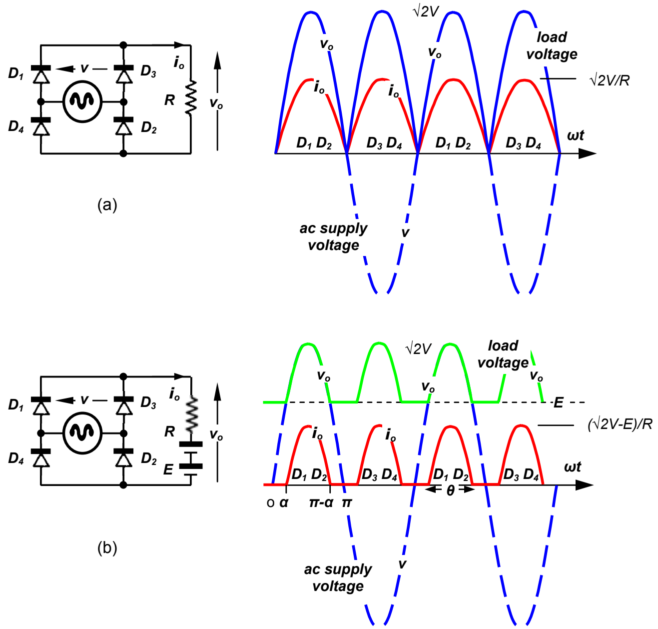


Figure 11.7. *Single-phase full-wave rectifiers:*
(a) purely resistive load, R and (b) resistive load R with back emf, E.

The load average and rms currents are

$$I_o = \frac{1}{R}\left[\frac{2\sqrt{2}V}{\pi}\sin ½\theta - E\frac{\theta}{\pi}\right] \qquad (11.51)$$

which is double the half-wave case and

$$I_{o\,rms} = \frac{1}{R}\left[\frac{V^2}{\pi}\sin\theta - \frac{4\sqrt{2}}{\pi}V\,E\sin ½\theta + \left(V^2 + E^2\right)\frac{\theta}{\pi}\right]^{½} \qquad (11.52)$$

which is √2 greater than the half-wave case.
The total power delivered to the load is (double the half-wave case):

$$P_o = P_R + P_E = I_{o\,rms}^2 R + E\,I_o \qquad (11.53)$$

Example 11.4: ***Full-wave rectifier with resistive and back emf load***

A dc motor, with series armature resistance of 10Ω and a back emf of 100V dc, is fed via a full-wave rectifier from the single-phase 230V 50Hz ac mains. Calculate

i. The average and rms motor voltages and currents, and diode maximum reverse voltage
ii. The supply power factor and motor efficiency

Solution

With a 100V back emf, the circuit and waveforms in figure 11.7b are applicable.
The current starts conducting when

$$\omega t = \alpha = \sin^{-1}\frac{E}{\sqrt{2}V} = \sin^{-1}\frac{100\text{V}}{\sqrt{2}\times 230\text{V}} = 17.9°$$

The current conducts for a period $\theta = \pi - 2\alpha = 180° - 2\times 17.9 = 144.2°$, ceasing at $\omega t = \pi - \alpha = 162.1°$.

i. The average and rms load currents and voltages are given by equations (11.49) to (11.52).

$$V_o = 2E\frac{\alpha}{\pi} + \frac{2}{\pi}\sqrt{2}V\cos\alpha$$

$$= 2\times 100\text{V}\frac{17.9°}{180°} + \frac{2}{\pi}\sqrt{2}\times 230\text{V}\times\cos 17.9° = 216.9\text{V}$$

$$I_o = \frac{1}{R}\left[\frac{2\sqrt{2}V}{\pi}\sin ½\theta - E\frac{\theta}{\pi}\right]$$

$$= \frac{1}{10\Omega}\left[\frac{2\sqrt{2}\times 230\text{ V}}{\pi}\sin ½\times 144.2° - 100\text{V}\times\frac{144.2°}{180°}\right] = 11.7\text{A}$$

$$V_{o\,rms} = \left[2\frac{\alpha}{\pi}E^2 + V^2\left(1 - 2\frac{\alpha}{\pi} + \frac{1}{\pi}\sin 2\alpha\right)^2\right]^{½}$$

$$= \left[2\times 100^2\frac{17.9°}{180°} + 230^2\left(1 - 2\times\frac{17.9°}{180°} + \frac{1}{\pi}\sin 2\times 17.9°\right)^2\right]^{½} = 231.4\text{V}$$

$$I_{o\,rms} = \frac{1}{R}\left[\frac{V^2}{\pi}\sin\theta - \frac{4\sqrt{2}}{\pi}V\,E\sin ½\theta + \left(V^2 + E^2\right)\frac{\theta}{\pi}\right]^{½}$$

$$= \frac{1}{10\Omega}\left[\frac{230^2}{\pi}\sin 144.2° - \frac{4\sqrt{2}}{\pi}\times 230\text{V}\times 100\text{V}\times\sin ½\times 144.2° + \left(230^2 + 100^2\right)\frac{144.2°}{180°}\right]^{½}$$

$$= 14.43\text{A}$$

The diode maximum reverse voltage is √2×230+100=425.3V.

ii. The motor loss is the loss in the 10Ω resistance in the dc motor equivalent circuit

$$P_R = I_{o\,rms}^2 R = 14.43^2\times 10\Omega = 2082.2\text{W}$$

The back emf represents the source of electrical energy converted to mechanical energy

$$P_E = E\times I_o = 100\text{V}\times 11.7\text{A} = 1170\text{W}$$

The supply power factor is defined as the ratio: supply power delivered to apparent supply power

$$pf = \frac{P}{S} = \frac{P_R + P_E}{V\times I_{o\,rms}} = \frac{2082.2\text{W} + 1170\text{W}}{230\text{V}\times 14.43\text{A}} = 0.98$$

The motor efficiency is

$$\eta = \frac{P_E}{P_R + P_E} = \frac{1170\text{W}}{2082.2\text{W} + 1170\text{W}}\times 100 = 36.0\%$$

♣

11.1.8 Single-phase, full-wave bridge rectifier circuit with an R-L load

Single-phase full-wave diode bridge circuits are shown in figures 11.8a and 11.8b. Both circuits appear identical as far as the load and supply are concerned. It will be seen in part b that two fewer diodes can be employed but this circuit requires a centre-tapped secondary transformer where each secondary has only a 50% copper utilisation factor. For the same output voltage, each of the secondary windings in figure 11.8b must have the same rms voltage rating as the single secondary winding of the transformer in figure 11.8a. The rectifying diodes in figure 11.8b experience twice the reverse voltage, (2√2 V), as that experienced by each of the four diodes in the circuit of figure 11.8a, (√2 V).
Figure 11.8c shows bridge circuit voltage and current waveforms. Assuming a 1:1(:1) transformer turns ratio, and with an inductive passive load, (no back emf) continuous load current flows, which is given by

$$i_o(\omega t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) + \frac{2\sin\phi}{1 - e^{-\pi/\tan\phi}}\times e^{-\omega t/\tan\phi}\right] \qquad 0 \le \omega t \le \pi \qquad (11.54)$$

Appropriate integration of the load current squared, gives the rms load (and ac supply) current:

$$I_{rms} = \frac{V}{Z}\left[1 + 4\sin^2\phi\tan\phi \times \left(1 + e^{-\pi/\tan\phi}\right)\right]^{1/2} = I_s \qquad (11.55)$$

The load experiences the transformer secondary rectified voltage which has a mean voltage (thence mean load current) of

$$V_o = \tfrac{1}{\pi}\int_0^{\pi}\sqrt{2}V\sin\omega t\; d\omega t = \bar{I}_o R = {2\sqrt{2}V}\big/{\pi} \quad = \quad 0.90V \qquad \text{(V)} \qquad (11.56)$$

Since the average inductor voltage is zero, the average resistor voltage equals the average *R-L* voltage.

The rms value of the load circuit voltage v_0 is

$$V_{rms} = \sqrt{\tfrac{1}{2\pi}\int_0^{2\pi}\left(\sqrt{2}V\sin\omega t\right)^2 d\omega t} \;=\; V \qquad \text{(V)} \qquad (11.57)$$

From the load voltage definitions in section 11.4, the load voltage form factor is constant:

$$FF_v = \frac{V_{rms}}{V_o} = \frac{V}{2\sqrt{2}V\big/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11 \qquad (11.58)$$

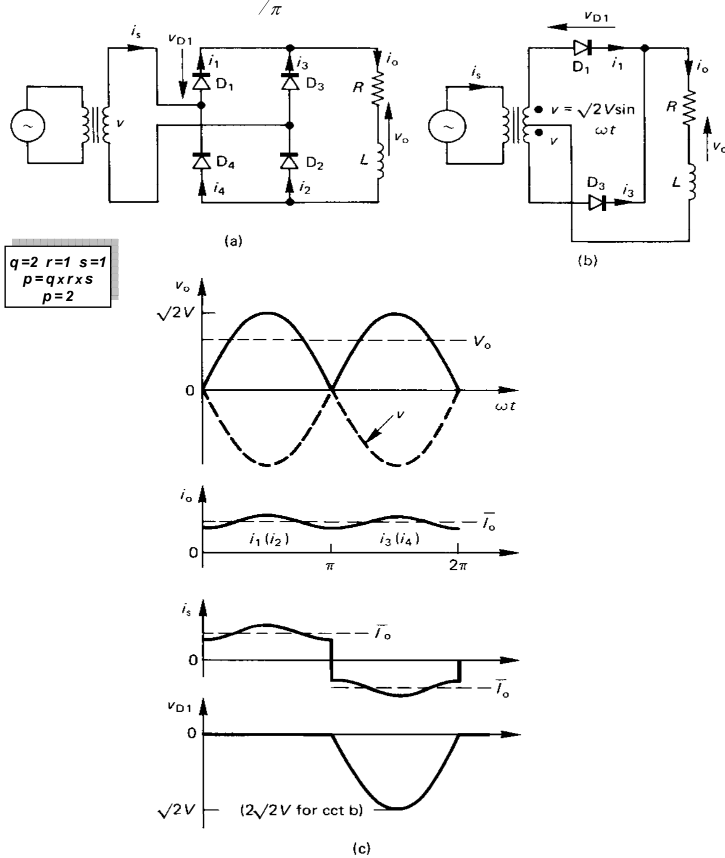


Figure 11.8. *Single-phase full-wave rectifier bridge: (a) circuit with four rectifying diodes; (b) circuit with two rectifying diodes; and (c) circuit waveforms.*

The load ripple voltage is

$$\begin{aligned} V_{RI} &\triangleq \sqrt{V_{rms}^2 - V_o^2} \\ &= \sqrt{V^2 - \left({2\sqrt{2}}/{\pi}\right)^2 V^2} = V\sqrt{1 - {8}/{\pi^2}} = 0.435\,V \qquad \text{(V)} \end{aligned} \qquad (11.59)$$

hence the load voltage ripple factor is

$$\begin{aligned} RF_v &\triangleq V_{RI} / V_o = \sqrt{FF_v^2 - 1} \\ RF_v &= \sqrt{1 - \left({2\sqrt{2}}/{\pi}\right)^2} \;\Big/\; {2\sqrt{2}}\big/{\pi} \;=\; \sqrt{{\pi^2}/{8} - 1} = 0.483 \qquad FF_v = \pi / 2\sqrt{2} = 1.11 \end{aligned} \qquad (11.60)$$

which is significantly less (better) than the half-wave rectified value of 1.211 from equation (11.32).

The output voltages and currents (rms and average) can be derived from the voltage Fourier expansion in equation (11.46):

$$v_o(\omega t) = \frac{2\sqrt{2}V}{\pi} + \frac{2\sqrt{2}V}{\pi}\sum_{n=2,4,6}^{\infty}\frac{2}{n^2-1}\cos n\omega t \qquad (11.61)$$

The first term is the average output voltage, as given by equation (11.56). Note the harmonic magnitudes decrease rapidly with increased order, namely $\frac{2}{3} : \frac{2}{15} : \frac{2}{35} : \frac{2}{63} : \cdots$. The output voltage is therefore dominated by the dc component and the harmonic at 2ω.

The output current can be derived by dividing each voltage component by the appropriate load impedance at that frequency. That is

$$\begin{aligned} \bar{I}_o &= \frac{V_o}{R} = \frac{2\sqrt{2}V}{\pi R} \\ I_n &= \frac{V_n}{Z_n} = \frac{2\sqrt{2}V}{\pi} \times \frac{\frac{2}{n^2-1}}{\sqrt{R^2 + (n\omega L)^2}} \qquad \text{for} \qquad n = 2, 4, 6.. \end{aligned} \qquad (11.62)$$

The load rms current whence load power, critical load inductance, and power factor, are given by

$$\begin{aligned} I_{rms} &= \sqrt{I_o^2 + \sum_{n=2,4,}^{\infty} \tfrac{1}{2} \times I_n^2} \qquad P_L = I_{rms}^2 R \\ pf &= \frac{P_L}{V\,I_{rms}} = \frac{I_{rms}R}{V} \qquad L_{critical} = \frac{R}{3 \times \omega} \quad \text{(see equation 11.67)} \end{aligned} \qquad (11.63)$$

Each diode rms current is $I_{rms} / \sqrt{2}$. For the circuit in figure 11.8a, the transformer secondary winding rms current is I_{rms}, while for the centre-tapped transformer, for the same load voltage, each winding has an rms current rating of $I_{rms} / \sqrt{2}$. The primary current rating is the same for both transformers and is related to the secondary rms current rating by the turns ratio. Power factor is independent of turns ratio.

11.1.8i - Single-phase full-wave bridge rectifier circuit with an output L-C filter

A – with an output L-C filter and continuous inductor current

Table 11.1 shows three typical single-phase, full-wave rectifier output stages, where part c is a typical output filtering stage used to obtain a near constant dc output voltage.

If it is assumed that the load inductance is large and the load resistance small such that continuous load current flows, then the bridge average output voltage $\bar{V}_o$ is the same as the average voltage across the load resistor since the average voltage across the filter inductor is zero. From equation (11.61), the dominant load voltage harmonic is due to the second harmonic therefore the ac current is predominately the second harmonic current, $I_{o,ac} \approx I_{o,2}$. By neglecting the higher order harmonics, the various circuit currents and voltages can be readily obtained as shown in table 11.1. From equation (11.61) the output voltage is given by

$$\begin{aligned} v_o(\omega t) &= \bar{V}_o + V_{o,2}\cos 2\omega t \\ &= \frac{2\sqrt{2}V}{\pi} + \frac{2\sqrt{2}V}{\pi} \times \frac{2}{n^2-1}\cos n\omega t \qquad \text{for } n = 2 \\ &= \frac{2\sqrt{2}V}{\pi} + \frac{2\sqrt{2}V}{\pi} \times \frac{2}{3}\cos 2\omega t \\ &= 0.90V + 0.60V \times \cos 2\omega t \end{aligned} \qquad (11.64)$$

With the filter capacitor across the load resistor, the average inductor current is equal to the average resistor current, since the average capacitor current is zero.

With continuous inductor current, the inductor current is

$$\begin{aligned} i_o(\omega t) &= \bar{I}_o + I_{o,2}\cos 2\omega t \\ &= \frac{\bar{V}_o}{R} + \frac{2}{3}\frac{\bar{V}_o}{Z_2}\cos 2\omega t = \frac{0.90V}{R} + \frac{0.60V}{\sqrt{R^2 + (2\omega L)^2}} \times \cos 2\omega t \end{aligned} \qquad (11.65)$$

From equation (11.65) for continuous inductor current, the average current must be larger than the peak second harmonic current magnitude, that is

$$\bar{I}_o > \left| I_{o,2} \right|$$

$$\frac{\bar{V}_o}{R} > \frac{2}{3}\frac{\bar{V}_o}{Z_2} \tag{11.66}$$

Since the load resistance must be low enough to ensure continuous inductor current, then $2\omega L > R$ such that $Z_2 = \sqrt{R^2 + (2\omega L)^2} \approx 2\omega L$. Equation (11.66) therefore gives the following load identity for continuous inductor current

$$\frac{1}{R} > \frac{2}{3}\frac{1}{Z_2} = \frac{1}{3\omega L} \quad \text{that is } L/R > 1/3\omega \qquad \text{generally } \left[L/R > 1\big/ m\left(m^2-1\right)\omega \right] \tag{11.67}$$

The load and supply (peak) ac currents are $I_{o,\,ac} = I_{s,\,ac} = I_{o,\,2}$. The output and supply rms currents are

$$I_{o,\,rms} = I_{s,\,rms} = \sqrt{\bar{I}_o^2 + \tfrac{1}{2} I_{o,\,ac}^2} = \sqrt{\bar{I}_o^2 + \tfrac{1}{2} I_{o,\,2}^2} \tag{11.68}$$

and the power delivered to resistance R in the load is

$$P_R = I_{o,rms}^2 R \tag{11.69}$$

B – with an output L-C filter and discontinuous inductor current

If the inductor current reduces to zero, at angle β, all the load current is provided by the capacitor. Its voltage falls to V_o ($<\sqrt{2}\,V$) and inductor current recommences when

$$v_L = \sqrt{2} V \sin\omega t - V_o \tag{11.70}$$

at an angle

$$\alpha = \sin^{-1}\frac{V_o}{\sqrt{2}\,V} \tag{11.71}$$

By integrating $v = L\, di/dt$ for i, the inductor current is of the form

$$i_L(\omega t) = \frac{1}{\omega L}\left(\sqrt{2}\,V\left(\cos\alpha - \cos\omega t\right) - V_o\left(\omega t - \alpha\right)\right) \tag{11.72}$$

where $\alpha \le \omega t \le \beta$. The voltage V_o is found from equation (11.72) by iterative techniques.

11.1.8ii Single-phase, full-wave bridge rectifier circuit with an R-L-E load

An *R-L* load incorporating a back emf *E*, is shown in Table 11.1.

For **continuous** output current

When continuous load current flows, the rectified supply is continuously impressed across the series *L-R-E* load, therefore the average and rms output voltages respectively are

$$\bar{V}_o = \frac{1}{\pi}\int_o^{\pi}\sqrt{2}V_s\sin\omega t\, d\omega t = \frac{2\sqrt{2}}{\pi}V_s$$
$$V_o = \sqrt{\frac{1}{\pi}\int_o^{\pi}\left(\sqrt{2}V_s\sin\omega t\right)^2 d\omega t} = V_s \tag{11.73}$$

Hence the output voltage form and ripple factors are

$$FF_v = \frac{V_o}{\bar{V}_o} = \frac{\pi}{2\sqrt{2}}$$
$$RF_v = \sqrt{V_{FF}^2 - 1} = \sqrt{\frac{\pi^2}{8} - 1} \tag{11.74}$$

If the input current is approximated by its fundamental, $4\bar{I}_o/\pi$, the following input characteristics are realised

$$\text{input displacement factor} = DPF = \cos\phi_i = \cos 0° = 1$$

$$\text{distortion factor} = DF_{i1} = \frac{I_{i1}}{\bar{I}_o} = \frac{2\sqrt{2}}{\pi}$$

$$\text{power factor} = pf = DPF \times DF_{i1} = \frac{2\sqrt{2}}{\pi}$$

$$THD_{i1} = \sqrt{\frac{1 - DF_{i1}^2}{DF_{i1}^2}} \times 100 = \sqrt{\frac{\pi^2}{8} - 1} \times 100$$

The output current is found by solving

$$\sqrt{2}V_s\sin\omega t = Ri_o + L\frac{di_o}{dt} + E \tag{11.75}$$

which in steady state yields

$$i_o(t) = I_o e^{-\frac{\omega t}{\tan\theta}} + \sqrt{2}V_s\left[\sin(\omega t - \theta) - \frac{\sin\alpha}{\cos\theta}\right]$$
$$\text{where } \tan\theta = \frac{\omega L}{R}; \quad Z = \sqrt{R^2 + \omega^2 L^2}; \quad \sin\alpha = \frac{E}{\sqrt{2}V_s} \tag{11.76}$$

and the boundary conditions give

$$I_o = \frac{\sqrt{2}V_s}{Z}\frac{2\sin\theta}{1 - e^{\frac{-\pi}{\tan\theta}}}$$

For continuous conduction, $i_o(\omega t = \theta) \ge 0$ in equation (11.76) gives the condition

$$\frac{2\sin\theta}{1 - e^{\frac{-\pi}{\tan\theta}}}e^{\frac{-\alpha}{\tan\theta}} \ge \sin(\theta - \alpha) + \frac{\sin\alpha}{\cos\theta} \tag{11.77}$$

If the left hand side is less than the right hand side, **discontinuous** current flows in the load, and if the current extinction angle is β, then the average output voltage is given by

$$\bar{V}_o = \frac{1}{\pi}\left[\int_\alpha^\beta \sqrt{2}V_s\sin\omega t\, d\omega t + \int_\beta^{\pi+\alpha} E\, d\omega t\right]$$
$$\bar{V}_o = \frac{\sqrt{2}V}{\pi}\left[\cos\alpha - \cos\beta + (\pi + \alpha - \beta)\sin\alpha\right] \tag{11.78}$$

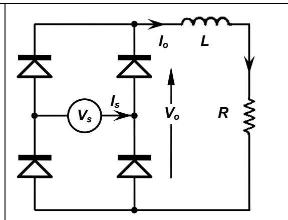
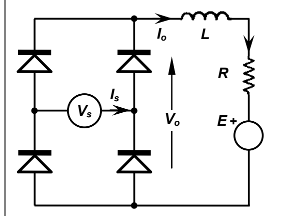
In the general solution to the circuit differential equation in equation (11.76), for discontinuous output current, (zero current boundary conditions), I_o for equation (11.76) becomes (during conduction)

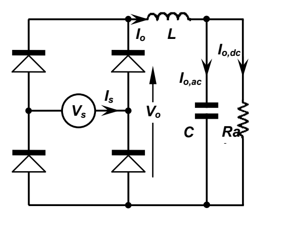
$$I_o = \frac{\sqrt{2}V_s}{Z}\left[\sin(\theta - \beta) + \frac{\sin\alpha}{\cos\theta}\right]$$

The conduction period β is found by iteratively solving

$$\sin(\beta - \alpha) = \left[1 - e^{\frac{\alpha-\beta}{\tan\theta}}\right] \times \frac{\sin\alpha}{\cos\theta} - e^{\frac{\alpha-\beta}{\tan\theta}} \times \sin(\theta - \alpha) \tag{11.79}$$

Table 11.1: Single-phase full-wave uncontrolled rectifier circuits – continuous inductor current

Full-wave rectifier circuit		2nd harmonic current	average output current	output power
		$I_{o,2}$	$\bar{I}_o$	P_R+P_E
Load	circuit	(A)	(A)	(W)
(a) ***R-L*** *see section 11.1.8i and 12.2.3* $\alpha = 0$		$\dfrac{V_{o,2}}{\sqrt{R^2 + (2\omega L)^2}}$	$\dfrac{\bar{V}_o}{R} = \dfrac{2\sqrt{2}V}{\pi R}$	$I_{o,\,rms}^2 R$
(b) ***R-L-E*** *see section 11.8ii and 12.2.4* $\alpha = 0$		$\dfrac{V_{o,2}}{\sqrt{R^2 + (2\omega L)^2}}$	$\dfrac{\bar{V}_o - E}{R} = \dfrac{1}{R}\left(\dfrac{2\sqrt{2}V}{\pi} - E\right)$	$I_{o,\,rms}^2 R + \bar{I}_o E$

(c) ***L-R//C*** *see section 11.1.8i*		$\frac{V_{o,2}}{2\omega L}$	$\frac{\bar{V}_o}{R}$ $=\frac{2\sqrt{2}V}{\pi R}$	$I^2_{o,\,rms}R = \bar{I}^2_o R$

Example 11.5: ***Full-wave diode rectifier with an L-C filter and continuous load current***

A single-phase, full-wave, diode rectifier is supplied from a 230V ac, 50Hz voltage source and uses an *L-C* output filter with a resistor load, as shown in the last circuit in Table 11.1. The average inductor current is 10A with a 4A rms ripple current dominated by the 100Hz component. Ignoring diode voltage drops and initially assuming the output voltage is ripple free, determine

i. dc output voltage, hence load resistance and power
ii. dc filter inductance and its average voltage, whilst neglecting any capacitor voltage ripple
iii. dc filter capacitance if its peak-to-peak ripple voltage is 5% the average voltage
iv. diode average, rms, and peak current
v. supply power factor

Solution

Since $\sqrt{2}\,I_{o\,rms,2} < \bar{I}_o \left(\sqrt{2}\times 4\text{A} < 10\text{A}\right)$, the output current is continuous.

i. The dc output voltage is $\bar{V}_o$ = 0.9×230V = 207V. Assuming the 207V is ripple free, that is, V_{rms} = V_{dc}, then the load resistance and power dissipated are

$$R = \frac{\bar{V}_o}{\bar{I}_o} = \frac{207\text{V}}{10\text{A}} = 20.7\Omega$$

$$P_R = \bar{V}_o \times \bar{I}_o = 207\text{V}\times 10\text{A} = 2070\text{W}$$

ii. The 100Hz voltage component in the output voltage is given by equation (11.64), that is

$$V_{o,\,2} = \frac{2\sqrt{2}V}{\pi}\times\frac{2}{n^2-1}\cos n\omega t$$
$$= \frac{2\sqrt{2}V}{\pi}\times\frac{2}{3}\cos 2\omega t$$
$$= 0.60\times 230\text{V}\times\cos 2\omega t = 138\times\cos 2\omega t$$

which has an rms value of 138/√2 = 97.6V. The 100Hz rms current $I_{o,2}/\sqrt{2}$ produced by this voltage is 4A thus

$$\text{from}\quad \frac{I_{o,\,2}}{\sqrt{2}} = \frac{V_{o,\,2}}{2\omega L}$$

$$L = \frac{V_{o,\,2}}{2\omega I_{o,\,2}} = \frac{97.6\text{V}}{2\times 2\pi 50\text{Hz}\times 4\text{A}} = 38.8\text{mH}$$

The average inductor voltage is zero.

iii. From part *i*, the dc output voltage is 207V. The peak-to-peak ripple voltage is 5% of 207V, that is 10.35V. This gives an rms value of 10.35V /2√2 = 3.66V. From

$$V_{o,\,2}\big/\sqrt{2} = I_{o,\,2}\big/\sqrt{2}\times X_{c,\,100\text{Hz}} = \frac{I_{o,\,2}\big/\sqrt{2}}{2\omega C}$$

$$\Rightarrow C = \frac{I_{o,\,2}}{2\omega\times V_{o,\,2}} = \frac{4\text{A}}{2\times 2\pi 50\text{Hz}\times 3.66\text{V}} = 1.7\text{mF}$$

iv. The diode currents are

$$I_{D,\,rms} = I_{o,\,rms}/\sqrt{2} = \sqrt{\bar{I}^2_o + \tfrac{1}{2}I^2_{o,2}}\,/\sqrt{2} = \sqrt{10\text{A}^2+4\text{A}^2}\,/\sqrt{2} = 10.8\text{A}/\sqrt{2} = 7.64\text{A}$$

$$I_{D,\,rms} = I_{o,\,rms}/\sqrt{2} = \sqrt{\bar{I}^2_o + \tfrac{1}{2}I^2_{o,2}} \qquad \hat{I}_D = \bar{I}_o + I_{o,2} = 10\text{A}+\sqrt{2}\times 4\text{A} = 15.7\text{A}$$

v. The input and output rms current is

$$I_s = I_{o,rms} = \sqrt{\bar{I}^2_o + \tfrac{1}{2}I^2_{o,2}} = \sqrt{10\text{A}^2+4\text{A}^2} = 10.8\text{A}$$

Assuming the input power equals the output power, then from part *i*, $P_o = P_i$ = 2070W. The supply power factor is

$$pf = \frac{P_i}{S} = \frac{P_i}{V_s I_s} = \frac{2070\text{W}}{230\text{V}\times 10.8\text{A}} = 0.83$$

♣

11.1.8iii - Single-phase full-wave bridge rectifier with highly inductive load– constant load current

With a highly inductive load, which is the usual practical case, virtually constant load current flows, as shown dashed in figure 11.8c. The bridge diode currents are then square wave 180º blocks of current of magnitude $\bar{I}_o$. The diode current ratings can now be specified and depend on the pulse number *p*. For this full-wave single-phase application each input cycle comprises two 180º output current pulses, hence *p* = 2.

