EIEN15 Electric Power Systems – Formulas

Allowed at the exam: This formula sheet, formula book like TEFYMA and calculator.

Δ-Y transformation

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_C} \qquad Z_A = \frac{Z_{AB} Z_{AC}}{Z_{AB} + Z_{BC} + Z_{AC}}$$

$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_A} \qquad Z_B = \frac{Z_{AB} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}}$$

$$Z_{AC} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_B} \qquad Z_C = \frac{Z_{AC} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}}$$

Three-phase per-unit base definitions according to Glover-Sarma-Overbye textbook:

$$S_{base3\phi} = 3S_{base1\phi} \qquad I_{base} = \frac{S_{base1\phi}}{V_{baseLN}} = \frac{S_{base3\phi}}{\sqrt{3}V_{baseLL}}$$

$$V_{baseLN} = V_{baseLL} / \sqrt{3} \qquad Z_{base} = \frac{V_{baseLN}}{I_{base}} = \frac{V_{baseLN}^2}{S_{base1\phi}} = \frac{V_{baseLL}^2}{S_{base3\phi}}$$

$$Y_{base} = 1 / Z_{base}$$

---

rms short-circuit current unloaded synchronous machine

Symmetrical (ac)

$$I_{ac}(t) = E_g \left[\left(\frac{1}{X''_d} - \frac{1}{X'_d} \right) e^{-t/T''_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d} \right) e^{-t/T'_d} + \frac{1}{X_d} \right]$$

Asymmetrical (ac+dc)

$$I_{rms}(t) = \sqrt{I_{ac}(t)^2 + i_{dc}(t)^2}$$

Asymmetrical (ac+maximum dc)

$$I_{rms}(t) = \sqrt{I_{ac}(t)^2 + \left[\sqrt{2} I'' e^{-t/T_A} \right]^2} \quad \text{where } I'' = \frac{E_g}{X''_d}$$

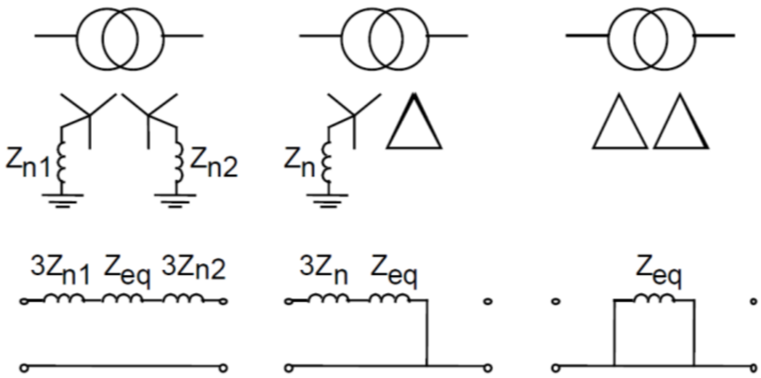
Salient pole rotor synchronous generator

$$P_e(\delta) = \frac{E_q V}{X_d} \sin\delta + \frac{V^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta$$

$$Q_e(\delta) = \frac{E_q V}{X_d} \cos\delta - V^2 \left(\frac{\cos^2\delta}{X_q} + \frac{\sin^2\delta}{X_d} \right)$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = A \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \text{ with } a = e^{j\frac{2\pi}{3}};\ A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Zero sequence networks of transformers, neglecting phase shift and shunt admittance:



Zero, positive and negative sequence fault currents (I_{n-0}, I_{n-1}, I_{n-2}) for a fault at bus n

Three-phase fault
$$I_{n-1} = \frac{V_F}{Z_{nn-1}};\ I_{n-0} = I_{n-2} = 0$$

Single line-to-ground, phase a
$$I_{n-0} = I_{n-1} = I_{n-2} = \frac{V_F}{Z_{nn-0} + Z_{nn-1} + Z_{nn-2} + 3Z_F}$$

Line-to-line fault, phases b and c
$$I_{n-1} = -I_{n-2} = \frac{V_F}{Z_{nn-1} + Z_{nn-2} + Z_F};\ I_{n-0} = 0$$

Double line-to-ground fault, phases b and c
$$I_{n-1} = \frac{V_F}{Z_{nn-1} + \frac{Z_{nn-2}(Z_{nn-0} + 3Z_F)}{Z_{nn-2} + Z_{nn-0} + 3Z_F}}$$
$$I_{n-2} = -I_{n-1}\frac{Z_{nn-0} + 3Z_F}{Z_{nn-2} + Z_{nn-0} + 3Z_F}$$
$$I_{n-0} = -I_{n-1}\frac{Z_{nn-2}}{Z_{nn-2} + Z_{nn-0} + 3Z_F}$$

Z_{nn-0}, Z_{nn-1} and Z_{nn-2} denote element (n,n) of zero, positive and negative sequence bus impedance matrix Z_{bus}.

If prefault currents are neglected, bus voltages during fault at bus *n*,

$$V_{bus} = V_F - Z_{bus} I_F$$

where I_F only has one nonzero element being I_n – the fault current leaving bus *n*.

Newton-Raphson solution of $y=f(x)$, iteration i:

$$x(i+1) = x(i) + J^{-1}(i)\{y - f[x(i)]\}$$

Matrix element $J_{mn}(i) = \left.\frac{\partial f_m}{\partial x_n}\right|_{x=x(i)}$

Complex power injected at bus k calculated using bus admittance matrix and bus voltage vector:

$$S_k = P_k + jQ_k = V_k I_k^* \Rightarrow \begin{cases} P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}) \\ Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}) \end{cases}$$

where $Y_{kn} e^{j\theta_{kn}} = Y_{bus}(k,n)$ and $V_k e^{j\delta_k} = V_{bus}(k)$