The mean current in each diode is

$$\bar{I}_D = \tfrac{1}{p}\,\bar{I}_o = \tfrac{1}{2}\bar{I}_o \qquad \text{(A)} \qquad (11.80)$$

and the rms current in each diode is

$$I_D = \tfrac{1}{\sqrt{p}}\,\bar{I}_o = \bar{I}_o/\sqrt{2} \qquad \text{(A)} \qquad (11.81)$$

whence the diode current form factor is

$$RF_{ID} = I_D/\bar{I}_D = \sqrt{p} = \sqrt{2} \qquad (11.82)$$

Since the load current is approximately constant, power delivered to the load is

$$P_o \approx \bar{V}_o\bar{I}_o = \tfrac{8}{\pi^2}\times\tfrac{V^2}{R} \qquad \text{(W)} \qquad (11.83)$$

The supply power factor is $pf = V_o / V$ = 2√2/π = 0.90, since $\bar{I}_o = I_{rms}$.

11.1.8iv - Single-phase full-wave bridge rectifier circuit with a C-filter and resistive load

The capacitor smoothed single-phase full-wave diode rectifier circuit shown in figure 11.9a is a common power rectifier circuit used to obtain unregulated dc voltages. The circuit is simple and cheap but the input current has high peak and rms values, high harmonics, and a poor power factor. The full-wave rectified case is an extension of the half-wave case considered in section 11.1.4.
The capacitor reduces the ripple voltage, so large voltage-polarised capacitance is used to produce an almost constant dc output voltage. Isolation and voltage matching (step-up or step down) are obtained by using a transformer before the diode rectification stage as shown in figures 11.8a and b. The resistor *R* across the filter capacitor represents a resistive dissipative load.

As the ac supply voltage rises to its extremes each half cycle, as shown in figure 11.9b, a pair of rectifier diodes D_1-D_2 or D_3-D_4, alternately become forward biased at time $\omega t = \alpha$. The ac supply provides load resistor current and simultaneously charges the capacitor, its voltage having drooped whilst providing the load current during the previous diode non-conduction period. The capacitor charging current period θ_c around the ac supply extremes is short, giving a high peak to rms ratio of diode and supply current. When all the rectifier diodes are reverse biased at $\omega t = \beta$ because the capacitor voltage is greater than the instantaneous supply ac voltage, the capacitor supplies the load current and its voltage decreases with an *R-C* time constant until $\omega t = \pi+\alpha$. The output voltage and diode voltages, plus load current v_o/R, and capacitor current $C\,dv_o/dt$ are defined in Table 11.2.

The start of diode conduction, *α*, the diode current extinction angle, *β*, hence diode conduction period, θ_c, are specified by the following equations.
From $i_c + i_R = 0$ at $\omega t = \beta$:

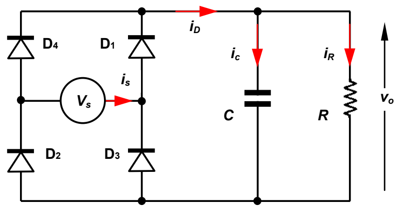
$$\frac{\sqrt{2}V}{X}\cos\beta + \frac{\sqrt{2}V}{R}\sin\beta = 0$$
$$\beta = \tan^{-1}\left(-\omega RC\right) = \pi - \tan^{-1}\left(\omega RC\right) \qquad \tfrac{1}{2}\pi \le \beta \le \pi \qquad (11.84)$$

By equating the two expression for output voltage at the boundary $\omega t = \pi+\alpha$ gives

$$\sqrt{2}V\left|\sin\left(\pi+\alpha\right)\right| = \sqrt{2}V\sin\beta\times e^{-(\pi+\alpha-\beta)/\tan\beta} \qquad (11.85)$$

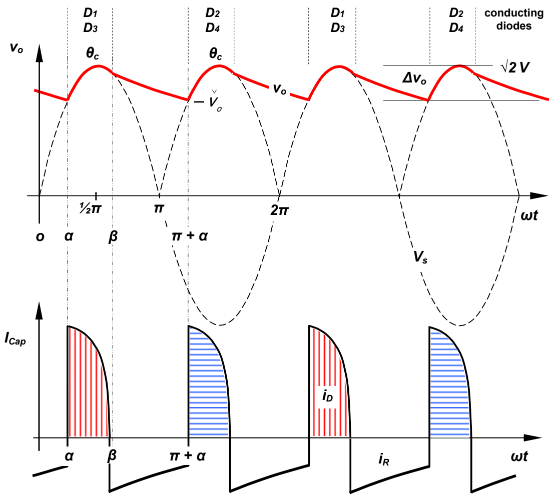
and a transcendental expression for *α* results:

$$\sin\alpha - \sin\beta\times e^{-(\pi+\alpha-\beta)/\omega RC} = 0 \qquad (11.86)$$



(a)

(a)



(b)

(b)

Figure 11.9. *Single-phase full-wave rectifier bridge: (a) circuit with C-filter capacitor and (b) circuit waveforms.*

Table 11.2: Single-phase, full-wave rectifier voltages and currents

$v_s(\omega t) = \sqrt{2}V\sin\omega t$		Diodes conducting	Diodes non-conducting
		$\alpha \le \omega t \le \beta$	$\beta \le \omega t \le \pi + \alpha$
Output voltage	$v_o(\omega t)$	$\sqrt{2}V\lvert\sin\omega t\rvert$	$\sqrt{2}V\sin\beta \times e^{-(\omega t-\beta)/\tan\beta}$
Diode voltage	$v_D(\omega t)$	0 and $-\sqrt{2}V\sin\omega t$	$-\sqrt{2}V\sin\beta \times e^{-(\omega t-\beta)/\tan\beta} + \sqrt{2}V\sin\omega t$
Capacitor current	$i_c(\omega t)$	$\frac{\sqrt{2}V}{X}\cos\omega t$	$\frac{\sqrt{2}V}{Z} \times e^{-(\omega t-\beta)/\tan\beta}$
Resistor current	$i_R(\omega t)$	$\frac{\sqrt{2}V}{R}\sin\omega t$	$\frac{\sqrt{2}V}{Z} \times e^{-(\omega t-\beta)/\tan\beta} = -i_c(\omega t)$
Diode bridge current	$I_D(\omega t) = i_c(\omega t) + i_R(\omega t)$	$\frac{\sqrt{2}V}{R\cos\phi} \times \sin(\omega t + \phi)$	0

The diode current conduction period θ_c is given by

$$\theta_c = \beta - \alpha \qquad (11.87)$$

When the diodes conduct, R and C are in parallel and $\tan\phi = \omega C R$.

When the diodes are not conducting, the output circuit current flows in a series *R-C* circuit with a fundamental impedance of:

$$Z = \sqrt{R^2 + X^2} \quad \text{and} \quad X = {}^1\!/_{\omega C}$$

The resistor average voltage and current are

$$\bar{V}_R = \frac{\sqrt{2}V}{\pi}\frac{(1-\cos\theta_c)}{-\cos\beta} = \bar{I}_R R \qquad (11.88)$$

The maximum output voltage occurs at ωt = ½π when $v_o = \hat{V}_o = \hat{V}_s = \sqrt{2}V$, while the minimum output voltage occurs at the end of the capacitor discharge period when $\omega t = \alpha$ and $v_o = \check{V}_o = \sqrt{2}V\sin\alpha$. The output peak-to-peak ripple voltage is therefore the difference:

$$\Delta V_o = \hat{V}_o - \check{V}_o = \sqrt{2}V - \sqrt{2}V\sin\alpha = \sqrt{2}V(1-\sin\alpha) \qquad (11.89)$$

By assuming $\alpha \approx$ ½π, $\beta \approx$ ½π, and a series expansion for the exponent

$$\Delta V_o \approx \frac{\sqrt{2}V\pi}{\omega RC} = \frac{V}{\sqrt{2}f\,RC} \qquad (11.90)$$

The ac source current is the sum of the diode currents, that is

$$i_s = i_{D1,2} - i_{D3,4} = i_R + i_c \qquad (11.91)$$

when $\alpha < \omega t < \beta$. Otherwise $i_s = 0$.

Since the capacitor voltage is in steady-state, the average capacitor current is zero, thus for full-wave rectification, the average diode current is half the average load current.
The peak capacitor current occurs at $\omega t = \alpha$, when the diodes first conduct. From the capacitor current equation in table 11.2:

$$\hat{I}_c = \sqrt{2}V\omega C\cos\alpha \qquad (11.92)$$

From table 11.2, the peak diode current occurs at the same time as the peak capacitor current, $\omega t = \alpha$:

$$\begin{aligned}\hat{I}_D &= i_c(\pi+\alpha) + i_R(\pi+\alpha)\\ &= \sqrt{2}V\omega C\cos\alpha + \frac{\sqrt{2}V}{R}\sin\alpha = \frac{\sqrt{2}V}{X}\cos\alpha + \frac{\sqrt{2}V}{R}\sin\alpha = \frac{\sqrt{2}V}{R}\frac{Z}{X}\sin(\alpha+\phi)\end{aligned} \qquad (11.93)$$

Similar expressions can be derived for the half-wave rectifier case. For the non-conduction period, $\beta = 2\pi + \alpha$. The output ripple voltage is about twice that given by equation (11.90) and the average resistor voltage in equation (11.88) (after modification), is reduced. The diode PIV rating is 2√2V in both cases.

Example 11.6: ***Single-phase full-wave bridge rectifier circuit with C-filter and resistive load***

A single-phase, full-wave, diode rectifier is supplied from a 230V ac, 50Hz voltage source and uses a capacitor output filter, 1000μF, with a resistor 100Ω load, as shown in Figure 11.9a. Ignoring diode voltage drops, determine

i. expressions for the output voltage
ii. output voltage ripple Δv_o and the % error in using the approximation equation (11.90)
iii. expressions for the capacitor current
iv. diode peak current
v. average load voltage and current

Assuming the output ripple voltage is triangular, estimate

vi. average output voltage and rms output ripple voltage
vii. capacitance C for Δv_o = 2% of the maximum output voltage

Solution

The supply voltage is $v_s = \sqrt{2}\times 230 \sin 2\pi\, 50t$, which has a peak value of $\hat{V}_s$ = 325.3V.

$$\omega RC = 2\pi 50\text{Hz} \times 100\Omega \times 1000\mu\text{F} = 31.416 \text{ rad}$$

Thus $X = 1/\omega C$ = 3.1831Ω and Z = 100.0507Ω.
(5 figure accuracy is used because of the sensitivity of the applicable equations around α = 90°.)

From equation (11.84) the diode current extinction angle β is

$$\beta = \pi - \tan^{-1}(\omega RC) = \pi - \tan^{-1}(31.416\text{rad}) = 1.6026\text{rad} = 91.8°$$

The diode current turn-on angle α is solve iteratively from equation (11.86), that is

$$\sin\alpha - \sin\beta \times e^{-(\pi+\alpha-\beta)/\omega RC} = 0$$

$$\sin\alpha - \sin 1.603 \times e^{-(\pi+\alpha-1.603)/31.416} = 0$$

gives α = 1.16095 rad or 66.5°. The diode conduction period is $\theta_c = \beta - \alpha$ =1.6026 -1.16095 = 0.44167rad or 25.3°.

i. From table 11.2, the output voltage, which is the capacitor voltage, is given by

$$v_o(\omega t) = \left|\sqrt{2}V\sin\omega t\right| = \left|325.27\text{V}\times\sin\omega t\right| \qquad 66.5° \le \omega t \le 91.8°$$

$$v_o(\omega t) = \sqrt{2}\times 230\text{V}\times\sin 1.6026\text{rad}\times e^{-(\omega t - 1.6026\text{rad})/31.416\text{rad}} = 325.13\times e^{-(\omega t-1.6026\text{rad})/31.416\text{rad}}$$

$$91.8° \le \omega t \le 246.5°$$

ii. The output voltage ripple Δv_o is given by equation (11.89), that is

$$\Delta V_o = \sqrt{2}V(1-\sin\alpha) = \sqrt{2}\,230\text{V}\times(1-\sin 1.16026) = 26.94\text{V p-p}$$

From equation (11.90)

$$\Delta V_o \approx \frac{V}{\sqrt{2}fRC} = \frac{230\text{V}}{\sqrt{2}\times 50\text{Hz}\times 100\Omega\times 1000\mu\text{F}} = 32.5\text{V}$$

The approximation predicts a higher ripple: a +21% over-estimate.

iii. From table 11.2, the capacitor current is

$$i_c(\omega t) = \sqrt{2}V\omega C\cos\omega t = \sqrt{2}\,230\text{V}\times 2\pi 50\text{Hz}\times 1000\mu\text{F}\times\cos\omega t = 102.2\times\cos\omega t$$

$$66.5° \le \omega t \le 91.8°$$

$$i_c(\omega t) = \frac{\sqrt{2}V\sin\beta}{R}\times e^{-(\omega t-\beta)/\omega RC} = \frac{\sqrt{2}230\text{V}\times\sin 1.16}{100\Omega}\times e^{-(\omega t-1.16)/31.4} = 3.0\times e^{-(\omega t-1.16)/31.4}$$

$$91.8° \le \omega t \le 246.5°$$

iv. The peak diode current is given by equation (11.93):

$$\hat{I}_D = \sqrt{2}V\omega C\cos\alpha + \frac{\sqrt{2}V}{R}\sin\alpha$$

$$= \sqrt{2}\,230\text{V}\times 2\pi 50\text{Hz}\times 1000\mu\text{F}\times\cos 1.16026 + \frac{\sqrt{2}\,230\text{V}}{100\Omega}\times\sin 1.16026$$

$$= 40.7\text{A} + 3\text{A} = 43.7\text{A}$$

The peak diode current is dominated by the capacitor initial charging current of 40.7A

v. The average load voltage and current are given by equation (11.88)

$$\overline{V}_R = \frac{\sqrt{2}V}{\pi}\frac{(1-\cos\theta_c)}{-\cos\beta}$$

$$= \frac{\sqrt{2}\,230\text{V}}{\pi}\times\frac{(1-\cos 0.4417)}{-\cos 1.603} = 312.3\text{V}$$

$$\overline{I}_R = \frac{\overline{V}_R}{R} = \frac{312.3\text{V}}{100\Omega} = 3.12\text{A}$$

vi. If the ripple voltage is assumed triangular then

(a) The average output voltage is the peak output voltage minus half the ripple voltage, that is

$\hat{V}_s$ - ½Δv_o = √2×230V - ½×26.9V = 311.8V

which is less than that given by the accurate equation (11.89), 312.3V.

(b) If the 26.9V p-p ripple voltage is assumed triangular then its rms value is ½×26.9/√3 =7.8V rms.

vii. Re-arrangement of equation (11.90), which under-estimates the capacitance requirement for 2% ripple, gives

$$C = \frac{V}{\sqrt{2}fR\times\Delta V_o} = \frac{\hat{V}_s}{2fR\times 2\%\text{ of }\hat{V}_s} = \frac{1}{2fR\times 2\%}$$

$$= \frac{1}{2\times 50\text{Hz}\times 100\Omega\times 0.02} = 5{,}000\mu\text{F}$$

♣

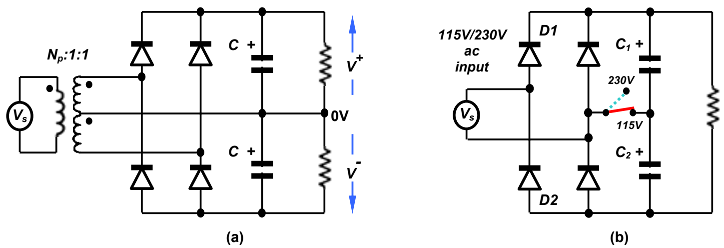


Figure 11.10. *Bridge rectifiers: (a) split rail dc supplies and (b) voltage doubler.*

11.1.8v - Other single-phase bridge rectifier circuit configurations

Figure 11.10a shows a transformer used to create a two-phase supply (each phase is 180° apart), which upon rectification produce equal split-rail dc output voltages, V^+ and V^-. The electrical characteristics can be analysed as in the case of the single-phase full-wave bridge rectifier circuit with a capacitive C-filter and resistive load, in section 11.1.8iv. In the split rail case, the rectifiers conduct every 180°, alternately feeding each output voltage rail capacitor. Thus the diode average and rms currents are increased by 2 and √2 respectively, above those of a conventional single phase rectifier.

The voltage doubler in figure 11.10b can be used in equipment that must be able to operate from both 115Vac and 230V ac voltage supplies, without the aid of a voltage-matching transformer. With the switch in the 115V position, the output is twice the peak of the input ac supply. The capacitor C_1 charges through diode D1, and when the supply reverses, capacitor C_2 charges through D2. Since C_1 and C_2 are in series, the output voltage is the sum $V_{C1}+V_{C2}$, where each capacitor is alternately charged (half-wave rectified) from the ac source V_s. The other, unused, two diodes remain reverse biased, and are only necessary if the dual input voltage function is required.

With the switch in the 230V ac position (open circuit), standard rectification occurs, with the two series capacitors charging simultaneously every half cycle. In dual frequency applications (110V ac, 60Hz and 230V ac, 50Hz), the capacitance requirements are based on the supply with the lower frequency, 50Hz.

11.2 Three-phase uncontrolled rectifier converter circuits

Single-phase supply circuits are adequate below a few kilowatts. At higher power levels, restrictions on unbalanced loading, line harmonics, current surge voltage dips, and filtering require the use of three-phase (or higher - polyphase) converter circuits. Generally it will be assumed that the output current is both continuous and smooth. This assumption is based on the dc load being highly inductive.

The characteristics of three-phase rectifiers with a purely resistive load are summarised in table 11.6.

11.2.1 Three-phase half-wave rectifier circuit with an inductive R-L load

Figure 11.11 shows a half-wave, three-phase diode rectifier circuit along with various circuit voltage and current waveforms. A transformer having a star connected secondary is required for neutral access, N.

The diode with the highest potential with respect to the neutral conducts a rectangular current pulse. As the potential of another diode becomes the highest, load current is transferred to that device, and the previously conducting device is reverse-biased and naturally (line) commutated. Note that the load voltage, hence current never reaches zero, when the load is passive (no opposing back emf).

In general terms, the mean output voltage for an *n*-phase *p*-pulse system is given by (see example 11.8)

$$V_o = \frac{1}{2\pi/p}\int_{-\pi/p}^{\pi/p}\sqrt{2}V\cos\omega t\,d\omega t \quad \text{(V)}$$
$$= \sqrt{2}V\frac{\sin(\pi/p)}{\pi/p} \quad \text{(V)} \qquad (11.94)$$

For a three-phase, half-wave circuit (p = 3) the mean output voltage, (thence average current) is

$$V_o = \overline{I}_oR = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6}\sqrt{2}V\sin\omega t\,d\omega t$$
$$= \sqrt{2}V\frac{\tfrac{1}{2}\sqrt{3}}{\pi/3} = 1.17\times V \quad \text{(V)} \qquad (11.95)$$

The rms load voltage is

$$V_{rms} = \sqrt{\frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6}\left(\sqrt{2}V\right)^2\sin^2\omega t\,d\omega t} = \sqrt{2}V\left[\frac{3}{2\pi}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)\right]^{\frac{1}{2}} = 1.19\times V \qquad (11.96)$$

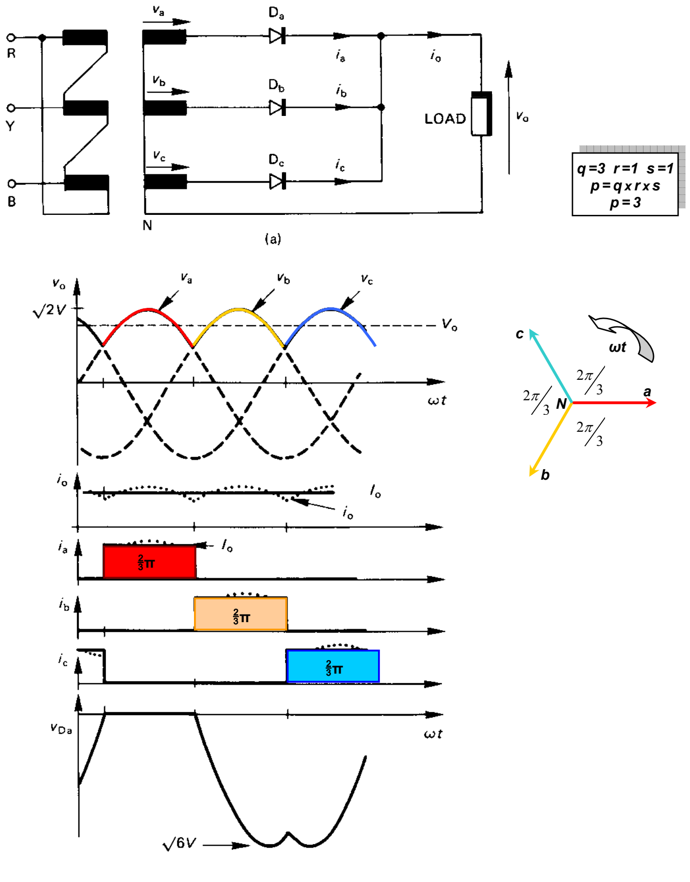


Figure 11.11. *Three-phase half-wave diode rectifier:*
(a) circuit diagram and (b) circuit voltage and current waveforms.

The load voltage form factor is

$$FF_v = {}^{V_{rms}}\!/_{V} = 1.19V\ /\ 1.17V = 1.01 \qquad (11.97)$$

$$\text{RF}_v = \text{ripple factor} = \frac{\text{ac voltage across the load}}{\text{dc voltage across the load}} = \sqrt{\left({}^{V_{rms}}\!/_{V}\right)^2 - 1} = 0.185 \qquad (11.98)$$

The diode conduction angle is 2π/*n*, namely ⅔π. The peak diode reverse voltage is given by the maximum voltage between any two phases, √3√2 *V* = √6 *V*.
From equations (11.80), (11.81), and (11.82), for a constant output current, $\overline{I}_o = I_{o\ rms}$, the mean diode current is

$$\overline{I}_D = {}^1\!/_n\,\overline{I}_o = {}^1\!/_3\,\overline{I}_o \qquad \text{(A)} \qquad (11.99)$$

and the rms diode current is

$$I_D = {}^1\!/_{\sqrt{n}}\,I_{o\ rms} \approx {}^1\!/_{\sqrt{n}}\,\overline{I}_o = {}^1\!/_{\sqrt{3}}\,\overline{I}_o \qquad \text{(A)} \qquad (11.100)$$

The diode current form factor is

$$FF_{ID} = I_D\ /\ \overline{I}_D = \sqrt{3} \qquad (11.101)$$

The input displacement factor cos*Φ* is unity and the input power factor (and displacement factor), assuming diode square currents, is

$$pf = \frac{V_o I_o}{3V_s I_{irms}} = \frac{\frac{3\sqrt{6}}{2\pi}V_s\,I_o}{3V_s\,\frac{I_o}{\sqrt{3}}} = \frac{3}{\sqrt{2}\,\pi} \qquad (11.102)$$

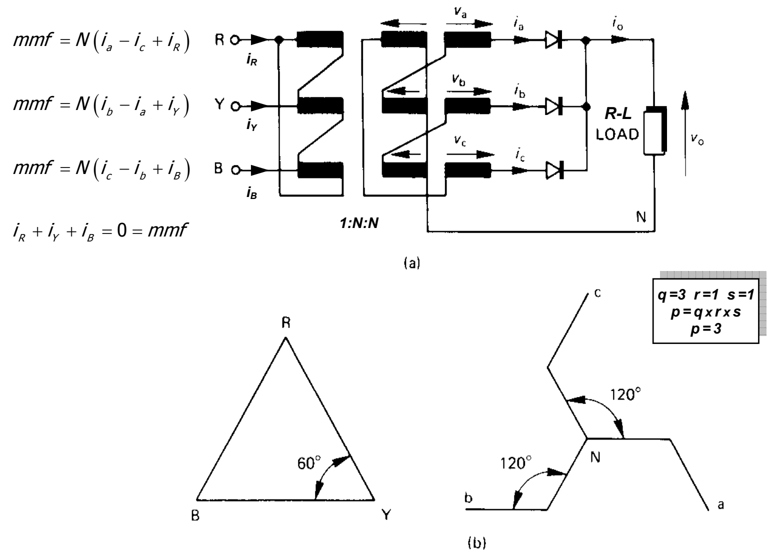


Figure 11.12. *Three-phase zig-zag interconnected star winding, with three windings per limb, 1:N:N:*
(a) transformer connection showing zero dc mmf in each limb (phase) and
(b) phasor diagram of transformer primary and secondary voltages.

If neutral is available, a transformer is not necessary. Then the full load current is returned via the neutral supply. This neutral current is generally not acceptable other than at low power levels. The simple delta-star connection of the supply in figure 11.11a is not appropriate since the unidirectional current in each phase is transferred from the supply to the transformer. This may result in increased magnetising current and iron losses if dc magnetisation occurs. As discussed in section 11.3.5, this problem is avoided in most cases by the special interconnected star winding, called zig-zag, shown in figure 11.12a and discussed in section 11.3.7. Each transformer limb has two equal voltage secondaries which are connected such that the magnetising forces balance. The resultant phasor diagram is shown in figure 11.12b. 15% more turns are needed than with a star connection. This transformer mmf problem resulting from half-wave rectification is considered in section 11.3.

As the number of phases increases, the windings become less utilised per cycle since the diode conduction angle decreases, from π for a single-phase circuit, to ⅔π for the three-phase case.

11.2.2 Three-phase full-wave rectifier circuit with an inductive R-L load

Figure 11.13a shows a three-phase full-wave rectifier circuit where no neutral is necessary and it will be seen that two series diodes (not in the same bridge leg) are always conducting. One diode (one of D_1, D_3, or D_5, at the highest potential) can be considered as being in the feed circuit, while the other (one of D_2, D_4, or D_6, at the lowest potential) is in the return circuit. As such, the line-to-line voltage is impressed across the load. Given no two series connected bridge leg diodes conduct simultaneously, there are six possible diode pair combinations. The rectifier circuit waveforms in figure 11.13b show that the load ripple frequency is six times the supply. Each diode conducts for ⅔π and experiences a reverse voltage of the peak line voltage, √2 V_L.

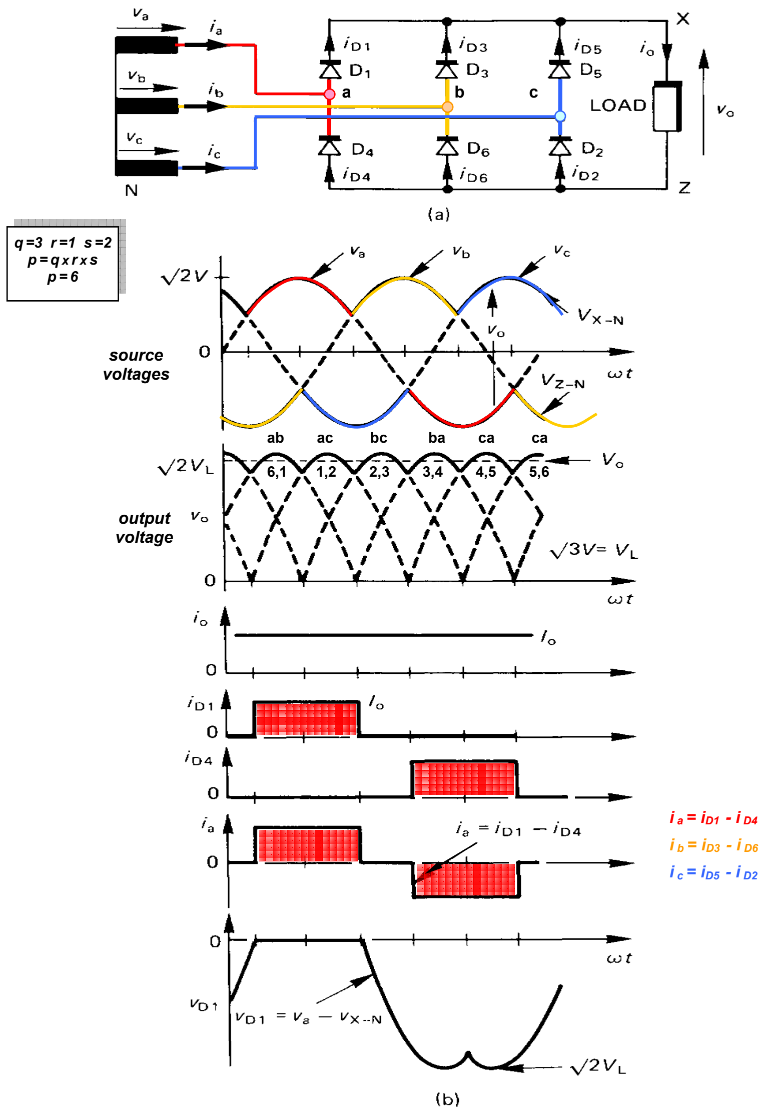


Figure 11.13. *Three-phase full-wave bridge rectifier:*
(a) circuit connection and (b) voltage and current waveforms.

The mean load voltage is given by twice equation (11.95), that is

$$\begin{aligned} V_o = I_o R &= \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} \sqrt{2}\, V_L \sin\omega t \; d\omega t \qquad \text{(V)} \\ &= \sqrt{2}V\,\frac{\sqrt{3}}{\pi/3} = \frac{3}{\pi}\sqrt{2}\, V_L = 1.35 V_L = 2.34 V \end{aligned} \qquad (11.103)$$

where V_L is the line-to-line rms voltage ($V_L = \sqrt{3}V$).

Generally the peak-to-peak ripple voltage for n-phases is $\sqrt{2}V - \sqrt{2}V\cos\pi/n$. (see table 11.4)

The critical load inductance (see figure 12.12) for continuous load current, is $L_{critical} = \frac{R}{\tfrac{3}{2}\omega \times p\left(p^2-1\right)}$.

The output harmonics of a p-pulse voltage output are

$$\begin{aligned} V_{an} &= -1^{n/p} \times \frac{\sqrt{2}V}{\pi/p} \sin \pi/p \, \frac{2}{\left[n^2-1\right]} \\ &= -1^{n/p} \times V_o \times \frac{2}{\left[n^2-1\right]} \end{aligned} \qquad (11.104)$$

where $n = mp$ and $m = 1,\ 2,\ 3,\ \ldots$ and V_o is the mean output voltage given by equation (11.94).

The output voltage harmonics for $p = 6$ are given by

$$V_{o\,n} = \frac{6\,\hat{V}_L}{\pi\left(n^2-1\right)} \qquad (11.105)$$

for n = 6, 12, 18, ..

The rms output voltage is given by

$$\begin{aligned} V_{rms} &= \left(\frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} \sqrt{2}V_L \sin^2 \omega t \; d\omega t\right)^{1/2} \\ &= V_L\sqrt{1+\frac{3\sqrt{3}}{2\pi}} = 1.352 V_L \end{aligned} \qquad (11.106)$$

Generally, for a p-pulse rectifier output, the rms output voltage is

$$V_{rms} = V_L\sqrt{1+\frac{p}{2\pi}\sin 2\pi/p} \qquad (11.107)$$

The load voltage form factor = 1.352/1.35 = 1.001 and the ripple factor = √form factor -1 = 0.06.

11.2.2i Three-phase full-wave bridge rectifier circuit with continuous load current

If it is assumed that the load inductance is large, then (even with a load back emf), continuous load current flows and the dominate load current harmonic is due to the sixth harmonic current, that is let $I_{o,ac} = I_{o,6}$. By neglecting the higher order harmonics, the various circuit currents and voltages can be readily obtained as shown in table 11.3. From equations (11.103) and (11.105) the output voltage is given by

$$\begin{aligned} v_o(\omega t) &= \overline{V}_o \quad + \quad V_{o,6}\cos 6\omega t \\ &= \frac{3}{\pi}\sqrt{2}V_L + \frac{3}{\pi}\sqrt{2}V_L\,\frac{2}{\left(n^2-1\right)}\cos n\omega t \qquad \text{for } n = 6 \\ &= \frac{3}{\pi}\sqrt{2}V_L + \frac{3}{\pi}\sqrt{2}V_L \times \frac{2}{35}\cos 2\omega t \\ &= 1.35 V_L \;+\; 0.077 V_L \cos 2\omega t \end{aligned} \qquad (11.108)$$

The fundamental voltage, hence current, V_o/R, is therefore much larger than the sixth harmonic current, $V_{o,6} / Z_6$, that is $\overline{I}_o > \overline{I}_{o,6}$. The load and supply ac currents are $I_{o,\,ac} = I_{s,\,ac} = I_{o,\,6}$. The output and supply rms currents are

$$I_{o,\,rms} = I_{s,\,rms} = \sqrt{\overline{I}_o^2 + I_{o,\,ac}^2} = \sqrt{\overline{I}_o^2 + I_{o,\,6}^2} \qquad (11.109)$$

and the power delivered to resistance R in the load is

$$P_R = I_{o,rms}^2 R \qquad (11.110)$$

11.2.2ii Three-phase full-wave bridge circuit with highly inductive load – constant load current

For a highly inductive load, that is a constant load current the average output voltage and current are given by equation (11.103), the rms output voltage by equation (11.106), and:

- the mean diode current is

$$\overline{I}_D = \tfrac{1}{n}\overline{I}_o = \tfrac{1}{3}\overline{I}_o \qquad \text{(A)} \qquad (11.111)$$

- and the rms diode current is

$$I_{D\,rms} = \tfrac{1}{\sqrt{n}}\, I_{o\,rms} \approx \tfrac{1}{\sqrt{n}}\, \bar{I}_o = \tfrac{1}{\sqrt{3}}\, \bar{I}_o \qquad \text{(A)} \qquad (11.112)$$

- and the power factor for a constant load current is

$$pf = \frac{3}{\pi} = 0.995 \qquad (11.113)$$

The rms input line currents are

$$I_{L\,rms} = \sqrt{\frac{2}{3}}\, I_{o\,rms} \qquad (11.114)$$

The diode current form factor is

$$FF_{ID} = I_{D\,rms} \,/\, \bar{I}_D = \sqrt{3} \qquad (11.115)$$

The diode current ripple factor is

$$RF_{ID} = \sqrt{FF_{ID}^2 - 1} = \sqrt{2} \qquad (11.116)$$

A phase voltage and current are given by

$$v_a = \sqrt{2}\, V \sin\omega t \qquad (11.117)$$

$$i_a = \frac{2\sqrt{3}}{\pi}\, \bar{I}_o \left[\sin\omega t + \frac{\sin(n-1)\omega t}{n-1} + \frac{\sin(n+1)\omega t}{n+1} \right] \qquad n = 6, 12, 18, .. \qquad (11.118)$$

with phases b and c shifted by ⅔π. That is substitute *ωt* in equations (11.117) and (11.118) with *ωt*±⅔*π*.

Each load current harmonic *n* produces harmonics *n+1* and *n-1* on the input current.

The total load instantaneous power is given by

$$p(\omega t) = 3 \times \sqrt{2}\, V\, \bar{I}_o \times \left(\tfrac{1}{2} - \frac{\cos n\omega t}{n^2 - 1} \right) \qquad (11.119)$$

The supply apparent power is

$$S = \sqrt{3}\, V_L I_{s\,rms} \qquad (11.120)$$

while the ac power, in terms of apparent ac resistance, is

$$P_{ac} = 3 \times \frac{V^2}{R_{ac}} \qquad (11.121)$$

Using the output voltage from equation (11.103), the output power is

$$P_{dc} = \left(\sqrt{2} V \frac{\sqrt{3}}{\pi / 3} \right)^2 / \, R_{dc} \qquad (11.122)$$

Since $P_{ac} = P_{dc}$, then $R_{dc} = 2\left(\frac{9}{\pi^2}\right) R_{ac} \approx 2R_{ac}$.

At the ac input, for a constant load current:

The rms value of the fundamental line current

$$I_{L1,rms} = \frac{2\sqrt{3}}{\pi} I_o \,/\, \sqrt{2} = \frac{\sqrt{6}}{\pi} I_o = 0.78 I_o$$

The input distortion, *DF*, is

$$DF = \frac{I_{o,1}}{I_o} = \frac{\sqrt{6}/_{\pi}\, I_o}{\sqrt{2/3}\, I_o} = \frac{3}{\pi} = 0.955 \qquad (11.123)$$

The input power factor, with unity displacement power factor, is therefore

$$pf = DF \times DPF = \frac{3}{\pi} = 0.955 \qquad (11.124)$$

The input total harmonic distortion is

$$THD = \frac{\sqrt{I_L^2 - I_{L,1}^2}}{I_{Ls,1}} = \frac{\sqrt{\tfrac{2}{3} I_o^2 - \tfrac{6}{\pi^2} I_o^2}}{\sqrt{6}/_{\pi}\, I_o} = 0.3108 \qquad (11.125)$$

11.2.2iii Three-phase full-wave bridge circuit with highly inductive load with an EMF source

With continuous load current, the output voltage and input characteristics are unaffected by a load back emf, with the average and rms output voltages given by equations (11.103) and (11.106) respectively. The input power factor and distortion factor are 3/π, as per equation (11.125).

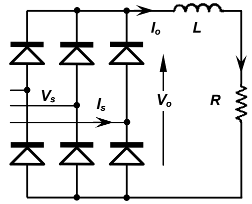
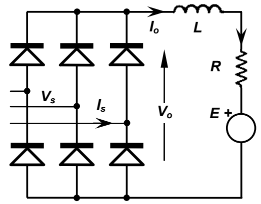
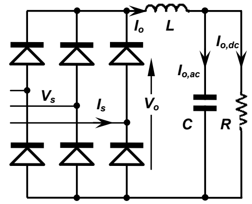
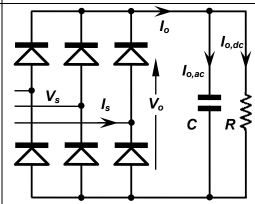
The output, that is, load current, is found from

$$L\frac{di_o}{dt} + Ri_o + E = \sqrt{2} V_s \sin\omega t \qquad \tfrac{1}{3}\pi \le \omega t \le \tfrac{2}{3}\pi$$

$$i_o(t) = I_o e^{-\frac{\omega t - \frac{1}{3}\pi}{\tan\phi}} + \frac{\sqrt{2} V_s}{Z}\left[\sin(\omega t - \phi) - \frac{\sin\alpha}{\cos\phi} \right] \qquad (11.126)$$

where $\tan\phi = \frac{\omega L}{R}$; $Z = \sqrt{R^2 + \omega^2 L^2}$; $\sin\alpha = \frac{E}{\sqrt{2} V_s}$; and $I_o = \frac{\sqrt{2} V_s}{Z} \frac{\sin\phi}{1 - e^{\frac{-\pi/3}{\tan\theta}}}$

Table 11.3: Three-phase full-wave uncontrolled rectifier circuits

Full-wave rectifier circuit		6th harmonic current	average output current	output power
		$I_{o,6}$	$\bar{I}_o$	$P_R + P_E$
load	circuit	(A)	(A)	(W)
(a) R-L *see section 11.2.2i*		$\frac{V_{o,6}}{\sqrt{R^2 + (6\omega L)^2}}$	$\frac{\bar{V}_o}{R}$	$I_{o,rms}^2 R$
(b) 11.2.2iii R-L-E		$\frac{V_{o,6}}{\sqrt{R^2 + (6\omega L)^2}}$	$\frac{\bar{V}_o - E}{R}$	$I_{o,rms}^2 R + \bar{I}_o E$
(c) R-L-C		$\frac{V_{o,6}}{6\omega L}$	$\frac{\bar{V}_o}{R}$	$I_{o,rms}^2 R = \bar{I}_o^2 R$
(d) 11.2.2iv R-C			$\frac{\bar{V}_o}{R}$	$I_{o,rms}^2 R$

11.2.2iv Three-phase full-wave bridge circuit with capacitively filtered load resistance

Part d in Table 11.1 shows a three-phase full-wave rectifier circuit with a parallel *R-C* load.

Interval $\alpha \le \omega t \le \beta$

In the interval $\alpha \le \omega t \le \beta$, two diodes are conducting connecting the supply voltage across the load. The input current provides both the resistive load and the output filter capacitor across the load.

$$i_s = i_o + i_C = \frac{v_o}{R} + C\frac{dv_o}{dt} \quad \text{where } v_o = V_s = \sqrt{2}V_s \sin\omega t \quad (V_s \text{ is the line-to-line voltage})$$

That is

$$i_s = i_o + i_C$$

$$\begin{aligned} i_s(t) &= \frac{\sqrt{2}V_s \sin\omega t}{R} + \sqrt{2}V_s\,\omega C\cos\omega t \\ &= \frac{\sqrt{2}V_s}{R}\sqrt{1+\omega^2C^2R^2}\cos(\omega t-\phi) = \frac{\sqrt{2}V_s}{R\cos\phi}\cos(\omega t-\phi) \end{aligned} \qquad (11.127)$$

where $\tan\phi = \frac{1}{\omega RC}$ and $\beta = ½\pi + \phi$

Interval $\beta \le \omega t \le \alpha + ⅓\pi$

In the interval $\beta \le \omega t \le \alpha + ⅓\pi$, the bridge diodes are all reverse biased, isolating the source from the load (discontinuous input current), and the load current is provided from the output capacitor.

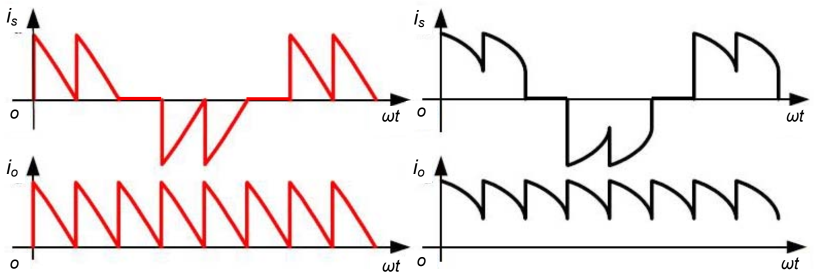
$$i_s = i_o + i_C = 0 = \frac{v_o}{R} + C\frac{dv_o}{dt}$$

In satisfying a boundary condition yields

$$\begin{aligned} v_o(t) &= v_C(t) = v_R(t) = \\ &= \sqrt{2}V_s\frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}e^{\frac{-(\omega t-\beta)}{\omega RC}} = \sqrt{2}V_s\frac{R\cos\phi}{X_C}\times e^{\frac{-(\omega t-\beta)}{\omega RC}} \\ &= i_o R \end{aligned} \qquad (11.128)$$

Equating the two output voltage expressions, equations (11.127) and (11.128), at the boundary $\omega t = \alpha + ⅓\pi$ yields an equation for determining α iteratively.

$$\sin\alpha = \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}e^{\frac{-(\alpha - ⅙\pi - \tan^{-1} 1/\omega RC)}{\omega RC}}$$



ωCR=√3 (a) ωCR<√3 (b)

Figure 11.14. *Three-phase full-wave bridge rectifier a capacitive output filter: (a) verge of discontinuous conduction and (b) continuous current conduction.*

Example 11.7: *Three-phase full-wave rectifier*

The full-wave three-phase dc rectifier in figure 11.13a has a three-phase 415V 50Hz source (240V phase), and a 10Ω, 50mH, series load. During the problem solution, verify that the only harmonic that need be considered is the sixth.

Determine

i. average output voltage and current
ii. rms load voltage and the ac output voltage
iii. rms load current hence power dissipated and supply power factor
iv. load power percentage error in assuming a constant load current
v. diode average and rms current requirements

Solution

i. From equation (11.103) the average output voltage and current are

$$V_o = I_oR = 1.35V_L = 1.35\times 415\text{V} = 560.45\text{V}$$

$$I_o = \frac{V_o}{R} = \frac{560.45V}{10\Omega} = 56.045\text{A}$$

ii. The rms load voltage is given by equation (11.106)

$$V_{rms} = 1.352V_L = 1.352\times 415\text{V} = 560.94\text{V}$$

The ac component across the load is

$$\begin{aligned} V_{ac} &= \sqrt{V_{rms}^2 - V_o^2} \\ &= \sqrt{560.94\text{V}^2 - 560.447\text{V}^2} = 23.52\text{V} \end{aligned}$$

iii. The rms load current is calculated from the harmonic currents, which are calculated from the harmonic voltages given by equation (11.105).

harmonic n	$V_n = \frac{6\hat{V}_L}{\pi(n^2-1)}$	$Z_n = \sqrt{R^2+(n\omega L)^2}$	$I_n = \frac{V_n}{Z_n}$	$½I_n^2$
0	***(560.45)***	10.00	56.04	***(3141.01)***
6	32.03	94.78	0.34	0.06
12	7.84	188.76	0.04	0.00
Note the 12th harmonic current is not significant			$I_o^2 + \sum ½I_n^2 =$	3141.07

The rms load current is

$$\begin{aligned} I_{rms} &= \sqrt{I_o^2 + \sum ½I_n^2} \\ &= \sqrt{3141.07} = 56.05\text{A} \end{aligned}$$

The power absorbed by the 10Ω load resistor is

$$P_L = I_{rms}^2R = 56.05\text{A}^2\times 10\Omega = 31410.7\text{W}$$

The supply power factor is

$$pf = \frac{P_L}{V_{rms}I_{rms}} = \frac{P_L}{\sqrt{3}V_LI_L} = \frac{31410.7\text{W}}{\sqrt{3}\times 415\text{V}\times\sqrt{\frac{2}{3}}\times 56.05\text{A}} = 0.955$$

This power factor of 0.955 is as predicted by equation (11.113), $3/\pi$, for a constant current load.

iv. The percentage output power error in assuming the load current is constant is given by

$$1 - \frac{\tilde{P}_L}{P_L} = 1 - \frac{I_o^2R}{I_{rms}^2R} = 1 - \frac{56.045\text{A}^2\times 10\Omega}{56.05\text{A}^2\times 10\Omega} = 1 - \frac{31410.1\text{W}}{31410.7\text{W}} \simeq 0\%$$

v. The diode average and rms currents are given by equations (11.111) and (11.112)

$$\bar{I}_D = ⅓\bar{I}_o = ⅓\times 56.045 = 18.7\text{A}$$

$$I_{D\,rms} = \tfrac{1}{\sqrt{3}}I_{o\,rms} = \tfrac{1}{\sqrt{3}}\times 56.05 = 23.4\text{A}$$

Example 11.8: *Rectifier average load voltage*

Derive a general expression for the average load voltage of a *p*-pulse rectifier.

Solution

Figure 11.15 defines the general output voltage waveform where *p* is the output pulse number per cycle of the ac supply. From the output voltage waveform

$$\begin{aligned} V_o &= \frac{1}{2\pi/p}\int_{-\pi/p}^{\pi/p}\sqrt{2}V\cos\omega t\; d\omega t \\ &= \frac{\sqrt{2}V}{2\pi/p}(\sin(\pi/p) - \sin(-\pi/p)) = \frac{\sqrt{2}V}{2\pi/p}2\sin(\pi/p) \end{aligned}$$

$$V_o = \frac{\sqrt{2}V}{\pi / p}\sin(\pi / p) \qquad (V)$$

where

for p = 2 for the single-phase (n = 1) full-wave rectifier in figure 11.8.
for p = 3 for the three-phase (n = 3) half-wave rectifier in figure 11.11.
for p = 6 for the three-phase (n = 3) full-wave rectifier in figure 11.13.

♣

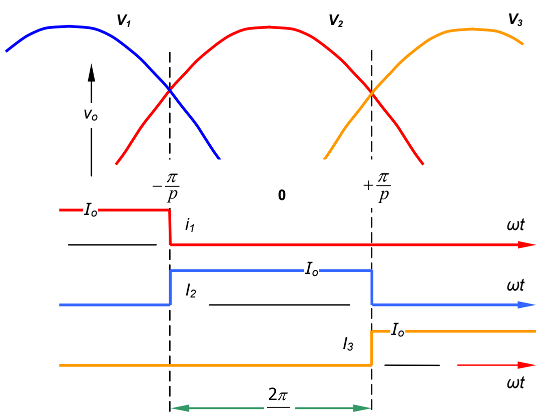


Figure 11.15. *A half-wave n-phase uncontrolled rectifier: output voltage and current waveforms.*

The output waveform smoothness, termed harmonic or ripple factor RF is defined by

$$\text{Ripple factor} = RF_v = \frac{\text{effective values of ac } V}{\text{average value of } V} = \frac{V_{ac}}{V_{dc}}$$

$$= \sqrt{\frac{V_{rms}^2 - V_{dc}^2}{V_{dc}^2}} = \sqrt{\frac{V_{rms}^2}{V_{dc}^2} - 1} = \sqrt{FF^2 - 1}$$

where $\qquad V_{ac} = \left[\sum_{n=1}^{\infty} \tfrac{1}{2}\left(V_{an}^2 + V_{bn}^2\right)\right]^{1/2}$

where FF is termed the form factor. RF_v is a measure of the voltage harmonics in the output voltage.

Ripple factors for constant output current rectifiers with different number of pulses, n

n	2	3	6	12	∞
%	48.2	18.27	4.18	0.994	0

11.3 DC *MMF*s in converter transformers

Half-wave rectification – whether controlled, semi-controlled or uncontrolled, is notorious for producing a dc *mmf* in transformers and triplen harmonics in the ac supply neutral of three-phase circuits. Generally, a transformer based solution can minimise the problem. In order to simplify the underlying concepts, a constant dc load current I_o is assumed, that is, the load inductance is assumed infinite. The transformer is assumed linear, no-load excitation is ignored, and the ac supply is assumed sinusoidal. Independent of the transformer and its winding connection, the average output voltage from a rectifier, when the rectifier bridge input rms voltage is V_B and there are q pulses in the output, is given by

$$V_o = \frac{\hat{V}_B}{2\pi / q}\int_{-\pi/q}^{\pi/q} \cos\omega t \, d\omega t = \hat{V}_B \frac{\sin \pi / q}{\pi / q} \qquad (11.129)$$

The rectifier bridge rms voltage output is dominated by the dc component and is given by

$$V_{o\,rms} = \frac{q}{2\pi}\int_{-\frac{\pi}{q}}^{+\frac{\pi}{q}} 2V_B^2 \cos^2(\omega t) \, d\omega t = V_B\sqrt{1 + \frac{q}{2\pi}\sin\frac{2\pi}{q}} \qquad (11.130)$$

The Fourier expression for the output voltage, which is also dominated by the dc component, is

$$v_o(\omega t) = V_o + V_o \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{k^2 n^2 - 1}\cos kn\omega t \qquad (11.131)$$

Table 11.4 summarizes the various rectifier characteristics that are independent of the transformer winding configuration.

Table 11.4: Rectifier characteristics with *q* phases (see section 11.6)

q phases	Parallel connected secondary windings		Series connected secondary windings
	Star, thus neutral always exists		Polygon, hence no neutral
	$v_1 = \sqrt{2}V\sin[\omega t]$ $v_2 = \sqrt{2}V\sin\left[\omega t - \frac{2\pi}{q}\right]$ . . $v_q = \sqrt{2}V\sin\left[\omega t - (q-1)\frac{2\pi}{q}\right]$		
	Half-wave	Full-wave	
V_o	$\frac{q}{\pi}\sqrt{2}V\sin\frac{\pi}{q}$	$2\frac{q}{\pi}\sqrt{2}V\sin\frac{\pi}{q}$	$\frac{q}{\pi}\sqrt{2}V$
Load harmonics	$n=q$	$n=q$ q even $n=2q$ q odd	$n=q$ q even $n=2q$ q odd
$\hat{V}_o$ $\check{V}_o$	$\sqrt{2}V$ $\sqrt{2}V\cos\frac{\pi}{q}$	$2\sqrt{2}V\cos\frac{\pi}{2q}$	
$\hat{V}_{DR}$	$\sqrt{2}V$ q even $\sqrt{2}V\cos\frac{\pi}{2q}$ q odd	$2\sqrt{2}V$ q even $2\sqrt{2}V\cos\frac{\pi}{2q}$ q odd	$\frac{\sqrt{2}V}{\sin\frac{\pi}{q}}$ q even $\frac{\sqrt{2}V}{2\sin\frac{\pi}{2q}}$ q odd
N° of diodes	q diodes	$2q$ diodes	$2q$ diodes
$\bar{I}_D$ $\quad I_{D\,rms}$	$\bar{I}_D = \frac{I_o}{q} \quad I_{D\,rms} = \frac{I_o}{\sqrt{q}}$		
I_s	$I_s = I_o\sqrt{\frac{1}{q}}$	$I_s = I_o\sqrt{\frac{2}{q}}$	$I_s = ½I_o$ q even $I_s = ½I_o\frac{\sqrt{q^2-1}}{q}$ q odd
$P_o = V_oI_o$ $S = qV_sI_s$ $pf_{load} = \frac{P_o}{S}$	$\frac{\sqrt{2q}}{\pi}\sin\frac{\pi}{q}$	$\frac{2\sqrt{q}}{\pi}\sin\frac{\pi}{q}$	$\frac{2\sqrt{2}}{\pi}$ q even $\frac{2\sqrt{2}}{\pi}\times\frac{q}{\sqrt{q^2-1}}$ q odd

11.3.1 Effect of multiple coils on multiple limb transformers

The transformer for a single-phase two-pulse half-wave rectifier has three windings, a primary and two secondary windings as shown in figure 11.16. Two possible transformer core and winding configurations are shown, namely shell and core. In each case the winding turns ratios are identical, as is the load voltage and current, but the physical transformer limb arrangements are different. One transformer, figure 11.16a, has three limbs (made up from E and I laminations), while the second, figure 11.16b, is made from a circular core (shown as a square core). The reason for the two possibilities is related to the fact that the circular core can use a single strip of wound cold-rolled grain-orientated silicon steel as lamination material. Such steels offer better magnetic properties than the non-oriented steel that must be used for E core laminations. Single-phase toroidal core transformers are attractive because of the reduced size and weight but manufacturers do not highlight their inherent limitation and susceptibility to dc flux biasing, particularly in half-wave type applications. Although the solution is simple, the advantageous features of the toroidal transformer are lost, as will be shown.

i. The E-I three-limb transformer (shell)

The key feature of the three-limb shell is that the three windings are on the centre limb, as shown in figure 11.16a. The area of each outer limb is half that of the central limb. Assuming a constant load current I_o and equal secondary turns, N_s, excitation of only the central limb yields the following *mmf* equation

$$mmf = i_p N_p + i_{s1} N_s - i_{s2} N_s \tag{11.132}$$

Thus the primary current i_p is

$$i_p = \frac{N_s}{N_p}\left(i_{s2} - i_{s1}\right) + \frac{mmf}{N_p} \tag{11.133}$$

From the waveforms in figure 11.16a, since $i_{s2} - i_{s1}$ is alternating, an average primary current of zero in equation (11.133) can only be satisfied by *mmf* = 0.

The various transformer voltages and currents are

$$\begin{aligned}
I_{s1} &= I_{s2} = I_s = \frac{I_o}{\sqrt{2}} \\
I_p &= I_o \frac{N_s}{N_p} \\
V_{s1} &= V_{s2} = V_s = V_p \frac{N_s}{N_p} = \frac{\pi}{2\sqrt{2}} V_o
\end{aligned} \tag{11.134}$$

Therefore the transformer input, output and average VA ratings are

$$\begin{aligned}
S_s &= V_{s1} I_{s1} + V_{s2} I_{s2} = \sqrt{2}\frac{N_s}{N_p} V_p I_o \left(= \frac{\pi}{2} P_o = 1.57 P_o \right) \\
S_p &= V_p I_p = \frac{N_s}{N_p} V_p I_o \left(= \frac{\pi}{2\sqrt{2}} P_o = 1.11 P_o \right) \\
\overline{S} &= ½\left(S_s + S_p\right) = \frac{N_p}{N_s} V_p I_o \frac{1+\sqrt{2}}{2}
\end{aligned} \tag{11.135}$$

The average output voltage, hence output power, are

$$\begin{aligned}
V_o &= \frac{2\sqrt{2}}{\pi} V_s = \frac{2\sqrt{2}}{\pi}\frac{N_s}{N_p} V_p = 0.9 \frac{N_s}{N_p} V_p \\
P_o &= I_o V_o
\end{aligned} \tag{11.136}$$

Thus

$$\overline{S} = \frac{1+\sqrt{2}}{4\sqrt{2}} \pi P_o = 1.34 P_o \tag{11.137}$$

Since the transformer primary current is the line current, the supply power factor is

$$pf = \frac{P_o}{S} = \frac{V_o I_o}{V_p I_p} = \frac{\frac{2}{\pi} V_s I_o}{\frac{N_p}{N_s} V_s \frac{N_s}{N_p} I_o} = \frac{2\sqrt{2}}{\pi} = 0.9 \tag{11.138}$$

ii. The two-limb strip core transformer

Figure 11.16b shows the windings equally split on each transformer leg. In practice the windings can all be on one leg and the primary is one coil, but separation as shown allows visual *mmf* analysis. The load and diode currents and voltages are the same as for the E-I core arrangement, as seen in the waveforms in figure 11.16b. The *mmf* analysis necessary to assess the primary currents and core flux, is based on analysing each limb.

$$\begin{aligned}
mmf_1 &= -i_p ½N_p + i_{s1} N_s \\
mmf_2 &= +i_p ½N_p + i_{s2} N_s \\
mmf_1 &= mmf_2 = mmf
\end{aligned} \tag{11.139}$$

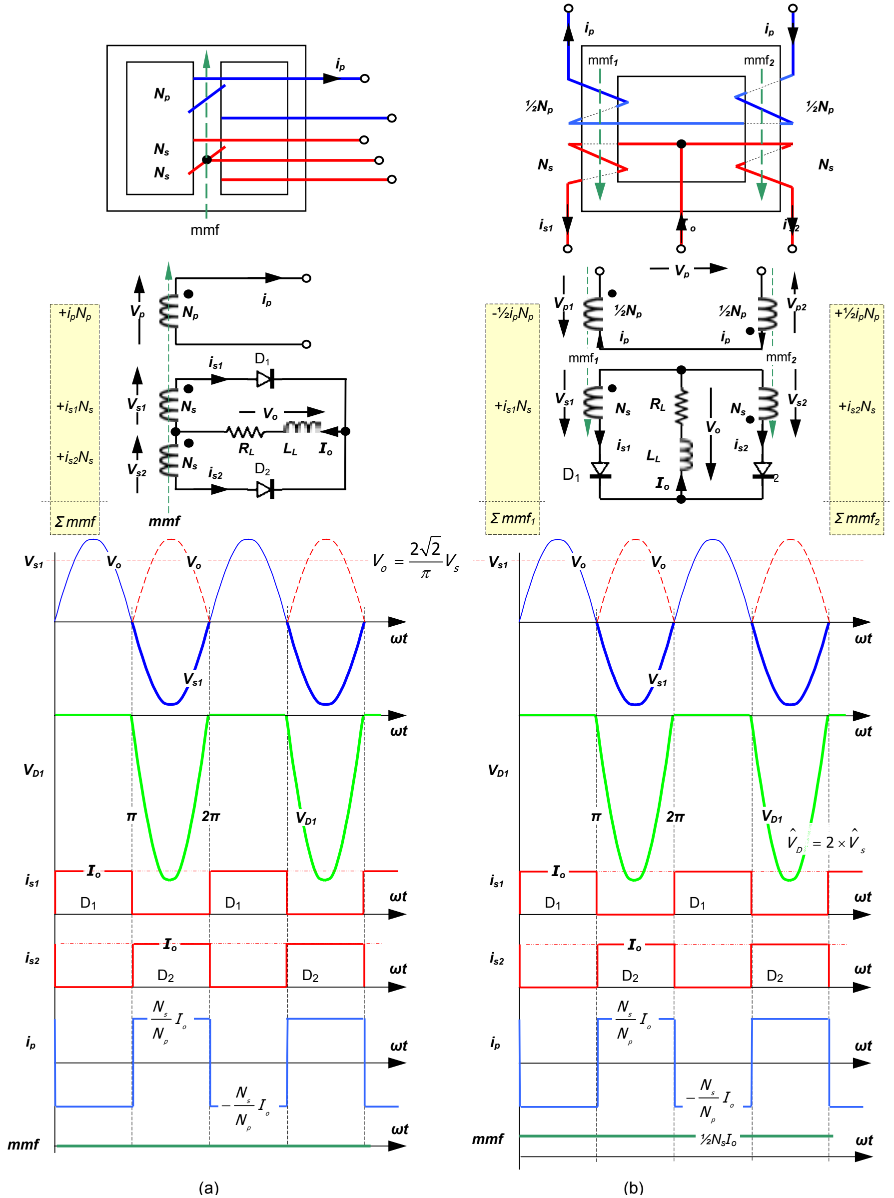


Figure 11.16. *Single-phase transformer core and winding arrangements: (a) E-I core with zero dc mmf bias and (b) square/circular core with dc mmf bias.*

These equations yield

$$mmf = N_s \tfrac{1}{2}(i_{s1} + i_{s2}) = N_s \tfrac{1}{2} I_o$$
$$i_p = \frac{N_s}{N_p}(i_{s1} - i_{s2}) \qquad (11.140)$$

These two equations are used every ac half cycle to obtain the plots in figure 11.16b. It will be noticed that the core has a magnetic *mmf* bias of $\tfrac{1}{2}N_sI_o$ associated with the half-wave rectification process.

The various transformer ratings are

$$I_{s1} = I_{s2} = I_s = \frac{I_o}{\sqrt{2}} \qquad I_p = I_o\frac{N_s}{N_p}$$
$$V_{p1} = V_{p2} = \tfrac{1}{2}V_p \qquad V_{s1} = V_{s2} = V_s = \tfrac{1}{2}V_p\frac{N_s}{\tfrac{1}{2}N_p} = V_p\frac{N_s}{N_p} \qquad (11.141)$$
$$V_p = \frac{N_p}{N_s}\frac{\pi}{2\sqrt{2}}V_o$$

Therefore the transformer VA ratings are

$$S_s = V_{s1}I_{s1} + V_{s2}I_{s2} = \sqrt{2}\frac{N_s}{N_p}V_pI_o \qquad \left(= \frac{\pi}{2}P_o = 1.57P_o\right)$$
$$S_p = V_{p1}I_p + V_{p2}I_p = \frac{N_s}{N_p}V_pI_o \qquad \left(= \frac{\pi}{2\sqrt{2}}P_o = 1.11P_o\right) \qquad (11.142)$$
$$\overline{S} = \tfrac{1}{2}(S_s + S_p) = \frac{1+\sqrt{2}}{2}\frac{N_s}{N_p}V_pI_o$$

The average output voltage, hence output power, are

$$V_o = \frac{2\sqrt{2}}{\pi}V_s = \frac{2\sqrt{2}}{\pi}\frac{N_s}{\tfrac{1}{2}N_p}\tfrac{1}{2}V_p = \frac{2\sqrt{2}}{\pi}\frac{N_s}{N_p}V_p \qquad (11.143)$$
$$P_o = I_oV_o$$

Thus

$$\overline{S} = \pi\frac{1+\sqrt{2}}{4\sqrt{2}}P_o = 1.34P_o \qquad (11.144)$$

and the supply power factor is $pf = P_o / \overline{S} = 0.9$.

The interpretation for equations (11.142) and (11.144) (and equations (11.135) and (11.137)) is that the transformer has to be oversized by 11% on the primary side and 57% on the secondary. From equation (11.144), in terms of the average VA, the transformer needs to be 34% larger than that implied by the rated dc load power. Further, the secondary is rated higher than the primary because of a dc component in the secondary. This core saturation aspect requires special attention when dimensioning the core size. Additionally, a component of the over rating requirement is due to circulating harmonics that do not contribute to real power output. This component is particularly relevant in three-phase delta primary or secondary connections when co-phasal triplens circulate. This discussion on apparent power aspects is relevant to all the transformer connections considered. Generally the higher the phase number the better the transformer core utilisation, but the poorer the secondary winding and rectifying diode utilisation since the percentage current conduction decreases with increased pulse number.

The fundamental ripple in the output voltage, at twice the supply frequency, is $\tfrac{2}{3}V_o$.

The two cores give the same rated transformer apparent power and supply power factor, but importantly, undesirably, the toroidal core suffers an *mmf* magnetic bias.

In each core case each diode conducts for 180º and

$$\overline{I}_D = \tfrac{1}{2}I_o \qquad I_{D\,rms} = \frac{I_o}{\sqrt{2}} \qquad \hat{V}_D = 2\sqrt{2}\frac{N_s}{N_p}V_p \qquad (11.145)$$

With a purely resistive load, a full-wave rectifier with a centre-tapped primary gives

$$\overline{I}_D = \tfrac{1}{2}I_o \qquad I_{D\,rms} = \tfrac{1}{4}\pi I_o \qquad S_s = 1.75P_o \qquad S_p = 1.23P_o \qquad \overline{S} = 1.49P_o \qquad (11.146)$$

11.3.2 Single-phase toroidal core mmf imbalance cancellation – zig-zag winding

In figure 11.17, each limb of the core has an extra secondary winding, of the same number of turns, N_s. *MMF* analysis of each limb in figure 11.17 yields

$$\text{limb1:} \quad mmf_o = -i_pN_p - i_{s2}N_s + i_{s1}N_s$$
$$\text{limb2:} \quad mmf_o = i_pN_p + i_{s2}N_s - i_{s1}N_s \qquad (11.147)$$

Adding the two mmf equations gives $mmf_o = 0$ and the resulting alternating primary current is given by

$$i_p = \frac{N_s}{N_p}(i_{s1} - i_{s2}) \qquad (11.148)$$

The transformer apparent and real power are rated by the same equation as for the previous winding arrangements, namely

$$\overline{S} = \tfrac{1}{2}(S_p + S_s) = \tfrac{1}{2}\left(\frac{\pi}{2\sqrt{2}}P_o + \frac{\pi}{2}P_o\right) = 1.34P_o$$
$$\text{where } P_o = V_oI_o \text{ and } V_o = \frac{N_s}{N_p}\frac{2\sqrt{2}}{\pi}V_p \qquad (11.149)$$

Since the transformer primary current is the ac line current, the supply power factor is $pf = P_o / \overline{S} = 0.9$.

The general rule to avoid any core dc *mmf* is, each core leg must be effectively excited by a net alternating current.

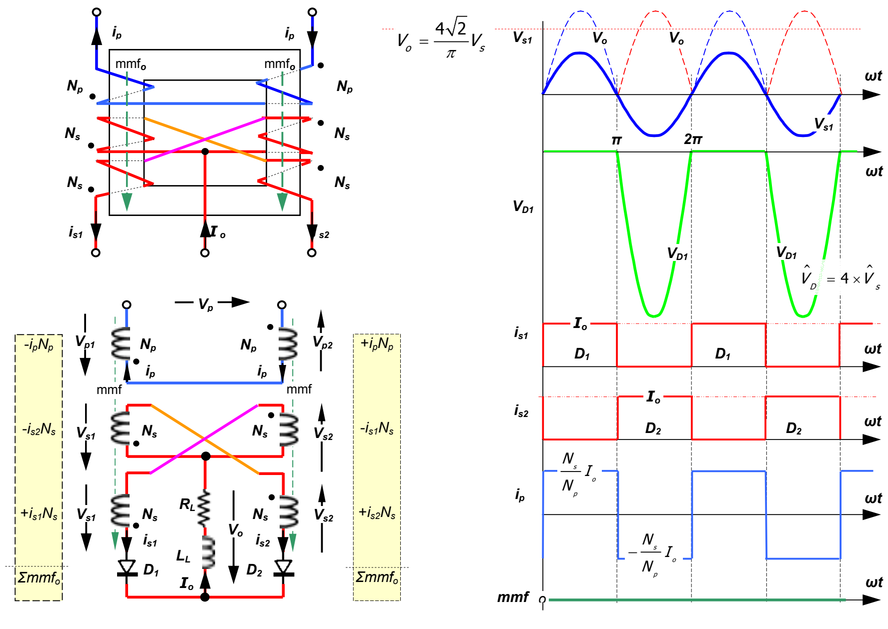


Figure 11.17. *Single-phase zig-zag transformer core and winding arrangement using square/circular core with zero dc mmf bias.*

11.3.3 Single-phase transformer connection, with full-wave rectification

The secondary current is ac with a zero average , thus no core mmf bias occurs.
The average output voltage and peak diode reverse voltage, in terms of the transformer secondary rms voltage, are

$$V_o = \frac{2\sqrt{2}}{\pi} V_s \qquad V_{Dr} = \sqrt{2} V_s \tag{11.150}$$

The rms output voltage is the bridge input rms voltage:

$$V_{o\,rms} = V_s \tag{11.151}$$

The various harmonic currents are

$$I_{s1} = \frac{2\sqrt{2}}{\pi} I_o = 0.9 I_o \qquad I_{sh} = \frac{I_{s1}}{h} \quad \text{for } h \text{ odd} \tag{11.152}$$

The power factor angle of the fundamental is unity, while the THD is 48.43%.

The transformer primary and secondary apparent powers are

$$S_p = S_s = \frac{\pi}{2\sqrt{2}} P_o = 1.11 P_o \tag{11.153}$$

The transformer average VAr rating is

$$\overline{S} = \frac{\pi}{2\sqrt{2}} P_o \tag{11.154}$$

Since the line current is the primary current, the supply power factor is

$$pf = \frac{P_o}{\overline{S}} = \frac{2\sqrt{2}}{\pi} \tag{11.155}$$

The fundamental ripple in the output voltage, at twice the supply frequency, is ⅔V_o.

With a purely resistive load (as opposed to a constant load current), a full-wave bridge rectifier gives

$$I_o = \frac{V_o}{R} \qquad I_{o\,rms} = \frac{V_s}{R}$$
$$\overline{I}_D = \tfrac{1}{2} I_o \qquad I_{D\,rms} = \tfrac{1}{4}\pi I_o \qquad S_s = 1.23 P_o \qquad S_p = 1.23 P_o \qquad \overline{S} = 1.23 P_o \tag{11.156}$$

11.3.4 Three-phase transformer connections

Basic three-phase transformers can have a combination of star (wye) and delta, primary and secondary winding arrangements. For a given line voltage and current, a wye connection reduces the phase winding voltage by √3, while a delta configuration reduces the phase winding current by √3.

i. Y - y (WYE-wye) is avoided due to imbalance and third harmonic problems, but with an extra delta winding, triplen problems can be minimised. The arrangement is used to interconnect high voltage networks, 240kV/345kV or when two neutrals are needed for grounding.

ii. Y - δ (WYE-delta) is commonly used for step-down voltage applications.

iii. Δ – δ (DELTA-delta) is used in 11kV medium voltage applications where neither primary nor neutral connection is needed.

iv. Δ - y (DELTA-wye) is used as a step-up transformer at the point of generation, before transmission.

Independent of the three-phase connection of the primary and secondary, for a balance three-phase load, the apparent power, VA, from the supply to the load is

$$S = \sqrt{3} V_{line} I_{line} = 3 V_{phase} I_{phase} \tag{11.157}$$

Also the sum of the primary and secondary line voltages is zero, that is

$$V_{AB} + V_{BC} + V_{CA} = 0$$
$$V_{ab} + V_{bc} + V_{ca} = 0 \tag{11.158}$$

where upper case subscripts refer to the primary and lower case subscripts refer to the secondary.

Y-y (WYE-wye) connection

Electrically, the *Y-y* transformer connection shown in figure 11.18, can be summarized as follows.

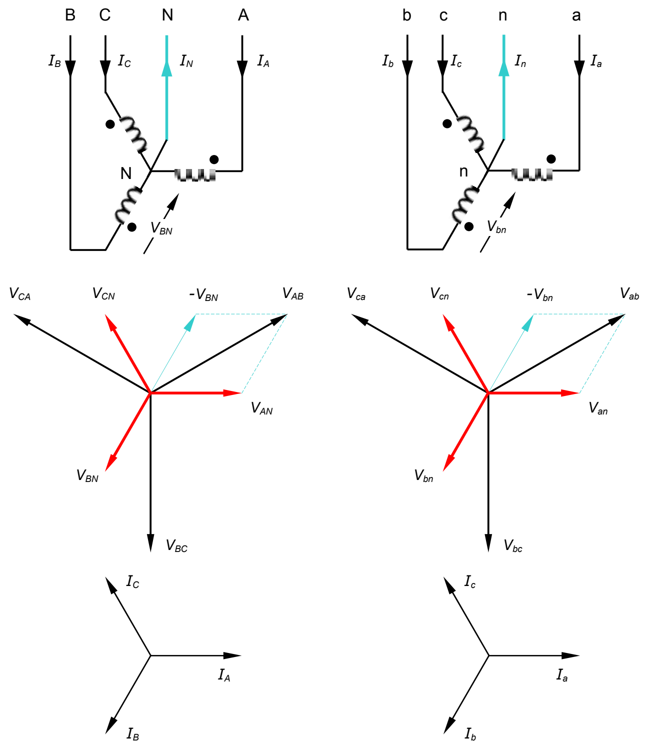
$$\eta_{Y-y} = \frac{N_p}{N_s} = \frac{V_{AN}}{V_{an}} = \frac{I_a}{I_A} = \frac{I_a}{I_{L1}} = \frac{V_{BN}}{V_{bn}} = \frac{I_b}{I_B} = \frac{V_{CN}}{V_{cn}} = \frac{I_c}{I_C} \tag{11.159}$$

$$\begin{aligned}
&V_{AB} = V_{AN} + V_{NB} = V_{AN} - V_{BN} = \sqrt{3} V_{AN} e^{j30^\circ} = \sqrt{3} V_{AN} \angle 30^\circ \\
&V_{BC} = V_{BN} + V_{NC} = V_{BN} - V_{CN} \quad V_{CA} = V_{CN} + V_{NA} = V_{CN} - V_{AN} \\
&V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} e^{j30^\circ} = \sqrt{3} V_{an} \angle 30^\circ \\
&V_{bc} = V_{bn} - V_{cn} \quad V_{ca} = V_{cn} - V_{an}
\end{aligned} \tag{11.160}$$

$$I_N = I_A + I_B + I_C \qquad I_n = I_a + I_b + I_c$$

The output current rating is (11.161)

$$I_Y = \frac{|S|/3}{V/\sqrt{3}} = \frac{|S|}{\sqrt{3} V}$$



(a) (b)

Figure 11.18. *Three-phase Y-y transformer: (a) winding arrangement and (b) phasor diagrams.*

Y- δ *(WYE-delta) connection*

The *Y*- δ transformer connection in figure 11.19 can be summarized as follows.

$$\eta_{Y-\delta} = \frac{N_p}{N_s} = \frac{V_{AN}}{V_{ab}} = \frac{I_{ba}}{I_A} = \frac{V_{BN}}{V_{bc}} = \frac{I_{cb}}{I_B} = \frac{V_{CN}}{V_{ca}} = \frac{I_{ac}}{I_C} \qquad (11.162)$$

$$V_{AB} = V_{AN} - V_{BN} = V_{AN} - V_{AN}e^{-j120^\circ} = \sqrt{3}\,V_{AN}e^{j30^\circ}$$

$$I_a = I_{ba} - I_{ac} = I_{ba} - I_{ba}e^{-j240^\circ} = \sqrt{3}\,I_{ba}\,e^{-j30^\circ}$$

$$I_a + I_b + I_c = 0$$

The output current rating is (11.163)

$$I_\Delta = \frac{|S|/3}{V} = \frac{|S|}{3V}$$

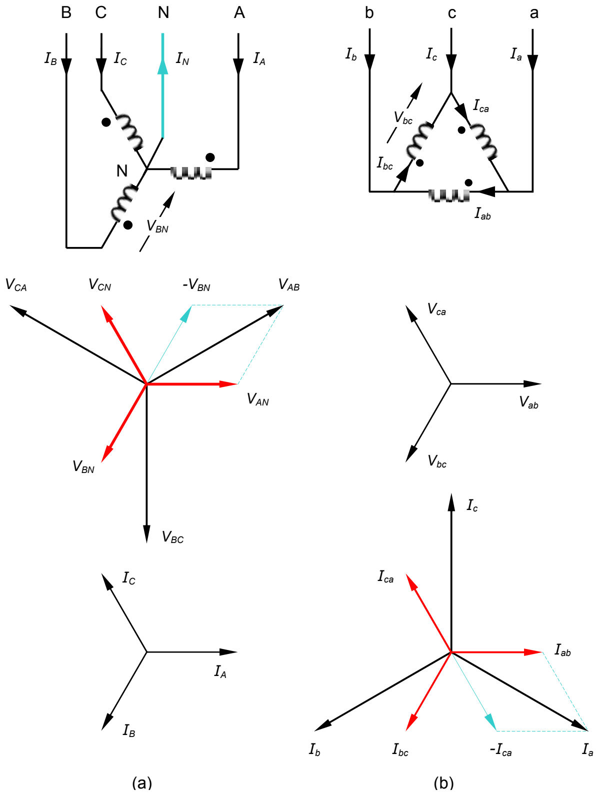


Figure 11.19. *Three-phase Y- δ transformer:*
(a) winding arrangement and (b) phasor diagrams.

Δ-δ (DELTA-delta) connection

In figure 11.20, the Δ - δ transformer connection can be summarized as follows.

$$\eta_{\Delta-\delta} = \frac{N_p}{N_s} = \frac{V_{AB}}{V_{ab}} = \frac{I_a}{I_A} = \frac{I_{ba}}{I_{AB}} = \frac{V_{BC}}{V_{bc}} = \frac{I_b}{I_B} = \frac{I_{cb}}{I_{BC}} = \frac{V_{CA}}{V_{ca}} = \frac{I_c}{I_C} = \frac{I_{cb}}{I_{CA}} \qquad (11.164)$$

$$I_A = I_{AB} - I_{CA} = \sqrt{3}\,I_{AB}e^{-j30^\circ} = \sqrt{3}I_{AB}\angle -30^\circ$$
$$I_B = I_{BC} - I_{AB} \qquad I_C = I_{CA} - I_{BC}$$
$$I_A + I_B + I_C = 0 \qquad (11.165)$$
$$I_a = I_{ab} - I_{ca} = \sqrt{3}\,I_{ab}e^{-j30^\circ} = \sqrt{3}I_{ab}\angle -30^\circ$$
$$I_b = I_{cb} - I_{ba} \qquad I_c = I_{ac} - I_{cb}$$
$$I_c + I_b + I_c = 0$$

The output current rating is

$$I_Y = \frac{|S|/3}{V/\sqrt{3}} = \frac{|S|}{\sqrt{3}V} \qquad (11.166)$$

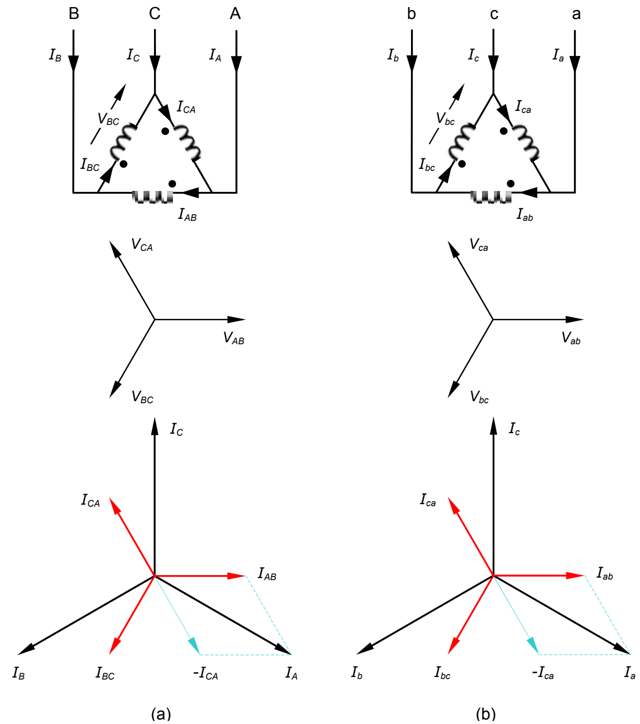


Figure 11.20. *Three-phase Δ-δ transformer:*
(a) winding arrangement and (b) phasor diagrams.

Δ-y (DELTA-wye) connection

The Δ-*y* transformer connection in figure 11.21 can be summarized as follows.

$$\eta_{\Delta-y} = \frac{V_{AB}}{V_{ab}} = \frac{V_{AB}}{V_{ab}}\frac{e^{-j30^\circ}}{\sqrt{3}} = \frac{V_{AN}}{V_{an}} = \frac{I^*_a}{I^*_A} = \frac{I^*_a}{\left(\sqrt{3}\,I_{AB}e^{-j30^\circ}\right)^*} \qquad (11.167)$$

$$I_A = I_{AB} - I_{CA} = I_{AB} - I_{AB}e^{-j240^\circ} = \sqrt{3}\,I_{an}e^{-j30^\circ}$$

The output current rating is

$$I_Y = \frac{|S|/3}{V/\sqrt{3}} = \frac{|S|}{\sqrt{3}V} \qquad (11.168)$$

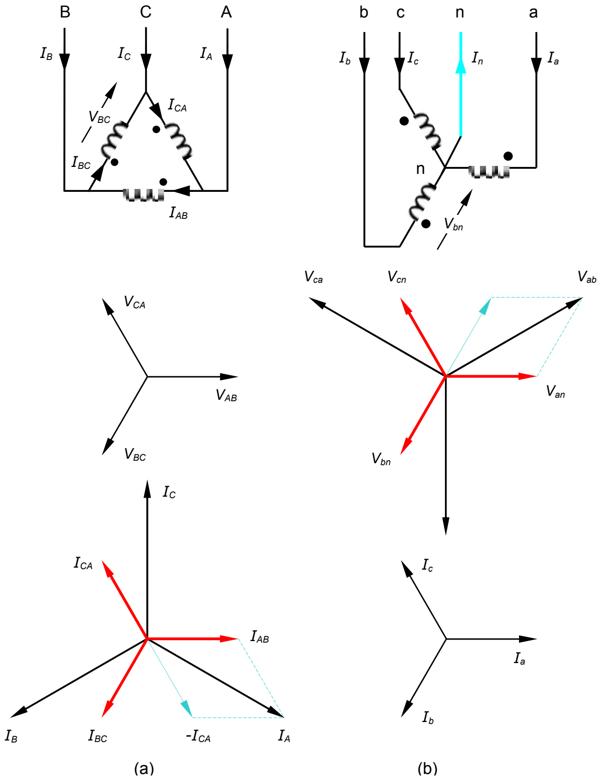


Figure 11.21. *Three-phase Δ-y transformer: (a) winding arrangement and (b) phasor diagrams.*

11.3.5 Three-phase transformer, half-wave rectifiers - core mmf imbalance

A delta secondary connection cannot be used for half-wave rectification since no physical neutral connection exists.

i. Star connected primary Y-y (WYE-wye)

The three-phase half-wave rectifier with a star-star connected transformer in figure 11.22a is prone to magnetic *mmf* core bias. With a constant load current I_o, each diode conducts for 120º. Each leg is analysed on an *mmf* basis, and the current and *mmf* waveforms in figure 11.22a are derived as follows.

$$\begin{aligned} mmf_o &= N_s i_{s1} - N_p i_{p1} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \\ mmf_o &= N_s i_{s3} - N_p i_{p3} \end{aligned} \qquad (11.169)$$

By symmetry and balance, the *mmf* in each leg must be equal.

If i_N is the neutral current then the equation for the currents is

$$i_{p1} + i_{p2} + i_{p3} = i_N \qquad (11.170)$$

The same *mmf* equations are obtained if the load is purely resistive.

Any triplens in the primary will add algebraically, while any other harmonics will vectorially cancel to zero. Therefore the neutral may only conduct primary side triplen currents. Any input current harmonics are due to the rectifier and the rectifier harmonics of the order $h = cp \pm 1$ where $c = 0,1,2,\ldots$ and p is the pulse number, 3. No secondary-side third harmonics can exist hence $h \neq 3k$ for $k = 1, 2, 3,$. Therefore no primary-side triplen harmonic currents exist to flow in the neutral, that is $i_N = 0$. In a balanced load condition, the neutral connection is redundant. The system equations resolve to

$$\begin{aligned} i_{p1} &= \frac{N_s}{N_p}\left(\tfrac{2}{3}i_{s1} - \tfrac{1}{3}i_{s2} - \tfrac{1}{3}i_{s3}\right) \\ i_{p2} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3}i_{s1} + \tfrac{2}{3}i_{s2} - \tfrac{1}{3}i_{s3}\right) \\ i_{p3} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3}i_{s1} - \tfrac{1}{3}i_{s2} + \tfrac{2}{3}i_{s3}\right) \\ mmf_o &= N_s\left(\frac{i_{s1} + i_{s2} + i_{s3}}{3}\right) = \tfrac{1}{3}N_s I_o \end{aligned} \qquad (11.171)$$

Specifically, the core has an mmf dc bias of $\tfrac{1}{3}N_s I_o$.

Waveforms satisfying these equations are show plotted in figure 11.22a. The various transformer currents and voltages are

$$\begin{aligned} I_{s1} &= I_{s2} = I_{s3} = I_s = \frac{I_o}{\sqrt{3}} \\ I_{p1} &= I_{p2} = I_{p3} = I_p = \frac{\sqrt{2}}{3}\frac{N_s}{N_p}I_o \\ V_{p1} &= V_{p2} = V_{p3} = V_p = V_o\frac{N_p}{N_s}\frac{2\pi}{3\sqrt{6}} \\ V_{s1} &= V_{s2} = V_{s3} = V_s = V_o\frac{2\pi}{3\sqrt{6}} = \frac{V_o}{1.17} \end{aligned} \qquad (11.172)$$

The fundamental ripple in the output voltage, at three times the supply frequency, is ¼V_o.

The various transformer VA ratings are

$$\begin{aligned} S_s &= V_{s1}I_{s1} + V_{s2}I_{s2} + V_{s3}I_{s3} = 3V_sI_s = \frac{\pi\sqrt{2}}{3}P_o = 1.48P_o \\ S_p &= V_{p1}I_p + V_{p2}I_p + V_{p3}I_p = 3V_pI_p = \frac{2}{3\sqrt{3}}P_o = 1.21P_o \\ \overline{S} &= ½\left(S_s + S_p\right) = P_o\frac{2 + \pi\sqrt{6}}{3\sqrt{3}} = 1.34P_o \end{aligned} \qquad (11.173)$$

The average output power is

$$P_o = I_oV_o \qquad (11.174)$$

Since with a wye connected transformer primary, the transformer primary phase current is the line current, the supply power factor is

$$pf = \frac{P_o}{\overline{S}} = \frac{V_oI_o}{3V_pI_p} = \frac{\frac{3}{\pi}\sqrt{2}V_s\frac{\sqrt{3}}{2}I_o}{3\frac{N_p}{N_s}V_s\frac{\sqrt{3}}{2}\frac{N_s}{N_p}I_o} = 0.827 \qquad (11.175)$$

Although the neutral connection is redundant for a constant load current, the situation is different if the load current has ripple at the three times the rectified ac frequency, as with a resistive load. Equations in (11.171) remain valid for the untapped neutral case. In such a case, when triplens exist in the load current, how they are reflected into the primary depends on whether or not the neutral is connected:

- No neutral connection – a triplen *mmf* is superimposed on the *mmf* dc bias of ⅓N_sI_o.
- Neutral connected – a dc current (zero sequence) flows in the neutral and the associated zero sequence line currents in the primary, oppose the generation of any triplen *mmf* onto the dc *mmf* bias of ⅓N_sI_o.

ii. Delta connected primary Δ-y (DELTA-wye)

The three-phase half-wave rectifier with a delta-star connected transformer in figure 11.18b is prone to magnetic *mmf* core bias. With a constant load current I_o each diode conducts for 120°. Each leg is analysed on an *mmf* basis, and the current and *mmf* waveforms in figure 11.22b are derived as follows.

$$\begin{aligned} mmf_o &= N_s i_{s1} - N_p i_{p1} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \\ mmf_o &= N_s i_{s3} - N_p i_{p3} \end{aligned} \qquad (11.176)$$

$$i_{L1} = i_{p1} - i_{p3} \qquad i_{L2} = i_{p2} - i_{p1} \qquad i_{L3} = i_{p3} - i_{p2}$$

The line-side currents have average values of zero and if it is assumed that the core *mmf* has only a dc component, that is no alternating component, then based on these assumptions

$$mmf_o = N_s\left(\frac{i_{s1} + i_{s2} + i_{s3}}{3}\right) = \frac{1}{3}N_sI_o \qquad (11.177)$$

The primary currents are then

$$\begin{aligned} i_{p1} &= \left(i_{s1} - \frac{1}{3}I_o\right)\frac{N_s}{N_p} = \frac{N_s}{N_p}\left(\frac{2}{3}i_{s1} - \frac{1}{3}i_{s2} - \frac{1}{3}i_{s3}\right) \\ i_{p2} &= \left(i_{s2} - \frac{1}{3}I_o\right)\frac{N_s}{N_p} = \frac{N_s}{N_p}\left(-\frac{1}{3}i_{s1} + \frac{2}{3}i_{s2} - \frac{1}{3}i_{s3}\right) \\ i_{p3} &= \left(i_{s3} - \frac{1}{3}I_o\right)\frac{N_s}{N_p} = \frac{N_s}{N_p}\left(-\frac{1}{3}i_{s1} - \frac{1}{3}i_{s2} + \frac{2}{3}i_{s3}\right) \end{aligned} \qquad (11.178)$$

These line-side equations are the same as for the star connected primary, hence the same real and apparent power equations are also applicable to the delta connected primary transformer, viz. equations (11.173) and (11.174).

The line currents are

$$\begin{aligned} i_{L1} &= \frac{N_s}{N_p}\left(i_{p1} - i_{p3}\right) \\ i_{L2} &= \frac{N_s}{N_p}\left(i_{p2} - i_{p1}\right) \\ i_{L3} &= \frac{N_s}{N_p}\left(i_{p3} - i_{p2}\right) \end{aligned} \qquad (11.179)$$

The waveforms for these equations are shown plotted in figure 11.22b, where

$$I_p = \frac{N_s}{N_p}\frac{\sqrt{3}}{2}I_o \text{ and } I_L = \frac{N_s}{N_p}\sqrt{\frac{3}{2}}I_o \qquad (11.180)$$

that is $I_L = \sqrt{3}\, I_p$

$$\overline{S} = 1.34P_o \qquad (11.181)$$

The supply power factor is

$$pf = \frac{V_oI_o}{\sqrt{3}V_pI_L} == 0.827 \qquad (11.182)$$

Although with a delta connected primary, the ac supply line currents are not the transformer primary currents, the supply power factor is the same as a star primary connection since the proportions of the input harmonics are the same.

The rms output voltage is

$$V_{o\,rms} = \sqrt{2}V_s\sqrt{\frac{3}{2\pi}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)} \qquad (11.183)$$

Each diode conducts for 120º and

$$\overline{I}_D = ⅓\, I_o \qquad I_{D\,rms} = \frac{I_o}{\sqrt{3}} \qquad \hat{V}_D = \sqrt{6}\frac{N_s}{N_p}V_p \qquad (11.184)$$

The primary connection, delta or wye, does not influence any ***dc*** mmf generated in the core, although the primary connection does influence if an ***ac*** mmf results.

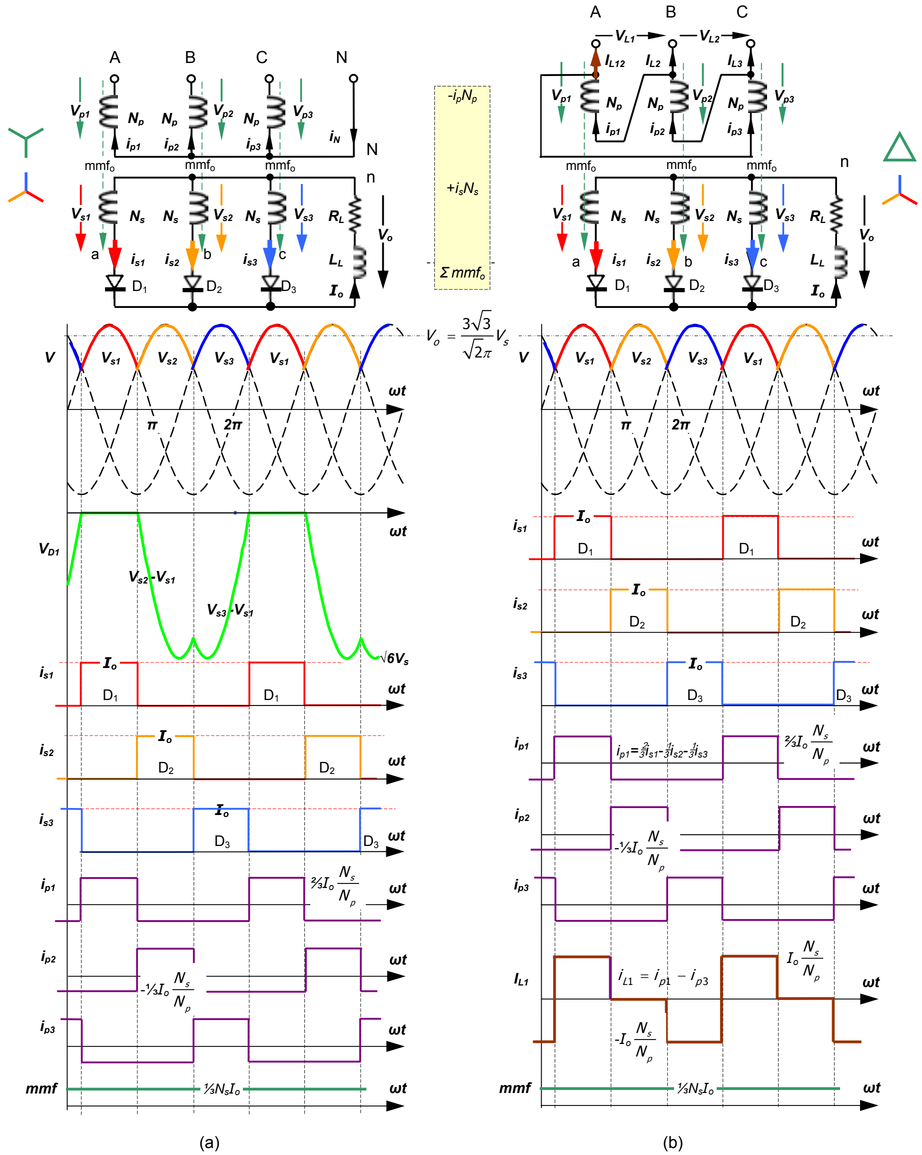


Figure 11.22. *Three-phase transformer winding arrangement with dc mmf bias: (a) star connected primary and (b) delta connected primary.*

11.3.6 Three-phase transformer with hexa-phase rectification, mmf imbalance

Figure 11.23 shown a tri-hexaphase half-wave rectifier, which can employ a wye or delta primary configuration, but only a star secondary connection is possible, since a neutral connection is required. The primary configuration can be shown to dictate core mmf bias conditions.

i. Y-y (WYE-wye) connection

The mmf balance for the wye primary connection in figure 11.23a is

$$\begin{aligned} &N_s i_{s1} - N_s i_{s4} - N_p i_{p1} = 0 \\ &N_s i_{s3} - N_s i_{s6} - N_p i_{p2} = 0 \\ &N_s i_{s5} - N_s i_{s2} - N_p i_{p3} = 0 \\ &i_{p1} + i_{p2} + i_{p3} = 0 \end{aligned} \qquad (11.185)$$

The primary currents expressed in terms of the secondary current are

$$\begin{aligned} i_{p1} &= \frac{N_s}{N_p}\left(\tfrac{2}{3} i_{s1} + \tfrac{1}{3} i_{s2} - \tfrac{1}{3} i_{s3} - \tfrac{2}{3} i_{s4} - \tfrac{1}{3} i_{s5} + \tfrac{1}{3} i_{s6}\right) \\ i_{p2} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3} i_{s1} + \tfrac{1}{3} i_{s2} + \tfrac{2}{3} i_{s3} + \tfrac{1}{3} i_{s4} - \tfrac{1}{3} i_{s5} - \tfrac{2}{3} i_{s6}\right) \\ i_{p3} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3} i_{s1} - \tfrac{2}{3} i_{s2} - \tfrac{1}{3} i_{s3} + \tfrac{1}{3} i_{s4} - \tfrac{2}{3} i_{s5} - \tfrac{1}{3} i_{s6}\right) \\ mmf &= N_s \frac{1}{3}\left(i_{s1} - i_{s2} + i_{s3} - i_{s4} + i_{s5} - i_{s6}\right) \end{aligned} \qquad (11.186)$$

These line side equations are plotted in figure 11.23a. Notice that an alternating mmf exists in the core related to the pulse frequency, $n = 2q = 6$.

The transformer primary currents and the line currents are

$$\begin{aligned} i_p &= \frac{\sqrt{2}}{3} I_o \\ i_L &= \frac{\sqrt{2}}{3} I_o \end{aligned} \qquad (11.187)$$

Note that because of the zero sequence current, triplens, in the delta primary that

$$i_L = \sqrt{2}\, i_p \quad \text{not} \quad i_L = \sqrt{3}\, i_p \qquad (11.188)$$

The transformer power ratings are

$$\begin{aligned} S_s &= 6\left(\frac{\pi}{3\sqrt{2}} V_o\right)\left(\frac{1}{\sqrt{6}} I_o\right) = \frac{\pi}{\sqrt{3}} P_o \\ S_p &= 3\left(\frac{\pi}{3\sqrt{2}} V_o\right)\left(\frac{\sqrt{2}}{3} I_o\right) = \frac{\pi}{3} P_o \\ \overline{S} &= ½\left(\frac{\pi}{\sqrt{3}} P_o + \frac{\pi}{3} P_o\right) = \frac{\pi}{6}\left(\sqrt{3}+1\right) P_o = 1.43 P_o \end{aligned} \qquad (11.189)$$

ii. Δ-y (DELTA-wye) connection

When the primary is delta connected, as shown in figure 11.23b, the mmf equations are the same as with a wye primary, namely

$$\begin{aligned} &N_s i_{s1} - N_s i_{s4} - N_p i_{p1} = 0 \\ &N_s i_{s3} - N_s i_{s6} - N_p i_{p2} = 0 \\ &N_s i_{s5} - N_s i_{s2} - N_p i_{p3} = 0 \end{aligned} \qquad (11.190)$$

but Kirchhoff's electrical current equation becomes of the following form for each phase:

$$mmf = \frac{1}{2\pi}\int_0^{2\pi} N_s\left(i_{s1} - i_{s4}\right) d\omega t = 0 \qquad (11.191)$$

Thus since each limb experiences an alternating current, similar to i_{s1} - i_{s4} for each limb, with an average value of zero, the line currents can be calculated from

$$i_{p1} = \frac{N_s}{N_p}\left(i_{s1} - i_{s4}\right) \qquad i_{p2} = \frac{N_s}{N_p}\left(i_{s3} - i_{s6}\right) \qquad i_{p3} = \frac{N_s}{N_p}\left(i_{s5} - i_{s2}\right) \qquad (11.192)$$

The line currents are

$$\begin{aligned} i_{L1} &= i_{p1} - i_{p3} = \frac{N_s}{N_p}\left(i_{s1} + i_{s2} - i_{s4} - i_{s5}\right) \\ i_{L2} &= i_{p2} - i_{p1} = \frac{N_s}{N_p}\left(-i_{s1} + i_{s3} + i_{s4} - i_{s6}\right) \\ i_{L3} &= i_{p3} - i_{p2} = \frac{N_s}{N_p}\left(-i_{s2} - i_{s3} + i_{s5} + i_{s6}\right) \end{aligned} \qquad (11.193)$$

The transformer primary currents and the line currents are

$$\begin{aligned} i_p &= \frac{1}{\sqrt{3}} I_o \\ i_L &= \sqrt{\frac{2}{3}}\, I_o \end{aligned} \qquad (11.194)$$

The transformer power ratings (which are relatively poor) are

$$\begin{aligned} S_s &= 6\left(\frac{\pi}{3\sqrt{2}} V_o\right)\frac{I_o}{\sqrt{6}} = \frac{\pi}{\sqrt{3}} P_o = 1.81 P_o \\ S_p &= 3\left(\frac{\pi}{3\sqrt{2}} V_o\right)\frac{I_o}{\sqrt{3}} = \frac{\pi}{\sqrt{6}} P_o = 1.28 P_o \\ \overline{S} &= ½\left(\frac{\pi}{\sqrt{3}} P_o + \frac{\pi}{\sqrt{6}} P_o\right) = ½\frac{\pi}{\sqrt{3}}\left(1 + \frac{1}{\sqrt{2}}\right) P_o = 1.55 P_o \end{aligned} \qquad (11.195)$$

The same primary and secondary apparent powers result for a purely resistive load.

The supply power factor is $pf = 3/\pi = 0.955$.

Independent of the primary connection, the average output voltage is

$$V_o = \frac{3\sqrt{2}}{\pi} V_s \qquad (11.196)$$

and the rms output voltage is

$$V_{o\,rms} = \sqrt{2}\, V_s \sqrt{\frac{6}{2\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} \qquad (11.197)$$

The diode average and rms currents are

$$I_D = \frac{I_o}{6} \qquad I_{D\,rms} = \frac{I_o}{\sqrt{6}} \qquad (11.198)$$

The maximum diode reverse voltage is

$$V_{Dr} = 2\sqrt{2}\, V_s \qquad (11.199)$$

The line currents are added to the waveforms in figure 11.23a and are also shown in figure 11.11b. The core mmf bias is zero, without any ac component associated with the 6-pulse rectification process. Zero sequence, triplen currents, can flow in the delta primary connection. A star connected primary is therefore not advisable.

If a single-phase inter-wye transformer is used between the neutrals of the two star rectifier groups, the transformer apparent power factors improve significantly, to

$$S_s = 1.48 P_o \quad S_p = 1.05 P_o \text{ giving } \overline{S} = 1.26 P_o \qquad (11.200)$$

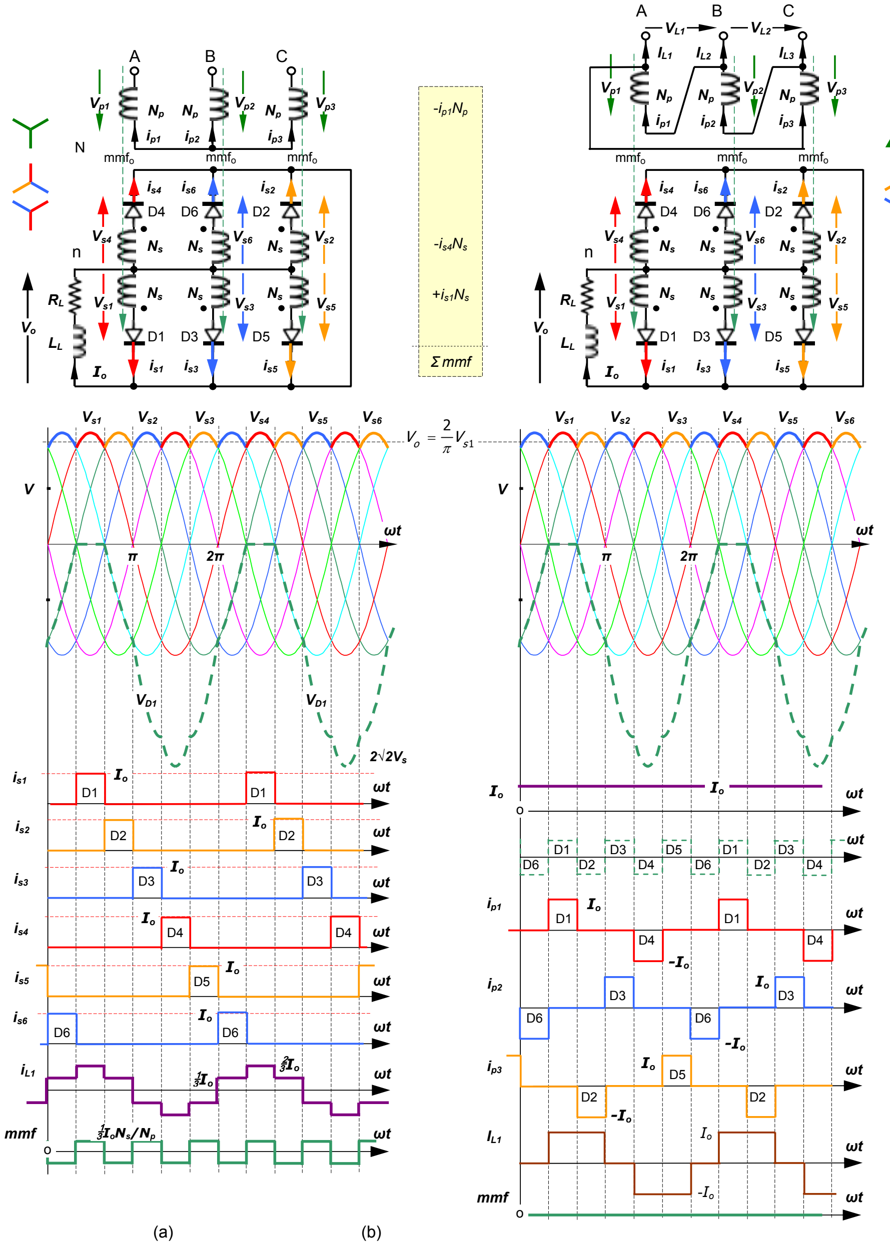


Figure 11.23. *Three-phase transformer winding arrangement with hexa-phase rectification: (a) star connected primary with dc mmf bias and (b) delta connected primary. (the transformer secondary and diode currents are the same in each case)*

11.3.7 Three-phase transformer mmf imbalance cancellation – zig-zag winding

In figures 11.24a and 11.25a, for balanced input currents and equal turns number N_s in the six windings

$$N_s\left(I_{aa'} + I_{nc'}\right) = N_s\left(I_{an} - I_{cn}\right)$$

whence (11.201)

$$N_s\left(I_{an} - I_{cn}\right) = \sqrt{3}\,N_s I_{an}\angle -30^\circ$$

If the same windings were connected in series in a Y configuration the mmf would be $2NI_{an}$. Therefore 1.15 times more turns (2/√3) are needed with the zig-zag arrangement in order to produce the same mmf.

Similarly for the output voltage, when compared to the same windings used in series in a Y secondary configuration:

$$\begin{aligned} V_{na} &= V_{na'} + V_{a'a} \\ &= -V_{a'n} + V_{a'a} \\ &= \sqrt{3}\,V_{a'a}\angle 30^\circ \end{aligned} \qquad (11.202)$$

That is, for a given line to neutral voltage, 1.15 times as many turns are needed as when Y connected.

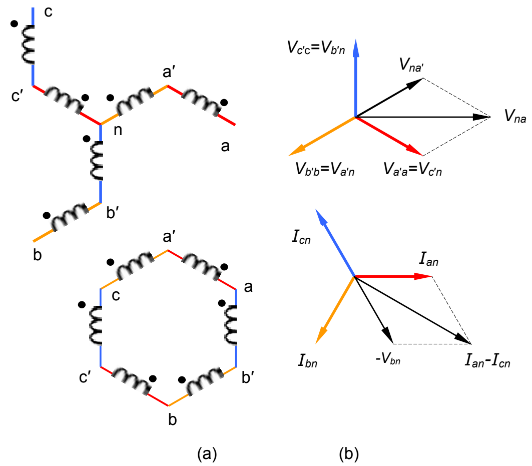


Figure 11.24. *Three-phase transformer secondary zig-zag winding arrangement: (a) secondary windings and (b) current and voltage phasors for the fork case.*

i. star connected primary Y-z (WYE-zigzag)

In figure 11.25, each limb of the core has an extra secondary winding, of the same number of secondary turns, N_s.

MMF analysis of each of the three limbs yields

$$\begin{aligned} &\text{limb 1:-} \quad mmf_o = -i_{p1}N_p + i_{s1}N_s - i_{s3}N_s \\ &\text{limb 2:-} \quad mmf_o = -i_{p2}N_p + i_{s2}N_s - i_{s1}N_s \\ &\text{limb 3:-} \quad mmf_o = -i_{p2}N_p + i_{s3}N_s - i_{s2}N_s \end{aligned} \qquad (11.203)$$

$$i_{p1} + i_{p2} + i_{p3} = 0$$

Adding the three *mmf* equations gives $mmf_o = 0$ and the alternating primary (and line) currents are

$$i_{p1} = \frac{N_s}{N_p}\left(i_{s1} - i_{s3}\right) \qquad i_{p2} = \frac{N_s}{N_p}\left(i_{s2} - i_{s1}\right) \qquad i_{p3} = \frac{N_s}{N_p}\left(i_{s3} - i_{s2}\right) \qquad (11.204)$$

These equations are plotted in figure 11.25a.

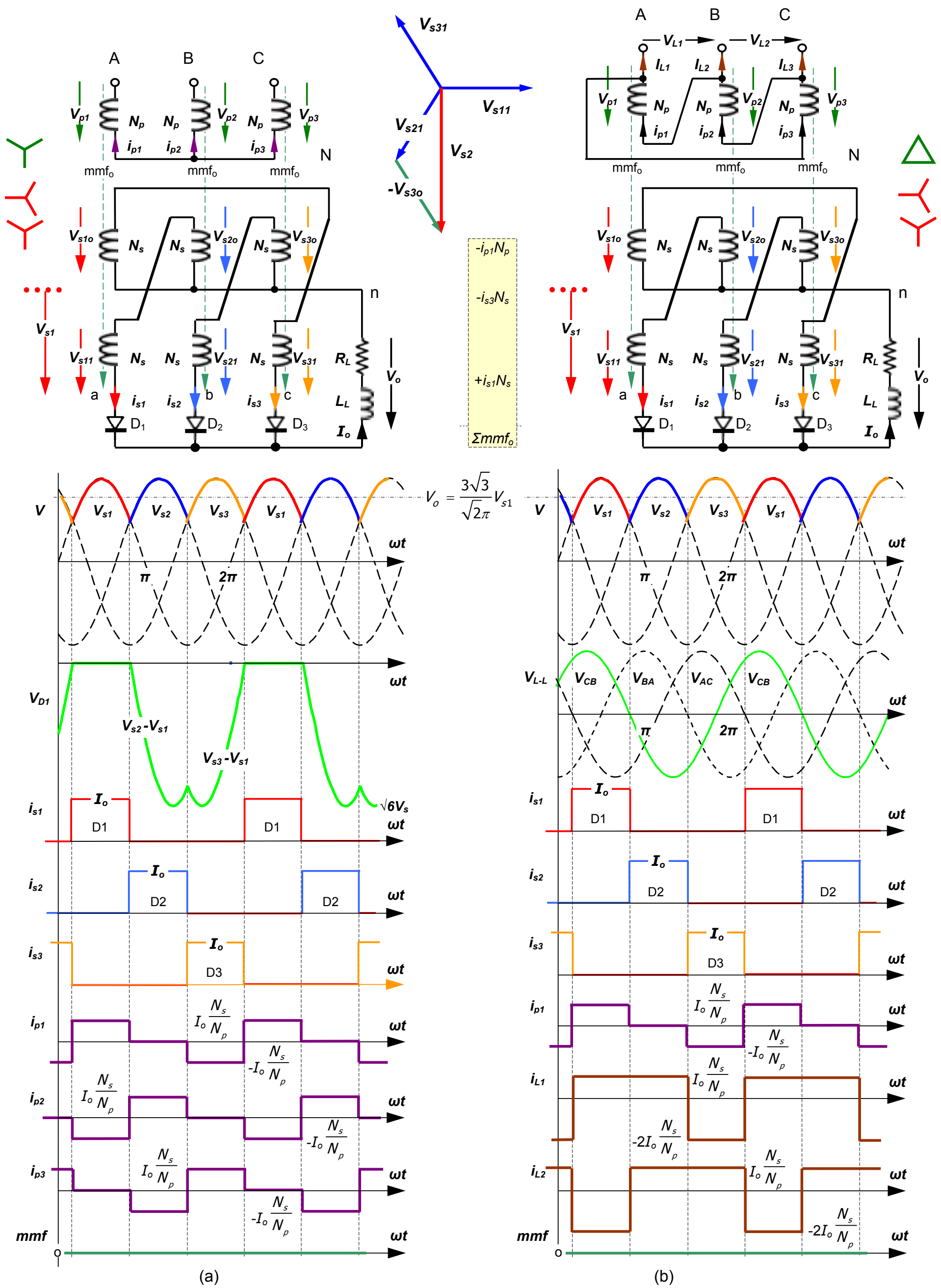


Figure 11.25. *Three-phase transformer winding zig-zag arrangement with no dc mmf bias: (a) star connected primary and (b) delta connected primary.*

If a 1:1:1 turns ratio is assumed, the power ratings of the transformer (which is independent of the turns ratio) involves the vectorial addition of the winding voltages.

$$\begin{aligned}\vec V_{s1} &= \vec V_{s11} - \vec V_{s2o}\\ \vec V_{s2} &= \vec V_{s21} - \vec V_{s3o}\\ \vec V_{s3} &= \vec V_{s31} - \vec V_{s1o}\end{aligned} \qquad (11.205)$$

The various transformer ratings are

$$S_s = 3I_sV_{s0} + 3I_sV_{s1} = 6I_sV_{so} = \frac{2\sqrt{2}\pi}{3\sqrt{3}}P_o \qquad S_p = 3I_pV_p = \frac{2\pi}{3\sqrt{3}}P_o$$

$$\bar S = ½\left(S_s + S_p\right) = P_o\frac{\pi}{3\sqrt{3}}\left(\sqrt{2}+1\right) = 1.46P_o \qquad (11.206)$$

ii. delta connected primary Δ-z (DELTA-zigzag)

Carrying out an *mmf* balancing exercise, assuming no alternating *mmf* component, and the mean line current is zero, yields

$$\begin{aligned} i_{L1} &= i_{p1} - i_{p3} = \frac{N_s}{N_p}\left(i_{s1} + i_{s2} - 2i_{s3}\right)\\ i_{L2} &= i_{p2} - i_{p1} = \frac{N_s}{N_p}\left(i_{s2} + i_{s3} - 2i_{s1}\right)\\ i_{L3} &= i_{p3} - i_{p2} = \frac{N_s}{N_p}\left(i_{s3} + i_{s1} - 2i_{s2}\right)\end{aligned} \qquad (11.207)$$

The primary and secondary currents are the same whether for a delta or star connected primary, therefore

$$\bar S = ½\left(S_s + S_p\right) = 1.46P_o \qquad (11.208)$$

If a 1:1:1 turns ratio is assumed, the line, primary and load current are related according to

$$I_L = \sqrt{\frac{2}{3}I_o^2 + \frac{4}{3}I_o^2} = \sqrt{2}I_o$$

$$I_p = \sqrt{\frac{2}{3}}I_o \qquad I_L = \sqrt{3}I_p \qquad (11.209)$$

A zig-zag secondary can be a Y-type fork for a possible neutral connection or alternatively, a Δ-type polygon when the neutral is not required.

Each diode conducts for 120º and

$$\bar I_D = ⅓ I_o \qquad I_{D\,rms} = \frac{I_o}{\sqrt{3}} \qquad \hat V_D = \sqrt{6}\frac{N_s}{N_p}V_p \qquad (11.210)$$

11.3.8 Three-phase transformer full-wave rectifiers – zero core mmf

Full-wave rectification is common in single and three phase applications, since, unlike half-wave rectification, the core *mmf* bias tends to be zero. In three-phase, it is advisable that either the primary or secondary be a delta connection. Any non-linearity in the core characteristics, namely hysteresis, causes triplen fluxes. If a delta connection is used, triplen currents can circulate in the winding, thereby suppressing the creation of triplen core fluxes. If a Y-y connection is used, a third winding set, delta connected, is usually added to the transformer in high power applications. The extra winding can be used for auxiliary type supply applications, and in the limit only one turn per phase need be employed if the sole function of the tertiary delta winding is to suppress core flux triplens.
The primary current harmonic content is the same for a given output winding configuration, independent of whether the primary is star or delta connected.

i. Star connected primary Y-y (Wye-wye)

The Y-y connection shown in figure 11.26a (with primary and secondary neutral nodes N, n respectively) is the simplest to analyse since each phase primary current is equal to a corresponding phase secondary current.

$$\begin{aligned} mmf_o &= N_s i_{s1} - N_p i_{p1} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \end{aligned} \quad (11.211)$$

Adding the three *mmf* equations gives

$$3 \times mmf_o = N_p \sum_{i=1}^{3} i_{pi} - N_s \sum_{i=1}^{3} i_{si} \quad (11.212)$$

but

$$i_{p1} + i_{p2} + i_{p3} = 0 \quad (11.213)$$

and the secondary currents always sum to zero, then $mmf_o = 0$.

Additionally

$$i_{L1} = i_{p1} = \frac{N_s}{N_p} i_{s1} \qquad i_{L2} = i_{p2} = \frac{N_s}{N_p} i_{s2} \qquad i_{L3} = i_{p3} = \frac{N_s}{N_p} i_{s3} \quad (11.214)$$

Generally

$$I_p = \frac{N_s}{N_p} I_s = \frac{N_s}{N_p} \sqrt{\frac{2}{3}} I_o \quad (11.215)$$

whence

$$\begin{aligned} S_p &= \frac{2\pi}{3\sqrt{3}} P_o = 1.21 P_o \qquad S_s = \frac{\sqrt{2}\pi}{3} P_o = 1.48 P_o \\ \overline{S} &= ½\left(\frac{2\pi}{3\sqrt{3}} P_o + \frac{\sqrt{2}\pi}{3} P_o\right) = 1.35 P_o \end{aligned} \quad (11.216)$$

The secondary harmonic currents are given by

$$I_{sh} = \frac{1}{h} I_{s1} = \frac{1}{h} \frac{\sqrt{6}}{\pi} I_o \quad \text{for } h = 6n \pm 1 \quad \forall \quad n > 0 \quad (11.217)$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, 1:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \quad (11.218)$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $0.057V_o$.

Since with a star primary the line currents are the primary currents, the supply power factor is

$$pf = \frac{P_o}{S} = \frac{3}{\pi} = 0.955 \quad (11.219)$$

ii. Delta connected primary Δ-y (Delta-wye)

The secondary phase currents in figure 11.26b are the same as for the Y-y connection, but the line currents are composed as follows

$$i_{L1} = i_{p1} - i_{p3} \qquad i_{L2} = i_{p2} - i_{p1} \qquad i_{L3} = i_{p3} - i_{p2} \quad (11.220)$$

Such that

$$\begin{aligned} I_p &= \frac{N_s}{N_p} I_s = \frac{N_s}{N_p} \sqrt{\frac{2}{3}} I_o \\ I_L &= \sqrt{3}\, I_p = \frac{N_s}{N_p} \sqrt{3}\, I_s = \frac{N_s}{N_p} \sqrt{2}\, I_o \end{aligned} \quad (11.221)$$

The secondary harmonic currents are given by

$$I_{sh} = \frac{1}{h} I_{s1} = \frac{1}{h} \frac{\sqrt{6}}{\pi} I_o \quad \text{for } h = 6n \pm 1 \quad \forall \quad n > 0 \quad (11.222)$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, √3:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \quad (11.223)$$

The transformer apparent power components are

$$S_s = 1.05 P_o \quad S_p = 1.05 P_o \quad \text{hence} \quad \overline{S} = 1.05 P_o \quad (11.224)$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $0.057V_o$.

The supply power factor is

$$pf = \frac{3}{\pi} = 0.955 \quad (11.225)$$

iii. Star connected primary Y-δ (Wye-delta)

In the Y-δ configuration in figure 11.27a, there are no zero sequence currents hence no *mmf* bias arises, $mmf_o = 0$, and both transformer sides have positive and negative sequence currents.

$$i_{p1} = \frac{N_s}{N_p} i_{s1} \qquad i_{p2} = \frac{N_s}{N_p} i_{s2} \qquad i_{p3} = \frac{N_s}{N_p} i_{s3} \quad (11.226)$$

and $i_{s1} + i_{s2} + i_{s3} = 0$

where

$$\begin{aligned} i_{L1} &= \frac{N_s}{N_p}\left(i_{s1} - i_{s2}\right) = i_{p1} - i_{p2} \\ i_{L2} &= \frac{N_s}{N_p}\left(i_{s2} - i_{s3}\right) = i_{p2} - i_{p3} \\ i_{L2} &= \frac{N_s}{N_p}\left(i_{s3} - i_{s1}\right) = i_{p3} - i_{p1} \end{aligned} \quad (11.227)$$

Thus the transformer currents are related to the supply line currents by

$$\begin{aligned} i_{p1} &= \frac{N_s}{N_p} i_{s1} = \frac{2}{3} i_{L1} - \frac{2}{3} i_{L2} \\ i_{p2} &= \frac{N_s}{N_p} i_{s2} = \frac{2}{3} i_{L2} - \frac{2}{3} i_{L3} \\ i_{p3} &= \frac{N_s}{N_p} i_{s3} = \frac{2}{3} i_{L3} - \frac{2}{3} i_{L1} \end{aligned} \quad (11.228)$$

where

$$i_{L1} + i_{L2} + i_{L3} = 0 \quad (11.229)$$

Generally

$$I_p = \frac{N_s}{N_p} I_s = \frac{N_s}{N_p} \frac{2\sqrt{2}}{3} ½ I_o \quad (11.230)$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, √3:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \quad (11.231)$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $2/5\times7 = 0.057V_o$.

The supply power factor is

$$pf = \frac{3}{\pi} = 0.995 \quad (11.232)$$

for an output power, $P_o = V_o I_o$,

iv. Delta connected primary Δ-δ (Delta-delta)

The phase primary and secondary voltages are in phase.

As shown in figure 11.27b the line currents are composed as follows

$$i_{L1} = i_{p1} - i_{p3} \qquad i_{L2} = i_{p2} - i_{p1} \qquad i_{L3} = i_{p3} - i_{p2} \tag{11.233}$$

The transformer primary and secondary currents are

$$i_{p1} = \frac{N_s}{N_p} i_{s1} \qquad i_{p2} = \frac{N_s}{N_p} i_{s2} \qquad i_{p3} = \frac{N_s}{N_p} i_{s3} \tag{11.234}$$

and

$$\begin{aligned} i_{p1} + i_{p2} + i_{p3} &= 0 \\ i_{s1} + i_{s2} + i_{s3} &= 0 \\ i_{L1} + i_{L2} + i_{L3} &= 0 \end{aligned} \tag{11.235}$$

Generally

$$I_p = \frac{N_s}{N_p} I_s \tag{11.236}$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, 1:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \tag{11.237}$$

The rms output voltage is

$$V_{o\,rms} = \sqrt{2} V_s \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}} \tag{11.238}$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $0.057V_o$.

The primary and secondary apparent powers are

$$S_p = S_s = \frac{\pi}{3} P_o = 1.05 P_o \tag{11.239}$$

Thus the supply power factor is

$$pf = \frac{P_o}{S} = \frac{3}{\pi} = 0.955 \tag{11.240}$$

for an output power, $P_o = V_o I_o$,

In summary, when the primary and secondary winding configurations are the same (Δ-δ or Y-y) the input and output line voltages are in phase, otherwise (Δ-y or Y-δ) the input and output line voltages are shifted by 30° relative to one another.

Independent of the transformer primary and secondary connection, for a specified input and output voltage, the following electrical equations hold.

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} \frac{N_s}{N_p} V_L \qquad pf = \frac{3}{\pi}$$

$$\overline{I}_D = \frac{1}{3} I_o \qquad I_{D\,rms} = \frac{1}{\sqrt{3}} I_o \qquad \hat{V}_{DR} = \sqrt{3}\sqrt{2} V_s$$

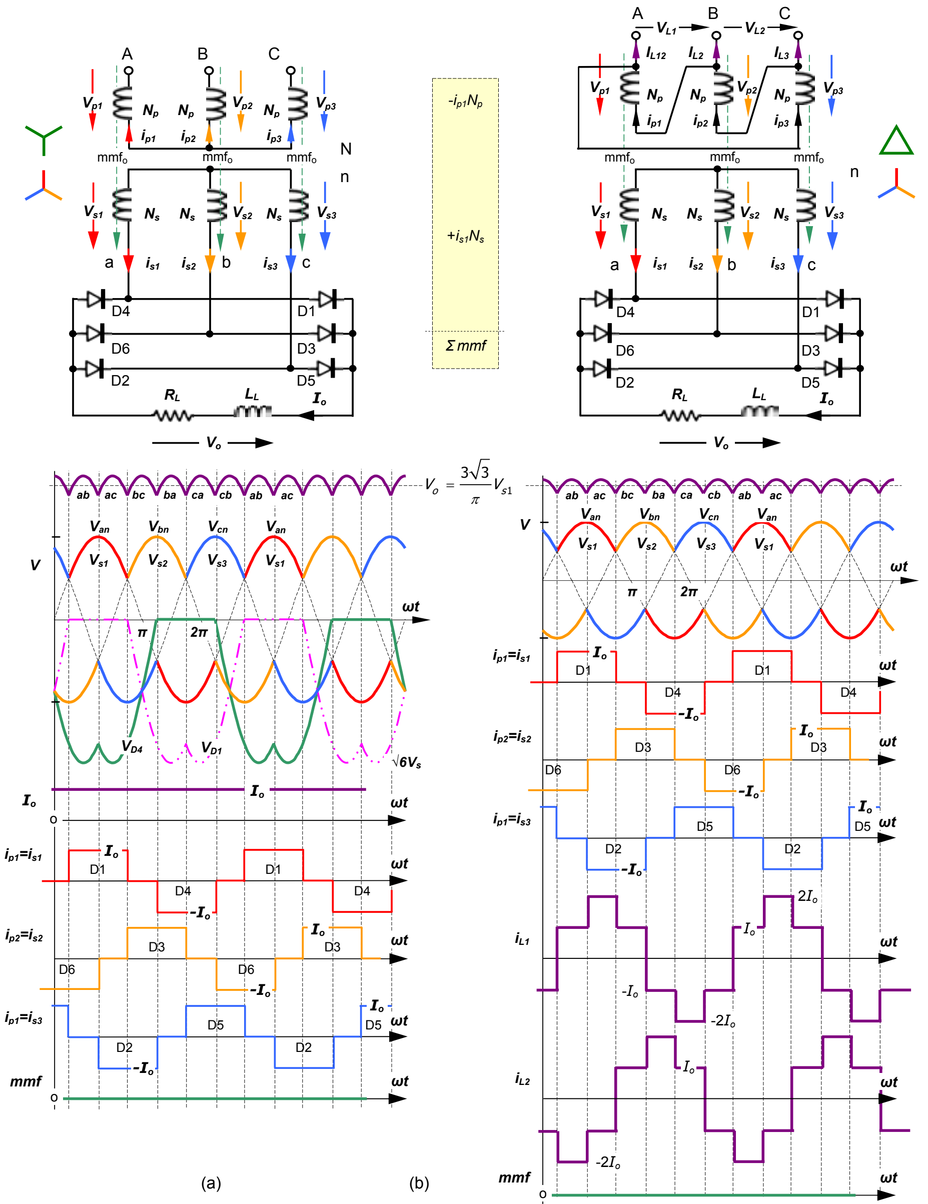


Figure 11.26. *Three-phase transformer wye connected secondary winding with full-wave rectification and no resultant dc mmf bias: (a) star connected primary Y-y and (b) delta connected primary Δ-y.*

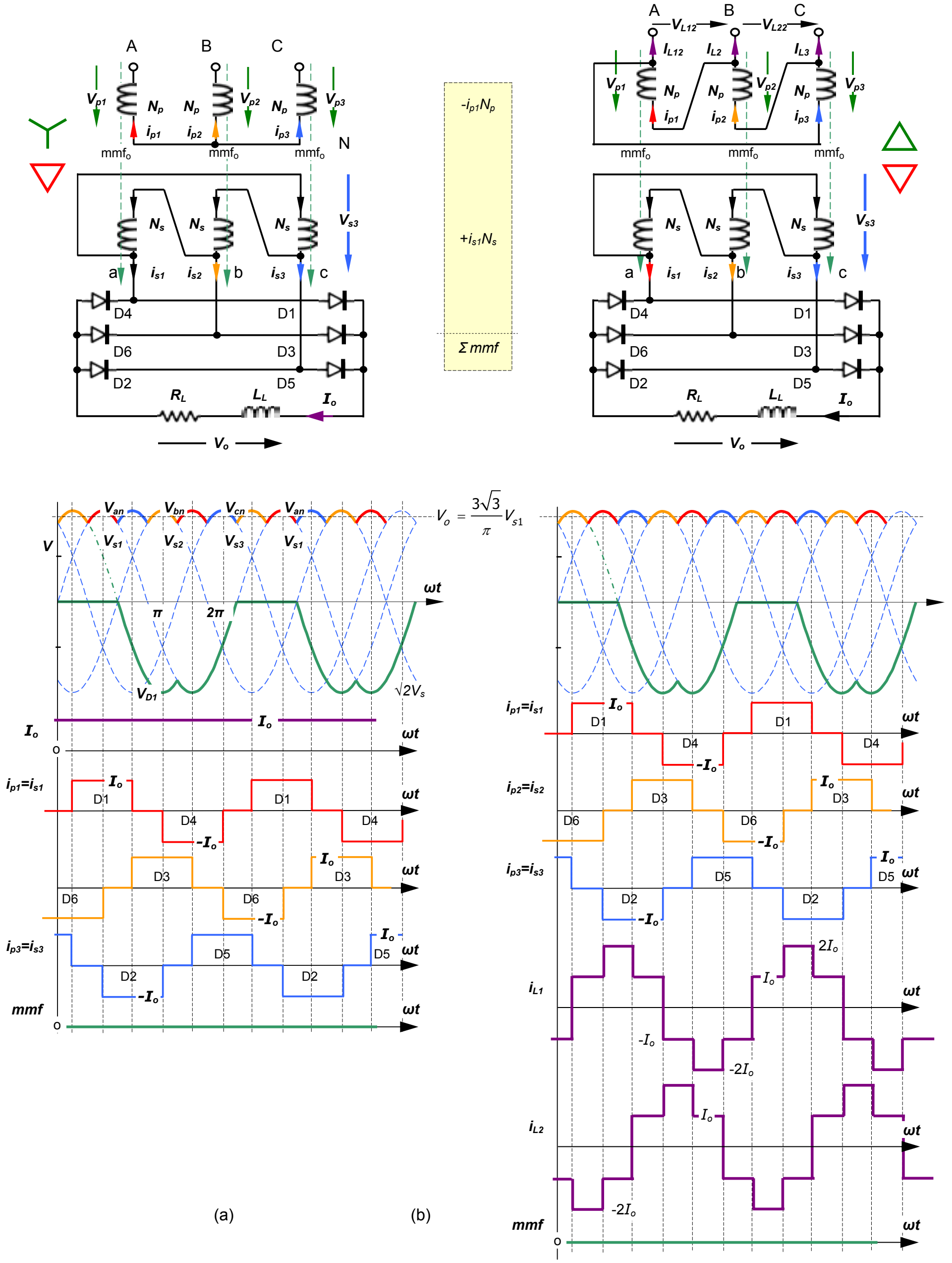


Figure 11.27. *Three-phase transformer with delta connected secondary winding with full-wave rectification and no resultant dc mmf bias: (a) star connected primary Y- δ and (b) delta connected primary Δ- δ.*

11.4 Voltage multipliers

Voltage multipliers are ac to dc power conversion circuits, comprised of diodes and capacitors that are interconnected so as to produce a high potential dc voltage from a lower voltage ac source. As in figure 11.28a, multipliers are made up of cascaded stages each comprised of a diode and a capacitor.

Voltage multipliers are a simple way to generate high voltages at relatively low currents. By using only capacitors and diodes, the voltage multipliers can step up relatively low voltages to extremely high values, while at the same time being far lighter and cheaper than transformers. The advantage of the circuit is that the voltage across each cascaded stage is only equal to twice the peak input voltage, so it requires relatively low cost components and is easy to insulate. An output can also be tapped from any stage, like a multi-tapped transformer.

The voltage multiplier has poor voltage regulation, that is, the voltage drops rapidly as a function the output current, as in figure 11.28b. The output *I-V* characteristic is approximately hyperbolic, so it is suitable for charging capacitor banks to high voltages at near constant charging power. Furthermore, the ripple on the output, particularly at high loads, is high. The output voltage is not isolated from the input voltage source, although transformer coupling provides general isolation.

The most commonly used multiplier circuit is the half-wave series multiplier. Other multiplier circuits can be derived from its operating principles.

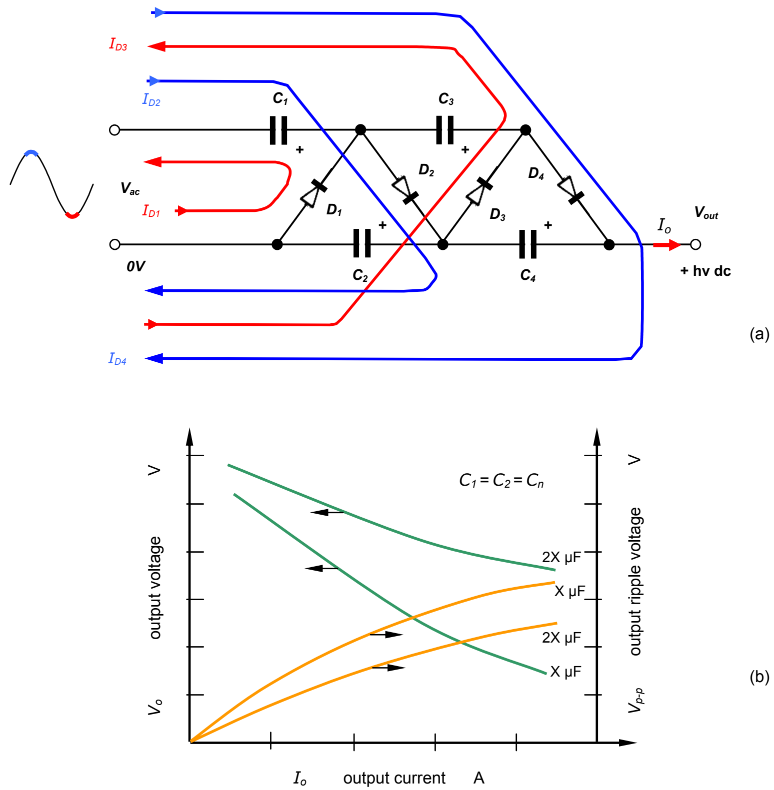


Figure 11.28. *Charging sequence of a half-wave series positive output voltage multiplier and output characteristics dependence on output current and stage capacitance.*

The following description for a two-stage series voltage multiplier assumes no losses and represents sequential reversals of polarity of the source transformer T_s in the figure 11.28a. The number of stages is equal to the number of smoothing capacitors between ground and V_{out}, which in this case is two, capacitors C_2 and C_4.

- V_{ac} = Negative Peak: C_1 charges through D_1 to V_{pk} by current I_{D1}
- V_{ac} = Positive Peak: V_{pk} of Ts adds arithmetically to existing potential C_1, thus C_2 charges to 2 V_{pk} thru D_2 by current I_{D2}
- V_{ac} = Negative Peak: C_3 is charged to $2V_{pk}$ through D_3 by current I_{D3}
- V_{ac} = Positive Peak: C_4 is charged to $2V_{pk}$ by current I_{D4} through D_4 then V_{pk}.

For *N* stages (series capacitors) the output voltage is $N\times V_{pk}$.

11.4.1 Half-wave series multipliers

The capacitors are in series, so effectively capacitance is as for series connected capacitors, *C*/*N*, but voltage rating is the cumulative sum of the series capacitors between the output terminals. This multiplier is the most common, and is versatile, being used in high-voltage, low-current applications. The basic charging sequence in figure 11.29 is as for the circuit shown in figure 11.28a, where the diodes conduct in the order D_1 to D_4, for both output polarity versions.

Half-wave series voltage multiplier features include:

- a wide range of multiplication stages
- low cost
- uniform stress per stage on diodes and capacitors, $2V_{pk}$ and V_{pk}

Any one capacitor can be eliminated from the capacitor filter bank if the load is capacitive. Whether full wave or half-wave, the series diodes prevent the output voltage from swinging negative. At high discharges, part of the output current is also drawn via a diode, hampering rapid high current discharge. Dual polarity output voltage is produced by connecting positive and negative multipliers as shown in the four stage circuit is shown in figure 11.29c, where an unlimited stage number can be cascaded. Since regulation is proportional to N^3, a large number of stages eventually becomes ineffective. A centre tapped capacitor string connection reduces the maximum voltage potential with respect to ground. An odd number of stages can be produced as well as an even number of stages. The output voltage may be tapped at any point on the capacitor series filter bank.

Once a load is connected at the output, the output voltage decreases due to the voltage regulation. Also, any small fluctuation of load impedance causes a large fluctuation in the multiplier output voltage due to the number of stages involved. For this reason, voltage multipliers are used only in special applications where the load is constant and has a high impedance or where voltage stability is not critical.

Half-wave Output Voltage

The open-circuit output voltage $V_{o/c}$ of each stage is nominally twice the peak input voltage V_{pk}. Assuming the ac input voltage and frequency are constant, for *N* cascaded stages, the output voltage is

$$V_{o/c} = 2N \times V_{pk} \tag{11.241}$$

In practice, several cycles are required to reach full output voltage. The output voltage follows an *RC* network exponential curve, where *R* is the output impedance of the ac source, whilst *C* is the effective dynamic capacitance of the voltage multiplier, *N*×C. This charging occurs only upon switch-on of the voltage multiplier from a discharged state, and does not repeat itself unless the output is short circuited. The most common input ac waveforms are sine waves and square waves.

Output Voltage Regulation

DC output voltage drops as the dc output current increases, as shown in figure 11.28b. Regulation is the drop in dc output voltage from the ideal at a specified dc output current (assuming the ac input voltage and input frequency are constant). The voltage drop under load is mostly reactive and is calculated as:

$$V_{reg} = I_o \times \frac{4N^3 + 3N^2 - N}{6f \times C} = I_o \times \frac{4N^2 + 3N^2 - 1}{6f \times {C}/{N}} \tag{11.242}$$

where:

I_o is the load or output dc current (A)
C is the stage capacitance (F)
f is the ac frequency (Hz)
N is the number of stages
C/*N* is the effective output capacitance (F).

Regulation voltage droop is not a power loss in a multiplier. Power losses are primarily diode forward conduction and rarely result in excessive multiplier temperatures at the low current loadings.
Substituting V_{reg} from equation (11.242):

$$V_{out} = V_{o/c} - V_{reg} = 2NV_{pk} - I_o \times \frac{4N^3 + 3N^2 - N}{6f \times C} \tag{11.243}$$

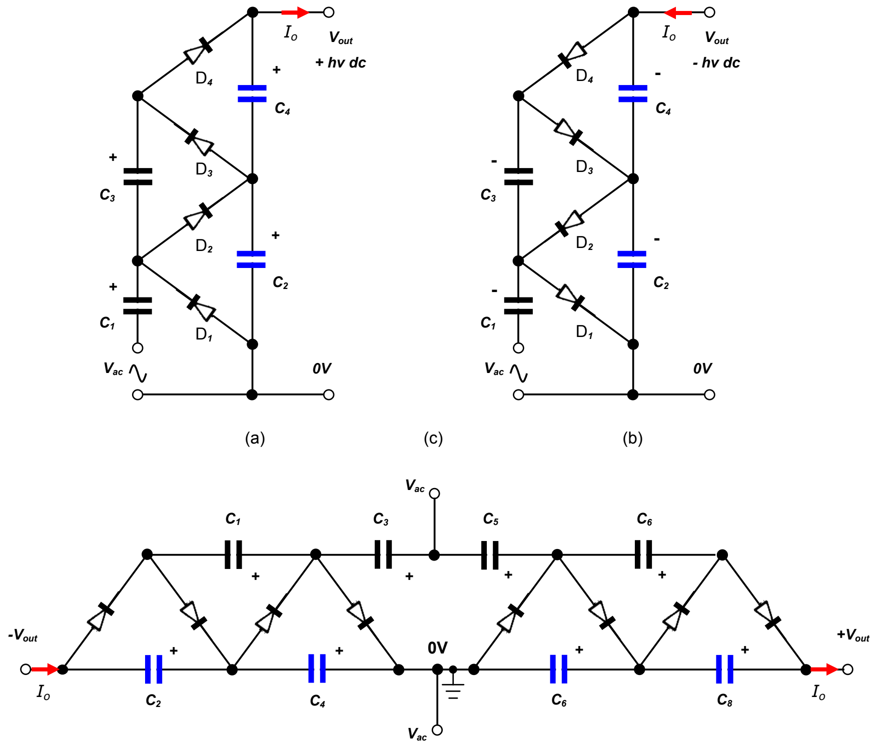


Figure 11.29. *Series half-wave voltage multipliers:*
(a) two stage positive hv output voltage; (b) two stage negative hv output voltage; and (c) four stage multiplier configured with ± output hv voltage.

Output Voltage Ripple

Ripple voltage is the magnitude of fluctuation in dc output voltage at a specific output current. This assumes the ac input voltage and frequency are maintained constant. The ripple voltage in the case where all stage capacitances, C_1 through C_{2N}, are equal, is:

$$V_{ripple} = I_o \times \frac{N^2 + N}{2f \times C} \tag{11.244}$$

The ripple grows rapidly as the number of stages increases, with *N* squared. A common modification to the design is to make the stage capacitances larger at the input, with $C_1 = C_2 = N\times C$, $C_3 = C_4 = (N-1)\times C$, and so forth. Then the ripple is:

$$V_{ripple} = \frac{I_o}{f \times C} \tag{11.245}$$

For a large number of stages, $N \geq 5$, the N^3 term in the voltage drop equation dominates. Differentiating the V_{out} equation without the negligible terms, with respect to the number of stages and equating to zero, gives an equation for the optimum (integer) number of stages N_{opt} for the equal valued capacitor design:

$$\frac{dV_{out}}{dN} = \frac{d}{dN}\left(2NV_{pk} - \frac{I_o}{6f \times C} \times 4N^3\right) = 0$$

:

$$N_{opt} = \text{int}\left[\left(\frac{V_{pk} f \times C}{I_o}\right)^{½}\right] \tag{11.246}$$

Increasing the frequency can dramatically reduce the ripple, and the voltage drop under load, which accounts for the popularity of driving a multiplier stack with a switching power supply.

If the driving voltage V_{pk} and the required output voltage $V_{o/c}$ are known, the optimum number of cascaded stages is:

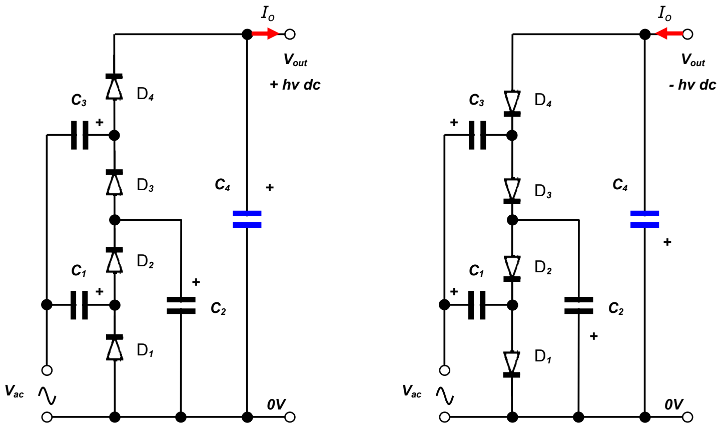
$$N_{opt} = \text{int}\left[\frac{3V_{out}}{4V_{pk}}\right] \tag{11.247}$$

11.4.2 Half-wave parallel multipliers

Opposite polarity half-wave parallel voltage multipliers are shown in figure 11.29. The output capacitors share a common connection but must have a high voltage rating. The output is usually low voltage but with high currents. The basic charging sequence in figure 11.30 is the same as shown in figure 11.28, where the diodes conduct in the order D_1 to D_4, for both output polarity versions.

Parallel multipliers offer the following features:

- uniform stress on diodes
- compact
- voltage stress on capacitors increases with successive stages by V_{pk}
- highly efficient



(a) (b)

Figure 11.30. *Parallel half-wave voltage multipliers: (a) two stage positive hv output voltage and (b) two stage negative hv output voltage.*

11.4.3 Full-wave series multipliers

Increasing the frequency can dramatically reduce the ripple, and the voltage drop under load, which can be achieved by driving a multiplier stack with a switched mode power supply.

Figure 11.31 shows a typical full-wave two-stage series voltage multiplier. It is comprised of two anti-phase ac input half-wave multipliers sharing a common series output capacitor string. This effectively doubles the number of charging cycles per second, and thus reduces the voltage drop and ripple factor. The input is usually fed from a centre-tapped ac transformer or MOSFET H-bridge circuit.

The full-wave series voltage multiplier has the following general features:

- uniform stress on components
- highly efficient
- high voltage
- high power capability
- easy to produce
- increased voltage stress on capacitors with successive stages
- wide range of multiplication stages

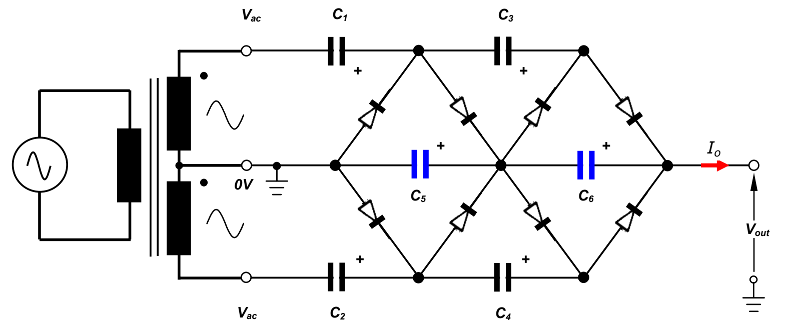


Figure 11.31. *Two-stage series full-wave voltage multiplier.*

Full-wave Output Voltage
As with the half-wave voltage multiplier, the full-wave voltage multiplier output voltage is given by:

$$V_{o/c} = 2NV_{pk} \tag{11.248}$$

Output Voltage Regulation
DC output voltage decreases as dc output current increases. Regulation is the drop in dc output voltage from the ideal at a specified dc output current, assuming constant ac input voltage and frequency. The voltage drop under load is mostly reactive and is:

$$V_{reg} = I_o \times \frac{N^3 + 2N}{6f \times C} = I_o \times \frac{N^2 + 2}{6f \times C/N} \tag{11.249}$$

where:

I_o is the load or output dc current (A)
C is the stage capacitance (F)
f is the ac frequency (Hz)
N is the number of stages
C/N is the effective output capacitance (F).

Regulation voltage droop is not a power loss in a multiplier. Power losses are primarily diode forward conduction and rarely result in excessive multiplier temperatures at the low current loadings. Substituting equation (11.249) for V_{reg}:

$$V_{out} = V_{o/c} - V_{reg} = 2N \times V_{pk} - I_o \times \frac{N^3 + 2N}{6f \times C} \tag{11.250}$$

Output Voltage Ripple
The ripple voltage, in the case where all stage capacitances are equal, is given by:

$$V_{ripple} = I_o \times \frac{N}{2f \times C} \tag{11.251}$$

If the driving voltage V_{pk} and the required output voltage $V_{o/c}$ are known, the optimum number of cascaded stages is:

$$N_{opt} = \text{int}\left[\frac{0.521V_{out}}{V_{pk}}\right] \tag{11.252}$$

Example 11.9: ***Half-wave voltage multiplier***

A three-stage half-wave series voltage multiplier, is driven by a 50kHz peak voltage of 10kV, with 1nF capacitances, and a load current of 10mA.

i. Calculate the open circuit output voltage, regulated output voltage, ripple voltage, and optimal number of stages for the required voltage transfer function.
ii. What is the capacitance and voltage rating of each stage of a parallel connected multiplier?
iii. What is the output ripple if progressively smaller capacitance is used?

Solution

i. In a three-stage voltage multiplier, the no load voltage $V_{o/c} = 2 \times N \times V_{pk} = 2 \times 3 \times 10\text{kV} = 60\text{kV}$

$$V_{reg} = I_o \frac{4N^3 + 3N^2 - N}{6f \times C} = 10\text{mA} \times \frac{3^3 + 3 \times 3^2 - 3}{6 \times 50\text{kHz} \times 1\text{nF}} = 1.7\text{kV}$$

V_{out} = 60kV - 1.7kV = 58.3kV
So the output voltage will swing between 60kV and 58.3kV, depending on the load current.
The output ripple voltage is

$$V_{ripple} = I_o \frac{N^2 + N}{2f \times C} = 10\text{mA} \frac{3^2 + 3}{2 \times 50\text{kHz} \times 1\text{nF}} = 3\text{kV}$$

The optimal number of stages, from equation (11.252), is

$$N_{opt} = \text{int}\left[\frac{0.521 \times V_{out}}{V_{pk}}\right] = \text{int}\left[\frac{0.521 \times 58.3\text{kV}}{10\text{kV}}\right] = 3$$

ii. An equivalent parallel multiplier would require each capacitor stage to equal the total series capacitance of the series capacitor bank.
In this case, the three capacitors in the dc bank would equal 1000pF/3 or 330pF. The parallel equivalent would require 330pF capacitors in each stage. However, each successive stage, from the input, would require a higher voltage capacitor, 20kV, 40kV and 60kV, respectively.

iii. When $C_1 = C_2 = N \times C = 3\text{nF}$, $C_3 = C_4 = (N-1) \times C = 2\text{nF}$, $C_5 = C_6 = (N-2) \times C = 1\text{nF}$.

$$V_{ripple} = \frac{I_o}{f \times C} = \frac{10\text{mA}}{50\text{kHz} \times 1\text{nF}} = 200\text{V}$$

This modification reduces the ripple voltage from 3kV to just 200V.

♣

Example 11.10: ***Full-wave voltage multiplier***

A three-stage full-wave parallel voltage multiplier, is driven by a 50kHz peak voltage of 10kV, with 1nF capacitances, and a load current of 10mA. Calculate the output voltage and ripple voltage.

Solution

In a three-stage voltage multiplier, the no load voltage $V_{o/c} = 2 \times N \times V_{pk} = 2 \times 3 \times 10\text{kV} = 60\text{kV}$.

$$V_{reg} = I_o \frac{N^3 + 2N}{6f \times C} = 10\text{mA} \frac{3^3 + 2 \times 3}{6 \times 50\text{kHz} \times 1\text{nF}} = 1.1\text{kV}$$

Full-wave rectification reduces the regulation voltage drop from 1.7kV in example 11.9, to 1.1kV. The output voltage is increased by 600V, from 58.3kV in example 11.9, to V_{out} = 60kV - 1.1kV = 58.9kV.
The ripple voltage reduces from 3kV for half-wave multiplication in example 11.9, to

$$V_{ripple} = I_o \frac{N}{2f \times C} = 10\text{mA} \frac{3}{2 \times 50\text{kHz} \times 1\text{nF}} = 200\text{V}$$

♣

11.4.4 Three-phase voltage multipliers

The full-wave multiplier in figure 11.32 is a special case of a poly-phase (0° and 180°) multiplier where more than one multiplier share a common series stack of load capacitors. In figure 11.32, the phase angle between phases is 0°, 120°, and 240°, respectively. The peak voltage supplied by each secondary winding is V_{pk}.
The three-phase circuit in figure 11.32b can be modified by disconnecting the centre point of the Y configuration from ground and omitting the first capacitor in each charging stack, as shown in figure 11.32c. As a result, the open-circuit dc voltage per stage is reduced from $2 \times V_{pk}$ to $\surd 3 \times V_{pk}$. The output impedance, however, decreases dramatically, so the output voltage under load may be even higher, depending on the load current. Therefore, this variant is preferred if the multiplier has to supply higher currents.

11.4.5 Series versus parallel voltage multipliers

The theory of operation is the same for both series and parallel connected voltage multipliers. Parallel multipliers require less capacitance per cascaded stage than their series counterparts, however parallel multipliers require higher capacitor voltage ratings on successive cascaded stages. The parallel multiplier output is easier to *RC* filter in applications requiring low output ripple voltage.

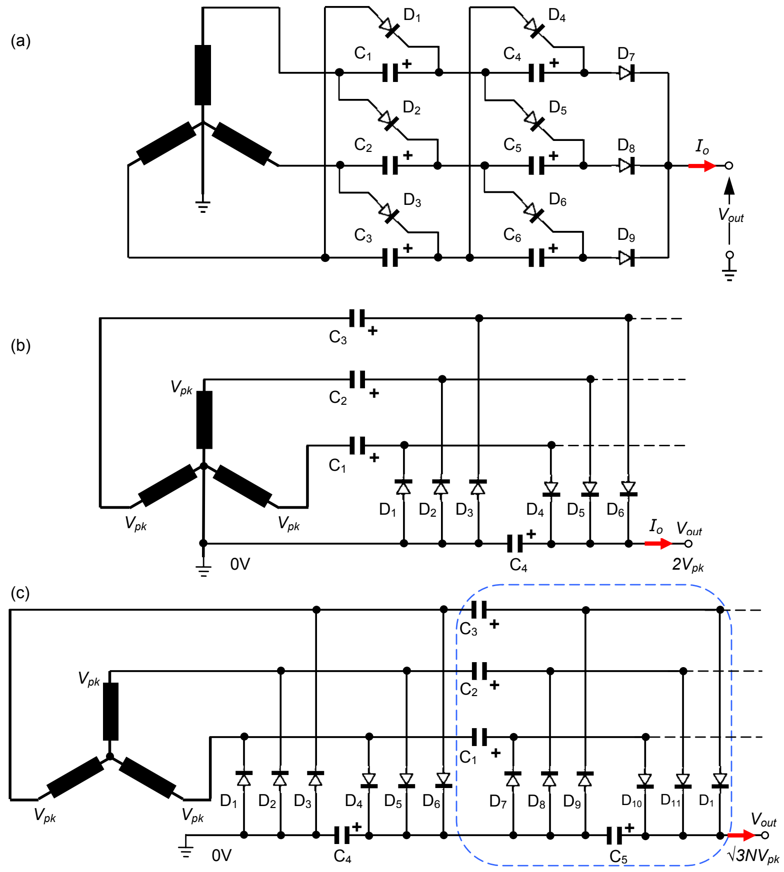


Figure 11.32. *Three-phase Y configuration voltage multipliers: (a) series diode output stage; (b) grounded centre point; and (b) floating centre point.*

11.5 Marx voltage generator

The Marx generator shown in figure 11.33, charges the energy storage capacitor of each stage in parallel with a relatively low voltage (1kV to 6kV), and then discharges them by means of active switches in series, into the load. The output voltage is then equal to the charging voltage multiplied by the number of stages. The series inductance of this type of generators is low, as a result the rise time and fall time of the output pulses can be less than 1μs. The pulse repetition rate can be more than 20kHz for short pulses, and the pulse length can be several ms.

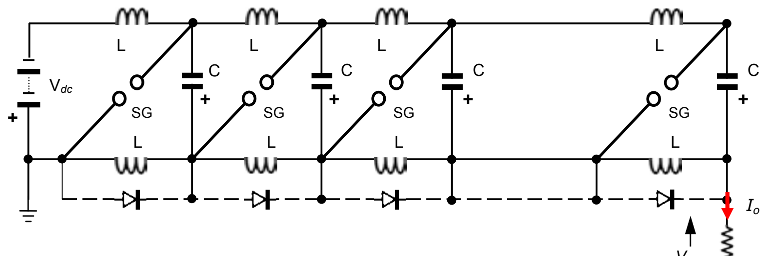


Figure 11.33. *The hv Marx generator.*

11.6 Definitions

$$v(\omega t)=\sum_{n=0}^{\infty}\sqrt{2}V_n\sin(n\omega t-\phi_n) \qquad i(\omega t)=\sum_{n=0}^{\infty}\sqrt{2}\,I_n\sin(n\omega t-\varphi_n)$$

total current $I_s^2=I_1^2+I_2^2+I_3^2+\ldots I_n^2$ distortion current harmonic current $I_{dis}^2=I_2^2+I_3^2+\ldots I_n^2$

total harmonic factor distortion factor $DF=k=\dfrac{I_{dis}}{I_s}$

total harmonic distortion $THD=\dfrac{I_{dis}}{I_1}$

displacement power factor $DPF=\cos\varphi_1=\lambda_1$

circuit power factor $pf=\lambda=\dfrac{\text{active input power}}{V_sI_s}$

$$=\frac{\cos\varphi_1}{\sqrt{1+THD^2}}$$

crest factor $cf=\dfrac{\hat{i}_s}{I_s}$

form factor $=\dfrac{\text{rms value}}{\text{average value}}$

The average (or mean or dc) rms (or effective) values, respectively, of a waveform, are defined by

$$V_o=\frac{1}{T}\int_o^T v_o(t)\,dt$$

and

$$V_{rms}=\sqrt{\frac{1}{T}\int_o^T v_o^2(t)\,dt}$$

V_o average output voltage $\quad \overline{I}_o$ average output current
V_{rms} rms output voltage $\quad I_{rms}$ rms output current
$\hat{V}$ peak output voltage $\quad \hat{I}$ peak output current

Load voltage form factor $=FF_v={V_{rms}}/{V_o}$ Load voltage crest factor $=CF_v={\hat{V}}/{V_{rms}}$

Load current form factor $=FF_i={I_{rms}}/{\overline{I}_o}$ Load current crest factor $=CF_i={\hat{I}}/{I_{rms}}$

$$\text{Waveform smoothness = Ripple factor}=RF_v=\frac{\text{effective values of ac } V \text{ (or } I)}{\text{average value of } V \text{ (or } I)}=\frac{V_{Ri}}{V_o}$$

$$=\sqrt{\frac{V_{rms}^2-V_o^2}{V_o^2}}=\sqrt{FF_v^2-1}$$

where $\quad V_{Ri}=\left[\sum_{n=1}^{\infty}\tfrac{1}{2}\left(v_{an}^2+v_{bn}^2\right)\right]^{1/2}$

similarly the current ripple factor is $RF_i=\dfrac{I_{Ri}}{\overline{I}_o}=\sqrt{FF_i^2-1}$

$RF_i=RF_v$ for a resistive load

$$\text{Rectification efficiency}=\eta=\frac{\text{dc load power}}{\text{ac load power + rectifier losses}}$$

$$=\frac{V_oI_o}{V_{rms}I_{rms}+\text{Loss}_{\text{rectifier}}}$$

Waveform fundamental and harmonic rms components are define by

$$V_1=\sqrt{1/2\left(V_{1a}^2+V_{1b}^2\right)}$$

where

$$V_{1a}=\frac{2}{T}\int_o^T v(t)\cos 2\pi\, {t}/{T}\,dt \qquad V_{1b}=\frac{2}{T}\int_o^T v(t)\sin 2\pi\, {t}/{T}\,dt$$

and for the k^{th} harmonic component

$$V_k=\sqrt{1/2\left(V_{ka}^2+V_{kb}^2\right)}$$

where

$$V_{ka}=\frac{2}{T}\int_o^T v(t)\cos 2\pi\, {kt}/{T}\,dt \qquad V_{kb}=\frac{2}{T}\int_o^T v(t)\sin 2\pi\, {kt}/{T}\,dt$$

Distortion factor is defined as

$$DF_v=\frac{V_1}{V_{rms}}$$

The total harmonic distortion is

$$THD_v=\sqrt{\sum_{\substack{k=0\\k\neq 1}}^{\infty}\left(\frac{V_k}{V_1}\right)^2}=\frac{\sqrt{1-DF_v^2}}{DF_v^2}$$

11.7 Output pulse number

Output pulse number p is the number of pulses in the output voltage that occur during one ac input cycle, of frequency f_s. The pulse number p therefore specifies the output harmonics, which occur at $p\times f_s$, and multiples of that frequency, $m\times p\times f_s$, for m = 1, 2, 3, ...

$$p=\frac{\text{period of input supply voltage}}{\text{period of minimum order harmonic in the output } V \text{ or } I \text{ waveform}}$$

The pulse number p is specified in terms of

- q the number of elements in the commutation group
- r the number of parallel connected commutation groups
- s the number of series connected (phase displaced) commutating groups

Parallel connected commutation groups, r, are usually associated with (and identified by) intergroup reactors (to reduce circulating current), with transformers where at least one secondary is effectively star connected while another is delta connected. The rectified output voltages associated with each transformer secondary, are connected in parallel.

Series connected commutation groups, s, are usually associated with (and identified by) transformers where at least one secondary is effectively star while another is delta connected, with the rectified output associated with each transformer secondary, connected in series.

$$\boxed{\begin{array}{c} q=3\ \ r=2\ \ s=2\\ p=q\times r\times s\\ p=12\end{array}}$$

The mean rectifier output voltage V_o can be specified by

$$V_o=s\frac{q}{\pi}\sqrt{2}V_\phi\times\sin\frac{\pi}{q} \tag{11.253}$$

For a full-wave, single-phase rectifier, r = 1, q = 2, and s = 1, whence p = 2

$$V_o=1\times\frac{2}{\pi}\sqrt{2}V_\phi\times\sin\frac{\pi}{2}=\frac{2\sqrt{2}V_\phi}{\pi}$$

For a full-wave, three-phase rectifier, r = 1, q = 3, and s = 2, whence p = 6

$$V_o=2\times\frac{3}{\pi}\sqrt{2}V_\phi\times\sin\frac{\pi}{3}=\frac{3\sqrt{2}V_\phi}{\pi}$$

11.8 AC-dc converter generalised equations

Alternating sinusoidal voltages

$$V_1 = \sqrt{2}V\sin\omega t$$
$$V_2 = \sqrt{2}V\sin\left(\omega t - \tfrac{2\pi}{q}\right)$$
$$.$$
$$V_q = \sqrt{2}V\sin\left(\omega t - (q-1)\tfrac{2\pi}{q}\right)$$

where q is the number of phases (number of voltage sources)

On the secondary or converter side of any transformer, if the load current is assumed constant I_o then the power factor is determined by the load voltage harmonics.

Voltage form factor

$$FF_v = \frac{V_{rms}}{V_o}$$

whence the voltage ripple factor is

$$RF_v = \frac{1}{V_o}\left[V_{rms}^2 - V_o^2\right]^{1/2} = \left[FF_v^2 - 1\right]^{1/2}$$

The power factor on the secondary side of any transformer is related to the voltage ripple factor by

$$pf = \frac{P_d}{S} = \frac{V_o I_o}{qVI_{rms}} = \frac{1}{\sqrt{RF_v^2 + 1}}$$

On the primary side of a transformer the power factor is related to the secondary power factor, but since the supply is assumed sinusoidal, the power factor is related to the primary current harmonics.
Relationship between current ripple factor and power factor

$$RF_i = \frac{1}{I_1}\sqrt{\sum_{h=3}^{\infty} I_h^2} = \frac{1}{I_1}\sqrt{I_{rms}^2 - I_1^2}$$
$$pf = \frac{I_1}{I_{rms}} = \frac{1}{\sqrt{RF_i^2 + 1}}$$

The supply power factor is related to the primary power factor and is dependent of the supply connection, star or delta, etc.

Half-wave diode rectifiers [see figures 11.2, 11.11]

Pulse number p=q. Pulse number is the number of sine crests in the output voltage during one input voltage cycle. There are q phases and q diodes and each diode conducts for $2\pi/q$, with q crest (pulses) in the output voltage

Mean voltage

$$V_o = \frac{q}{2\pi}\int_{1/2\pi - \pi/q}^{1/2\pi + \pi/q} \sqrt{2}V\sin\omega t \; d\omega t$$
$$= \frac{q}{\pi}\sqrt{2}V\sin\frac{\pi}{q}$$

RMS voltage

$$V_{rms} = \left[\frac{q}{2\pi}\int_{1/2\pi - \pi/q}^{1/2\pi + \pi/q}\left(\sqrt{2}V\sin\omega t\right)^2 d\omega t\right]^{1/2}$$
$$= \sqrt{2}V\left[\tfrac{1}{2} + \frac{q}{4\pi}\sin\frac{2\pi}{q}\right]^{1/2}$$

Normalised peak to peak ripple voltage

$$v_{p-p} = \sqrt{2}V - \sqrt{2}V\cos\frac{\pi}{q}$$
$$V_{n_{p-p}} = \frac{v_{p-p}}{V_o} = \frac{\sqrt{2}V - \sqrt{2}V\cos\frac{\pi}{q}}{\frac{q}{\pi}\sqrt{2}V\sin\frac{\pi}{q}} = \frac{\pi}{q}\;\frac{1-\cos\frac{\pi}{q}}{\sin\frac{\pi}{q}}$$

Voltage form factor

$$FF_v = \frac{V_{rms}}{V_o} = \frac{\left[\tfrac{1}{2} + \frac{q}{4\pi}\sin\frac{2\pi}{q}\right]^{1/2}}{\frac{q}{\pi}\sin\frac{\pi}{q}}$$

whence the voltage ripple factor is

$$RF_v = \frac{1}{V_o}\left[V_{rms}^2 - V_o^2\right]^{1/2} = \left[FF_v^2 - 1\right]^{1/2}$$

Diode reverse voltage

$$\hat{V}_{DR} = 2\sqrt{2}V \quad \text{if } q \text{ is even}$$
$$\hat{V}_{DR} = 2\sqrt{2}V\cos\frac{\pi}{2q} \quad \text{if } q \text{ is odd}$$

For a constant load current I_o, diode currents are

$$\hat{I}_D = I_o \qquad \bar{I}_D = \frac{I_o}{q} \qquad I_{D\,rms} = \frac{I_o}{\sqrt{q}}$$

For a constant load current I_o the output power is

$$P_d = V_o I_o$$

The apparent power is

$$S = qVI_{rms}$$

The power factor on the secondary side of any transformer is

$$pf = \frac{P_d}{S} = \frac{V_o I_o}{qVI_{rms}} = \frac{1}{\sqrt{RF_v^2 + 1}}$$
$$= \frac{\frac{q}{\pi}\sqrt{2}V\sin\frac{\pi}{q}\times I_o}{qV\times I_o\sqrt{\frac{1}{q}}} = \frac{\sqrt{2q}}{\pi}\sin\frac{\pi}{q}$$

The primary side power factor is supply connection and transformer construction dependant.

For two-phase half-wave p=q=2

$$pf_{1\phi,1/2} = \frac{V_o I_o}{V\,I_o} = \frac{2\sqrt{2}}{\pi} = 0.90$$

For three-phase half wave p=q=3

$$pf_{3\phi,1/2} = \frac{V_o I_o}{3V\,I_o} = \frac{3\sqrt{3}}{2\pi} = 0.827$$

For six-phase half-wave p=q=6

$$pf_{6\phi,1/2} = \frac{V_o I_o}{3V\,I_o} == \frac{3}{\pi} = 0.995 \quad \text{(Y conection)}$$

The short circuit ratio (ratio actual s/c current to theoretical s/c current) is

$$K_{s/c} = \frac{q\sqrt{2}V\Big/\omega L_c}{2\sqrt{2}V\Big/\omega L_c\,\sin\frac{\pi}{q}} = \frac{q}{2\sin\frac{\pi}{q}}$$

Commutation overlap angle

$$1 - \cos\mu = \frac{\omega L_c I_o}{\sqrt{2}V\sin\frac{\pi}{q}}$$

The commutation voltage drop

$$v_{com} = \frac{q}{2\pi}\omega L_c I_o \quad \text{where } 2L_c = L_{s/c}$$

$p=q=$	$I_{sec\ rms}$	RF_v	$\overline{V}_o$	$\hat{V}_D$	$\%V_{p\text{-}p}$	$K_{s/c}$	pf_{sec}	pf_{prim}
2	$I_o/\sqrt{2}$		0.90V	$2\sqrt{2}$ V	0.157	1	0.636	0.90
3	$I_o/\sqrt{3}$	0.68	1.17V	$\sqrt{6}$ V	0.604	1.73	0.675	0.827
6	$I_o/\sqrt{6}$	0.31	1.35V	$2\sqrt{2}$ V	0.140	6	0.55	0.995

For three-phase resistive load, with transformer turns ratio 1:N

$$I_o = \frac{\sqrt{2}V}{R}\frac{3\sqrt{3}}{2\pi} \qquad I_{o\,rms} = \frac{V}{R}\left[\frac{1}{3}+\frac{\sqrt{3}}{4\pi}\right]^{1/2}$$

$$FF_{i\,output} = \left[\frac{2\pi^2}{27}+\frac{\pi}{6\sqrt{3}}\right]^{1/2}$$

$$I_{p\Delta} = \frac{N}{1}\times\frac{V}{R}\left[\frac{1}{3}+\frac{\sqrt{3}}{4\pi}-\frac{3}{2\pi^2}\right]^{1/2} \qquad I_{L\Delta} = \frac{N}{1}\times\frac{V}{R}\left[\frac{2}{3}+\frac{\sqrt{3}}{2\pi}\right]^{1/2}$$

$$I_{LY} = \frac{N}{1}\times\frac{V}{R}\left[\frac{2}{9}+\frac{\sqrt{3}}{6\pi}\right]^{1/2}$$

Time domain half-wave single phase R-L-E load

$$i_o(\omega t) = -\frac{E}{R}+\frac{\sqrt{2}V}{Z}\left(\sin(\omega t-\phi)+\left[\frac{E}{R}\frac{Z}{\sqrt{2}V}-\sin(\omega t-\phi)\right]e^{-\frac{\omega t-\alpha}{\tan\phi}}\right)$$

$$v_o(\omega t) = V_o\left[1+\sum_{k=1}^{\infty}\frac{-2(-1)^k}{k^2q^2-1}\cos(kq\omega t)\right]$$

Full-wave diode bridge rectifiers - star [see figures 11.8, 11.13]

q phases and 2q diodes

Mean voltage

$$V_o = \frac{q}{\pi}\int_{\frac{1}{2}\pi-\pi/q}^{\frac{1}{2}\pi+\pi/q}\sqrt{2}V\sin\omega t\, d\omega t$$
$$= \frac{2q}{\pi}\sqrt{2}V\sin\frac{\pi}{q}$$

Pulse number

$p=q$ if q is even
$p=2q$ if q is odd

Diode reverse voltage

$$\hat{V}_{D_R} = 2\sqrt{2}V \quad \text{if } q \text{ is even}$$
$$\hat{V}_{D_R} = 2\sqrt{2}V\cos\frac{\pi}{2q} \quad \text{if } q \text{ is odd}$$

For a constant load current I_o, diode currents are

$$\hat{I}_D = I_o \qquad \overline{I}_D = \frac{I_o}{q} \qquad I_{D\,rms} = \frac{I_o}{\sqrt{q}}$$

The current and power factor are

$$I_{rms} = I_o\sqrt{\frac{2}{q}}$$

$$pf = \frac{P_d}{S} = \frac{V_oI_o}{qVI_{rms}} = \frac{\frac{2q}{\pi}\sqrt{2}V\sin\frac{\pi}{q}\times I_o}{qV\times I_o\sqrt{\frac{2}{q}}} = \frac{2\sqrt{q}}{\pi}\sin\frac{\pi}{q}$$

which is √2 larger than the half-wave case.

For single-phase, full-wave $p=q=2$

$$pf_{1\phi} = \frac{V_oI_o}{V\,I_o} = \frac{2\sqrt{2}}{\pi} = 0.90$$

For three-phase full-wave $p=2$ $q=6$

$$pf_{3\phi} = \frac{V_oI_o}{3VI_o} = \frac{\frac{6}{\pi}\sqrt{2}V\times\frac{1}{2}I_o}{3V\times\frac{\sqrt{2}}{3}I_o} = \frac{3}{\pi} = 0.955$$

p, q	I_{sec}	RF_v	$\overline{V}_o$	$\hat{V}_D$	$\%V_{p\text{-}p}$	$K_{s/c}$	$pf_{Y\,prim}$	pf_{sec}
$p=q=2$	I_o	0.483	1.80V	$2\sqrt{2}$ V	0.157	$2/\pi$	0.90	0.90
$p=2$ $q=6$	$\sqrt{2/3}\,I_o$	0.31	2.34V	$\sqrt{6}$ V	0.140	$6/\pi$	0.995	0.995

The short circuit ratio (ratio actual s/c current to theoretical s/c current) is

$$K_{s/c} = \frac{q}{2\pi\sin\frac{\pi}{q}}$$

which is smaller by a factor π than the half-wave case.

$$K_{s/c} = \frac{q}{\pi} \quad \text{for } q = 2$$

Relationship between current ripple factor and supply side power factor on the primary

$$RF_i = \frac{1}{I_1}\sqrt{\sum_{h=3}^{\infty}I_h^2} = \frac{1}{I_1}\sqrt{I_{rms}^2-I_1^2}$$

$$pf = \frac{I_1}{I_{rms}} = \frac{1}{\sqrt{1+RF_i^2}}$$

For single phase p=2

$$RF_i = \frac{1}{I_1}\sqrt{I_{rms}^2-I_1^2}$$

$$= \frac{\sqrt{I_o^2-\left(\frac{1}{\sqrt{2}}\frac{4}{\pi}I_o\right)^2}}{\frac{1}{\sqrt{2}}\frac{4}{\pi}I_o} = \sqrt{\frac{\pi^2-8}{8}} = 0.483$$

$$pf = \frac{1}{\sqrt{1+RF_i^2}} = \frac{1}{\sqrt{1+\frac{\pi^2-8}{8}}} = \frac{2\sqrt{2}}{\pi} = 0.90$$

The rms of the fundamental component is

$$I_1 = \frac{1}{\sqrt{2}}\frac{4}{\pi}I_o$$

The rms of the harmonic components are

$$I_h = \frac{I_1}{h} = \frac{I_1}{kp\pm1} \quad \text{for } k \geq 1,2,3\ldots$$

For p-pulse

$$RF_v = \sqrt{\frac{\frac{\pi^2}{p^2}}{\sin^2\frac{\pi}{p}}-1}$$

$$pf = \frac{1}{\sqrt{1+RF_v^2}} = \frac{p}{\pi}\sin\frac{\pi}{p}$$

Commutation overlap angle

$$1-\cos\mu = \frac{\omega L_cI_o}{\sqrt{2}V\sin\frac{\pi}{q}}$$

The commutation voltage drop

$$v_{com} = \frac{q}{\pi}\omega L_cI_o \quad \text{where } 2L_c = L_{s/c}$$

For p=q=2, only

$$1-\cos\mu = \frac{2\omega L_cI_o}{\sqrt{2}V}$$

$$v_{com} = \frac{4}{\pi}\omega L_cI_o$$

Load characteristics

$$\text{Current Form Factor} = FF_I = \frac{I_{o\,rms}}{I_o} = \frac{I_o\sqrt{\frac{2}{q}}}{I_o} = \sqrt{\frac{2}{q}}$$

Full-wave diode bridge rectifiers – delta

Same expression as for delta connected secondary, except supply voltages V are replaced by

$$\frac{V}{2\sin\frac{\pi}{q}}$$

For example in three-phase, V is replaced by $V/\sqrt{3}$, that is, $V_{L-L} = \sqrt{3}V_{L-N} = \sqrt{3}V_{phase}$

The mean output voltage is

$$V_o = \frac{2q}{\pi}\sqrt{2}V_\Delta \sin\frac{\pi}{q} = \frac{2q}{\pi}\sqrt{2}\frac{V}{2\sin\frac{\pi}{q}}\sin\frac{\pi}{q} = \frac{q}{\pi}\sqrt{2}V$$

Pulse number

$p=q$ if q is even

$p=2q$ if q is odd

diode reverse voltage and currents

$$\hat{V}_{D_R} = \frac{\sqrt{2}V}{\sin\frac{\pi}{q}} \quad \text{if } q \text{ is even}$$

$$\hat{V}_{D_R} = \frac{\sqrt{2}V}{2\sin\frac{\pi}{2q}} \quad \text{if } q \text{ is odd}$$

$$\hat{I}_D = I_o \qquad \bar{I}_D = I_o/q \qquad I_{D\,rms} = I_o/\sqrt{q}$$

rms current and power factor

$$I_{rms\,even} = \frac{I_o}{2} \qquad pf_{q\,even} = \frac{V_oI_o}{qVI_{rms}} = \frac{\frac{q}{\pi}\sqrt{2}V\,I_o}{qV\,½ I_o} = \frac{2\sqrt{2}}{\pi}$$

$$I_{rms\,odd} = \frac{I_o}{2}\frac{\left[q^2-1\right]^{½}}{q} \qquad pf_{q\,odd} = \frac{V_oI_o}{qVI_{rms}} = \frac{2\sqrt{2}}{\pi}\frac{q}{\left[q^2-1\right]^{½}}$$

Commutation angle and voltage

$$1-\cos\mu = \frac{\omega LI_o}{\sqrt{2}V} \qquad v_{com} = \frac{q}{2\pi}\omega L_cI_o \qquad q \text{ even}$$

$$1-\cos\mu = \frac{\omega LI_o}{\sqrt{2}V}\left(1-\frac{1}{q}\right) \qquad v_{com} = \frac{q}{2\pi}\omega L_cI_o\left(1-\frac{1}{q}\right) \qquad q \text{ odd}$$

The short circuit ratio (ratio actual s/c current to theoretical s/c current) is

$$K_{s/c\,even} = \frac{q}{\pi}\sin\frac{\pi}{q} \qquad K_{s/c\,odd} = \frac{q-1}{\pi}\sin\frac{\pi}{q}$$

For single-phase resistive load, with transformer turns ratio 1:N

$$I_o = \frac{\sqrt{2}V}{R}\frac{4}{\pi} \qquad I_{o\,rms} = \frac{2V}{R}$$

$$FF_{i\,output} = \frac{\pi}{2\sqrt{2}} \qquad RF_v = \sqrt{FF^2-1} = \sqrt{\frac{\pi^2}{8}-1}$$

$$I_p = \frac{N}{1}I_{sec} = \frac{N}{1}\times\frac{2V}{R} \qquad pf = \frac{1}{\sqrt{RF^2+1}} = \frac{2\sqrt{2}}{\pi}$$

Reading list

Dewan, S. B. and Straughen, A., *Power Semiconductor Circuits,* John Wiley and Sons, New York, 1975.

Sen, P.C., *Power Electronics*, McGraw-Hill, 5th reprint, 1992.

Shepherd, W *et al. Power Electronics and motor control*, Cambridge University Press, 2nd Edition 1995.

http://www.ipes.ethz.ch/

http://www.celnav.de/hv/hv9.htm

http://www.physiqueindustrie.com/custom_converter.php

http://www.voltagemultipliers.com/html/multdesign.html

Problems

11.1. Derive equations (11.35) and (11.36) for the circuit in figure 11.6.

11.2. Assuming a constant load current, derive an expression for the mean and rms device current and the device form factor, for the circuits in figure 11.8.

11.3. The single-phase full-wave uncontrolled rectifier is operated from the 415 V line-to-line voltage, 50 Hz supply, with a series load of 10 Ω + 5 mH + 40 V battery. Derive the load voltage expression in terms of a Fourier series. Determine the rms value of the fundamental of the load current.

11.4. A single-phase uncontrolled rectifier has a 24Ω resistive load a 240V ac 50Hz supply. Determine the average, peak and rms current and peak reverse voltage across each rectifier diode for
- i. an isolating transformer with a 1:1 turns ratio
- ii. centre-tapped transformer with turns ratio 1:1:1.

11.5. A single-phase bridge rectifier has an *R-L* of *R* = 20Ω and *L* = 50mH and a 240V ac 50Hz source voltage. Determine:
- i. the average and rms currents of the diodes and load
- ii. rms and average 50Hz source currents
- iii. the power absorbed by the load
- iv. the supply power factor

11.6. A single-phase, full-wave uncontrolled rectifier has a back emf E_b in its load. If the supply is 240Vac 50Hz and the series load is *R* = 20Ω, *L* = 50mH, and E_b = 120V dc, determine:
- i. the power absorbed by the dc source in the load
- ii. the power absorbed by the load resistor
- iii. the power delivered from the ac source
- iv. the ac source power factor
- v. the peak-to-peak load current variation if only the first ac term of the Fourier series for the load current is considered.

11.7. A three-phase uncontrolled rectifier is supplied from a 50Hz 415V ac line-to-line voltage source. If the rectifier load is a 75 Ω resistor, determine
- i. the average load current
- ii. the rms load current
- iii. the rms source current
- iv. the supply power factor.

11.8. A three-phase uncontrolled rectifier is supplied from a 50Hz 415V ac line-to-line voltage source. If the rectifier load is a series *R-L* circuit where *R* = 10Ω and *L* = 100mH, determine:
- i. the average and rms load currents
- ii. the average and rms diode currents
- iii. the rms source and power current
- iv. the supply power factor.

Table 11.5. Characteristics of single-phase rectifier circuits with a resistive load

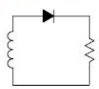
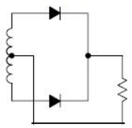
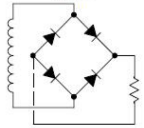
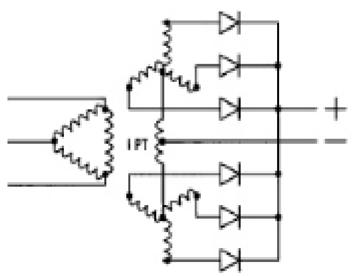
$P = I_o^2 R$ $V_o = I_o R$ $\eta = \dfrac{I_o^2}{I_{o\,rms}^2}$	Half–Wave 	Full–Wave Center–Tap 	Full–Wave Bridge 
Load Voltage and Current Waveshape Characteristic			
Diode Average Current $I_{F(AV)}/I_{L(DC)}$	1.00	0.50	0.50
Diode Peak Current $I_{FM}/I_{F(AV)}$	3.14	3.14	3.14
Form Factor of Diode $I_{F(RMS)}/I_{(DC)}$	1.57	1.57	1.57
Diode RMS Current $I_{F(RMS)}/I_{L(DC)}$	1.57	0.785	0.785
RMS Input Voltage Per Transformer Leg $V_i/V_{L(DC)}$	2.22	1.11	1.11
Peak Inverse Voltage $V_{RRM}/V_{L(DC)}$	3.14	3.14	1.57
Transformer Primary Rating VA/P_{DC}	3.49	1.23	1.23
Transformer Secondary Rating VA/P_{DC}	3.49	1.75	1.23
Total RMS Ripple, %	121	48.2	48.2
Lowest Ripple Frequency, f_r/f_i	1	2	2
Rectification Ratio (Conversion Efficiency), %	40.6	81.2	81.2

Table 11.6. Characteristics of three-phase rectifier circuits with a resistive load

	Half–wave Star	Bridge	Double Wye with Interphase Transformer	Full–wave Star	Wye–Delta Connections Parallel	Wye–Delta Connections Series
Average Current through Diode $I_{F(AV)}/I_{L(DC)}$	0.333	0.333	0.167	0.167	0.167	0.333
Peak Current through Diode $I_{FM}/I_{F(AV)}$	3.63	3.14	3.15	6.30	6.30	6.30
Form Factor of Current through Diode $I_{F(RMS)}/I_{(DC)}$	1.76	1.74	1.76	2.46	2.46	2.46
RMS Current through Diode $I_{F(RMS)}/I_{L(DC)}$	0.587	0.579	0.293	0.409	0.409	0.818
RMS Input Voltage Per Transformer Leg $V_i/V_{L(DC)}$	0.855	0.428	0.855	0.741	0.715	0.37
Diode Peak Inverse Voltage $V_{RRM}/V_{L(DC)}$	2.09	1.05	2.42	2.09	1.05	1.05
Transformer Primary Rating VA/P_{DC}	1.23	1.05	1.06	1.28	1.01	1.01
Transformer Secondary Rating VA/P_{DC}	1.50	1.05	1.49	1.81	1.05	1.05
Total RMS Ripple, %	18.2	4.2	4.2	4.2	1.0	1.0
Lowest Ripple Frequency, f_r/f_i	3	6	6	6	12	12
Rectification Ratio (Conversion Efficiency), %	96.8	99.8	99.8	99.8	100	100

Table 11.7. Characteristics of rectifiers with an *L-C* output filter

Rectifier Circuit Connection / Characteristic	Single–Phase Full–Wave Center–Tap	Single–Phase Full–Wave Bridge	Three Phase Hall–Wave Star	Three–Phase Full–Wave Bridge	Three–Phase Double Wye With Interphase Transformer
‡Average Current Through Diode $I_{F(AV)}/I_{L(DC)}$	0.500	0.500	0.333	0.333	0.167
‡Peak Current Through Diode $I_{FM}/I_{F(AV)}$	2.00	2.00	3.00	3.00	3.00
Form Factor of Current Through Diode $I_{F(RMS)}/I_{F(AV)}$	1.41	1.41	1.73	1.73	1.76
RMS Input Voltage Per Transformer Leg $V_i/V_{L(DC)}$	1.11*	1.11	0.855	0.428	0.885
Diode Peak Inverse Voltage (PIV) V_{RRM} $V_{L(DC)}$	3.14	1.57	2.09	1.05	2.42
Transformer Primary Rating V_A/P_{DC}	1.11	1.11	1.21	1.05	1.05
Transformer Secondary Rating V_A/P_{DC}	1.57	1.11	1.48	1.05	1.48
Ripple ($V_F/V_{L(DC)}$ Lowest frequency in rectifier output (f_r/f_1) Peak Value of Ripple	2	2	3	6	6
Components: Ripple frequency (fundamental) Second harmonic Third harmonic	 0.667 0.133 0.057	 0.667 0.133 0.057	 0.250 0.057 0.025	 0.057 0.014 0.006	 0.057† 0.014 0.006
Ripple peaks with reference to dc axis: Positive peak Negative peak	0.363 0.837	0.363 0.637	0.209 0.395	0.0472 0.0930	0.0472 0.0930



Three-phase double wye with a centre tapped inter-phase transformer.